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## Freedom of additional signals on genes: on the combination of DNA mechanics, genetics and translation speed

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## Appendix A

## The physics behind the mechanical nucleosome positioning code

## A. 1 Energy contributions of twist and cross terms

In section 2.2 we define the energy of a dinucleotide as the sum of the energy of roll and tilt. Keeping in mind the observation that the basic nucleosome positioning rules can be rationalized by discussing energy costs involved in the roll and tilt degrees of freedom [14], as well as our goal to reduce our model to its bare essentials, we chose to neglect the contribution of twist and of the cross terms between roll, tilt, and twist. In this appendix we will show that including the twist and cross terms does not change the qualitative agreement of our model with well-known positioning rules (see Fig. 2.3).

We start by defining the energy of twist and the cross-terms:

$$
\begin{equation*}
E^{\mathrm{twist}}(a, b) \equiv \frac{1}{2} Q^{\mathrm{twist}}(a, b)\left[q^{\mathrm{twist}}-\bar{q}^{\mathrm{twist}}(a, b)\right]^{2} \tag{A.1}
\end{equation*}
$$

and

$$
\begin{equation*}
E_{p}^{\text {cross }}(a, b) \equiv \sum_{\substack{i, j \in\{\text { roll,tiltt,twist }\} \\ i \neq j}} \frac{1}{2} Q^{i, j}(a, b)\left[q_{p}^{i}-\bar{q}^{i}(a, b)_{p}\right]\left[q_{p}^{j}-\bar{q}^{j}(a, b)_{p}\right] . \tag{A.2}
\end{equation*}
$$

The bp-step dependent stiffnesses are now given by $Q^{i}(a, b), i \in\{$ roll, tilt, twist $\}$ and the corresponding intrinsic values by $\bar{q}^{i}(a, b), i \in\{$ roll, tilt, twist $\}$. The cross terms depend on the cross stiffnesses $Q^{i, j}(a, b), i, j \in\{$ roll, tilt, twist $\}, i \neq j$. (Note that, because of the constant twist, the energy associated with twist does not depend on position $p$ but only on the dinucleotide step.) For the twist and cross terms, too, the hybrid parametrization [38] is used. We can redefine our energy as

$$
\begin{equation*}
E_{p}(a, b)=E_{p}^{\mathrm{roll}}(a, b)+E_{p}^{\mathrm{tilt}}(a, b)+E^{\mathrm{twist}}(a, b)+E_{p}^{\mathrm{cross}}(a, b) \tag{A.3}
\end{equation*}
$$

Fig. A. 1 was created using this redefined energy. We see that the relative behaviour of the dinucleotide probabilities at different positions is the same as without the cross terms, see Fig. 2.3.


Figure A.1: Same as Fig. 2.3 but with including the cross-terms (see Eq. A.3). The positioning rules (i.e. the relative behaviour of the dinucleotide probabilities at different position) has stayed the same.

## A. 2 Validity of the average neighbour energy approximation

The average neighbour energy approximation of the probability works extremely well. We checked it for all dinucleotides and found that the largest error of this approximation occurs for the probability distribution of dinucleotide AA. Fig. A. 2 depicts both the exact probability and its approximation for this dinucleotide. The difference between the values is always smaller than $3.5 \%$.

To understand why this error is so small, one needs to consider the function $C_{p}(x, y)$, defined in Eq. 2.28. The average neighbour energy approximation is exact if this function is a constant (i.e., independent of $x$ and $y$ for each $p$ ). The approximation works well if the function is almost constant. That this is true is best seen by inspecting the standard deviation of $C_{p}(x, y)$, divided by its mean, and checking whether this quantity is much smaller than one. Here the standard deviation and mean are defined as:

$$
\begin{equation*}
\operatorname{std}\left[C_{p}\right] \equiv \sqrt{\left\langle\left\{C_{p}(x, y)-\operatorname{mean}\left[C_{p}\right]\right\}^{2}\right\rangle_{x, y}} \tag{A.4}
\end{equation*}
$$

with

$$
\begin{equation*}
\operatorname{mean}\left[C_{p}\right] \equiv\left\langle C_{p}(x, y)\right\rangle_{x, y} \tag{A.5}
\end{equation*}
$$

Fig. A. 3 shows that this ratio is indeed much smaller than one for all dinucleotide positions.

## A. 3 Effect of temperature on the probability

The probabilities shown in the results section have all been obtained at room temperature $\beta=1 / k_{B} T_{r}$. Here we study how these probabilities change with temperature,


Figure A.2: The exact probability and its average neighbour energy approximation to find AA steps at all dinucleotide positions. The approximation introduces an error that is nowhere larger than $3.5 \%$.


Figure A.3: The standard deviation (Eq. A.4) divided by the mean (Eq. A.5) of $C_{p}(x, y)$. As this ratio is very small at all positions $p$ the function $C_{p}(x, y)$ is nearly constant, explaining the high accuracy of the average neighbour energy approximation.


Figure A.4: The probability to obtain AA at all dinucleotide positions at several temperatures.

— $\mathrm{AA}, \beta=0.1\left[1 / k_{B} T_{r}\right]$
----- $\mathrm{AA}, \beta=1\left[1 / k_{B} T_{r}\right]$
$\ldots \ldots \ldots \ldots . \quad \mathrm{AA}, \beta=10\left[1 / k_{B} T_{r}\right]$

Figure A.5: The first-order bounds of the probability of encountering an AA step are shown at five different temperatures. At low temperatures, the bounds become significantly far apart from each other and only provide a qualitative description of the behaviour of the probability as a function of position.
focusing on dinucleotide AA. Its exact probability distribution for different temperatures is shown in Fig. A.4. We find at temperature $\beta=0$ a constant value $1 / 16$ for the probability. This is the high temperature limit where all steps are equally probable. At low temperatures the probability varies between values close to 0 and 1 , reflecting the fact that the ground state sequences becomes exceedingly important.

We also evaluated the first- and second-order bounds of the AA probability distribution at five different temperatures: $\beta=0,0.1,1,10$, and 100 (in units of $\left[1 / k_{B} T_{r}\right]$ ), see Fig. A.5, and Fig. A.6. At high temperatures (low $\beta$ ) the bounds for both orders are very close to each other enclosing values close to $1 / 16$. With decreasing temperature the quality of the first-order bounds becomes poorer, giving only a rough qualitative estimate whereas the second-order bounds continue to work well for relatively low temperatures. Note that at $\beta=100$ the probability takes values close to 0 and 1 at most places.


Figure A.6: The second-order bounds of the probability of encountering an AA step are shown at five different temperatures. At all temperatures the method provides a quantitative description of the probability, clearly outperforming the second-order bounds.


Figure A.7: (a) First-order and (b) second-order bounds on the probability to find dinucleotide AA at several dinucleotide positions on a nucleosome, in the limit of zero temperature. Higher-order bounds get increasingly sharper. At zero temperature the only possible DNA sequences are ground state sequences, hence the bounds provide us statistics on the ground states of our system. When the lower and upper bounds are 0 and 1 , AA is part of an unknown number of ground states at this position. If AA is part of all possible ground states the bounds are 1 and 1 , and if it is not a part of the ground state they are 0 and 0 .

We finally take the limit $\beta \rightarrow \infty$, see Fig. A.7. This figure shows the second-order and third-order bounds on the probability to encounter AA at zero temperature. The only possible sequences are now ground state sequences (due to the high level of symmetry in our model we expect many different ground states). This explains why the probability of AA can take the values 0 and 1 : at several positions AA is not part of any ground state sequence (probability is zero), while at other positions AA is part of all possible ground state sequences (probability is 1). At some positions the method cannot determine the percentage of ground state sequences AA is part of, resulting in bounds of 0 and 1 .

The method of obtaining upper and lower bounds remains effective at all possible temperatures for our model, and even provides insight into the possible ground states. Going to higher-order bounds (i.e., taking neighbours that are further away into account as well) or using the exact probability should eliminate the discrepancy between the upper and lower value. However, the method employed in Chapter 3 (which uses a graph representation of all possible sequences in combination with a shortest path algorithm) is much more efficient in obtaining ground states.

## Appendix B

## Shortest paths through synonymous codons

## B. 1 Definition of the energy

In Chapter 3, we aim to find sequences with 'special' energies, e.g. the sequences with the lowest and highest possible energies. To calculate the energy of a sequence, we use the probabilistic trinucleotide model by Tompitak et al. [10] which is based on the sequence preferences of a coarse grained nucleosome model, parametrized by experimental parameters derived from protein-DNA crystals [32]. Because it is a trinucleotide model, we are able to represent the total energy of a sequence as a sum of 'conditional' trinucleotide energies, which function as the (main ingredients of the) weights in our graphs. Here we will formally define these energies.

Let $\mathcal{B}$ be the set of all nucleotides, $\mathcal{B}=\{A, T, C, G\}$. For the trinucleotide model, it is assumed that the probability of a nucleotide depends only on the previous two. Defining $S$ as a sequence of length $L$, consisting of nucleotides $S_{i} \in \mathcal{B}$ with $i$ from 1 to 147 , this gives a probability for the full sequence:

$$
\begin{equation*}
P(S)=\frac{\prod_{n=1}^{L-2} P_{n}\left(S_{n+2} \cap S_{n+1} \cap S_{n}\right)}{\prod_{n=1}^{L-3} P_{n}\left(S_{n+2} \cap S_{n+1}\right)} \tag{B.1}
\end{equation*}
$$

where $P_{n}\left(S_{n+2} \cap S_{n+1} \cap S_{n}\right)$ is the joint (trinucleotide) probability to obtain $S_{n+2}$, $S_{n+1}$, and $S_{n}$ at position $n$, and $P\left(S_{n+2} \cap S_{n+1}\right)$ the joint (dinucleotide) probability to obtain $S_{n+2}, S_{n+1}$ at position $n$. However, since the original trinucleotide model by Tompitak et al. does not enforce the symmetry of the coding and noncoding strand, we introduce symmetrized probabilities:

$$
\begin{align*}
P_{n}^{\prime}\left(S_{n} \cap S_{n-1} \cap S_{n-2}\right) & =\frac{1}{2}\left[P_{n}\left(S_{n} \cap S_{n-1} \cap S_{n-2}\right)\right] \\
& +\frac{1}{2}\left[P_{n}\left(S_{n-2}^{\prime} \cap S_{n-1}^{\prime} \cap S_{n}^{\prime}\right)\right] \tag{B.2}
\end{align*}
$$

and

$$
\begin{equation*}
P_{n}^{\prime}\left(S_{n} \cap S_{n-1}\right)=\frac{1}{2}\left[P_{n}\left(S_{n} \cap S_{n-1}\right)+P_{n}\left(S_{n-1}^{\prime} \cap S_{n}^{\prime}\right)\right] \tag{B.3}
\end{equation*}
$$

where

$$
S_{n}^{\prime} \equiv\left\{\begin{array}{l}
A_{148-n} \text { if } S_{n}=T  \tag{B.4}\\
T_{148-n} \text { if } S_{n}=A \\
C_{148-n} \text { if } S_{n}=G \\
G_{148-n} \text { if } S_{n}=C
\end{array}\right.
$$

such that

$$
\begin{equation*}
P_{n}^{\prime}(S)=\frac{\prod_{n=1}^{L-2} P_{n}^{\prime}\left(S_{n+2} \cap S_{n+1} \cap S_{n}\right)}{\prod_{n=1}^{L-3} P_{n}^{\prime}\left(S_{n+2} \cap S_{n+1}\right)} \tag{B.5}
\end{equation*}
$$

Following Tompitak et al., we use the probability to calculate a free energy, using $E(S)=-k_{B} T_{r} \ln [P(S)]+$ const. We rewrite the energy as:

$$
\begin{equation*}
E(S)=\sum_{n=1}^{L-2} E_{n}\left(S_{n+2}, S_{n+1}, S_{n}\right)+\text { const } \tag{B.6}
\end{equation*}
$$

where

$$
\begin{align*}
& E_{n}\left(S_{n}, S_{n+1}, S_{n+2}\right)= \\
& \qquad\left\{\begin{array}{l}
-k_{B} T_{r} \ln \left[P^{\prime}\left(S_{n+2} \cap S_{n+1} \cap S_{n}\right)\right] \text { if } n=1 \\
-k_{B} T_{r} \ln \left[\frac{P^{\prime}\left(S_{n+2} \cap S_{n+1} \cap S_{n}\right)}{P^{\prime}\left(S_{n+1} \cap S_{n}\right)}\right] \text { if } 1<n<146 \\
0 \text { else. }
\end{array}\right. \tag{B.7}
\end{align*}
$$

We define const. such that the energy $E$ is zero if $S$ is the ground state.
For $n=1, E_{n}$ is the energy cost related to the first three bases of a sequence $S$, for $1<n<146$, it is a 'conditional' energy, and it is zero elsewhere. We use these terms as weights of our graph, while keeping in mind that the sum of these weights will provide the well-defined total energy $E$.

## B. 2 Definition of the depth of a minimum

In the main text of Chapter 3, we use the depth of a minimum $\mathcal{D}$ as a measure for how well the nucleosome is positioned at this minimum. Here we will formally define $\mathcal{D}$.

Let $\mathcal{S}$ be some sequence of length greater than $L+10$ (with $L=147$ ). Let $S^{p}$ be a subsequence of $\mathcal{S}$ of length $L$ starting at position $p$.

We call a nucleosome positioned at $p$ if the energy $E\left(S^{p}\right)$ is lower than the energies at positions $p-5, p-4, \ldots, p+5$ (excluding $p$ ). We denote the energy corresponding to a nucleosome containing the sequence $S^{p}$ by $\mathscr{E}_{p} \equiv E\left(S^{p}\right)$. For a minimum at $p_{\min }$ of sequence $S$ we are interested in its depth, $\mathcal{D}\left(S^{p_{\text {min }}}\right)$. Now we can formally define the depth as

$$
\begin{equation*}
\mathcal{D}\left(S^{p_{\text {min }}}\right) \equiv \min \left[\mathscr{E}_{\text {left }}^{\max }\left(S^{p_{\text {min }}}\right), \mathscr{E}_{\text {right }}^{\max }\left(S^{p_{\text {min }}}\right)\right] \tag{B.8}
\end{equation*}
$$

where

$$
\begin{gather*}
\mathscr{E}_{\text {left }}^{\max }\left(S^{p_{\min }}\right) \equiv \max \left[\mathscr{E}_{p_{\min }-i}(S) \text { for } i \in\{1,2, \ldots, 5\}\right]-\mathscr{E}_{p_{\min }}(S),  \tag{B.9}\\
\mathscr{E}_{\text {right }}^{\max }\left(S^{p_{\min }}\right) \equiv \max \left[\mathscr{E}_{p_{\min }+i}(S) \text { for } i \in\{1,2, . ., 5\}\right]-\mathscr{E}_{p_{\min }}(S) . \tag{B.10}
\end{gather*}
$$

## B. 3 The deepest possible minimum

Here we show how to obtain the deepest possible minimum, with only a tiny possible error, by taking the shortest paths through the graphs $\mathcal{G}_{h, j}^{+}$defined in the main text.

A nucleosome is best positioned at a minimum $p_{\min }$ if $\mathcal{D}\left(S^{p_{\text {min }}}\right)$ is maximal. We assume that the deepest possible minimum $\mathcal{D}\left(S_{\text {deepest }}\right)$ is found for a sequence $S_{\text {deepest }}$. Furthermore, we assume that

$$
\begin{equation*}
\mathscr{E}_{\text {left }}^{\max }\left(S_{\text {deepest }}\right)=\mathscr{E}_{p_{\text {min }}+h}\left(S_{\text {deepest }}\right)-\mathscr{E}_{p_{\text {min }}}\left(S_{\text {deepest }}\right) \tag{B.11}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathscr{E}_{\text {right }}^{\max }\left(S_{\text {deepest }}\right)=\mathscr{E}_{p_{\text {min }}+j}\left(S_{\text {deepest }}\right)-\mathscr{E}_{p_{\text {min }}}\left(S_{\text {deepest }}\right) \tag{B.12}
\end{equation*}
$$

for $h \in\{-5,-4, . .,-1\}, j \in\{1,2 \ldots, 5\}$.
Let us denote the shortest path through $\mathcal{G}_{h, j}^{+}$by $S_{h, j}$ with the minimum at $p_{\text {min }}$. A shortest path through $\mathcal{G}_{h, j}^{+}$will minimize the quantity $2 \mathscr{E}_{p_{\min }}-\mathscr{E}_{p_{\min }+h}-\mathscr{E}_{p_{\min }+j}$. Because of this, we have

$$
\begin{equation*}
\mathscr{E}_{\text {left }}^{\max }\left(S_{h, j}^{p_{\min }}\right)+\mathscr{E}_{\text {right }}^{\max }\left(S_{h, j}^{p_{\min }}\right) \geq \mathscr{E}_{\text {left }}^{\max }\left(S_{\text {deepest }}^{p_{\min }}\right)+\mathscr{E}_{\text {right }}^{\max }\left(S_{\text {deepest }}^{p_{\min }}\right) \tag{B.13}
\end{equation*}
$$

Since $S_{\text {deepest }}^{p_{\text {min }}}$ is the sequence with the greatest depth, we have

$$
\begin{align*}
& \min \left[\mathscr{E}_{\text {left }}^{\max }\left(S_{h, j}^{p_{\text {min }}}\right), \mathscr{E}_{\text {right }}^{\max }\left(S_{h, j}^{p_{\text {min }}}\right)\right] \leq  \tag{B.14}\\
& \quad \min \left[\mathscr{E}_{\text {left }}^{\max }\left(S_{\text {deepest }}^{p_{\text {min }}}\right), \mathscr{E}_{\text {right }}^{\mathscr{m a x}_{\text {max }}}\left(S_{\text {deepest }}^{p_{\min }}\right)\right] .
\end{align*}
$$

Combining Eq. B. 13 and B. 14 leads to bounds on the depth of the deepest possible minimum:

$$
\begin{align*}
& \min \left[\mathscr{E}_{\text {left }}^{\mathscr{o m a x}_{\max }}\left(S_{h, j}^{p_{\text {min }}}\right), \mathscr{E}_{\text {right }}^{\max }\left(S_{h, j}^{p_{\min }}\right)\right] \leq \\
& \quad \mathscr{E}_{\text {Ieft }}^{\max }\left(S_{\text {deepest }}^{p_{\text {mi }}}\right)  \tag{B.15}\\
& \quad \leq \frac{1}{2}\left[\mathscr{E}_{\text {left }}^{\max }\left(S_{h, j}^{p_{\text {min }}}\right)+\mathscr{E}_{\text {right }}^{\max }\left(S_{h, j}^{p_{\text {min }}}\right)\right] . \tag{B.16}
\end{align*}
$$

We took the shortest path through all graphs $\mathcal{G}_{h, j}^{+}$for all $h \in\{-5,-4, . .,-1\}$, $j \in\{1,2 \ldots, 5\}$. Of all the graphs, $\mathcal{G}_{-5,5}^{+}$provided the deepest minimum. Using the above equation, we obtained $83.47 \pm 0.03 k_{B} T_{r}$ as the deepest possible minimum.

## B. 4 Graphs

We have defined the graphs $\mathcal{G}_{h, j}^{+}$, extensions of $\mathcal{G}$ with differently assigned weights, for $h, j \in\{-5,-4, . .,-1,1,2 \ldots, 5\}$. A visual depiction is shown by Fig. B.1. The graph $\mathcal{G}_{\text {gene }}^{+}$, an extended version of $\mathcal{G}_{\text {gene }}$, is depicted by Fig. B.2.


Figure B.1: Visualisation of a graph $\mathcal{G}_{h, j}^{+}$


Figure B.2: Visualisation of a graph $\mathcal{G}_{\text {gene }}$. This graph corresponds to creating a minimum at the $7^{\text {th }}$ nucleosome position on the gene YAL002W of yeast.

## B. 5 Create local minima on top of genes

To create local minima at a position on a gene, we came up with a specifically tailored method where we alter the values of the constants $c_{i}$ with each iteration.

| iteration | starting conditions | action |
| :--- | :---: | :---: |
| all iterations | $c_{0}=1$ | minimum and depth check: |
|  | $c_{i}=0$ for $i \notin\{-5,0,5\}$ | $\mathcal{D} \geq 10 k_{B} T_{r}$ |
| $1-20$ | $c_{-5}=c_{5}=-0.3$ | regular decrement |
| $21-40$ | $c_{-5}=c_{5}=-0.3$ | neighbor decrement |
| $41-60$ | $c_{-5}=c_{5}=-0.2$ | regular decrement |
| $61-80$ | $c_{-5}=c_{5}=-0.2$ | neighbor decrement |
| $81-100$ | $c_{-5}=c_{5}=-0.1$ | regular decrement |
| $101-120$ | $c_{-5}=c_{5}=-0.1$ | neighbor decrement |
| $121-140$ | $c_{-5}=c_{5}=0$ | regular decrement |
| $141-160$ | $c_{-5}=c_{5}=0$ | regular decrement |
| if all fail | - | take best solution |

Table B.2: Schematic form of specifically tailored method to create deep local minima at a position on a gene. The method works by altering the weights $w_{i}^{\prime}$ of graph $\mathcal{G}^{\text {gene }}$ by changing the constants $c_{i}$, see Eq. 4 of the main text.

This will result in a differently weighted graph each iteration and different shortest paths. The algorithm uses at most 160 iterations per position. The iterations are grouped in eight parts, with differing starting conditions and different increment rules. See Table B. 2 for an overview of this method.

All iterations start with $c_{0}=1, c_{i}=0$ for $i \notin\{-5,0,5\}$. Iterations 1-20 start with $c_{-5}=c_{5}=-0.3$. At the start of iteration 21-40, all constants are reset and we again begin with $c_{-5}=c_{5}=-0.3$. Iterations 41-60 and 61-80 have $c_{-5}=c_{5}=-0.2,81-100$ and 100-120 have $c_{-5}=c_{5}=-0.1$, and 121-140 and 141160 have $c_{-5}=c_{5}=0$. The different starting conditions are intended to first try to create deep minima through a larger incentive to have high walls, but if this fails, settle for lower minima.

At the beginning of each and every iteration a check is performed. The energy landscape corresponding to the shortest path is evaluated to find whether a local minimum has been created at the right position. If there is such a local minimum, we evaluate how deep it is. If it is deeper than $10 k_{B} T_{r}$, we accept the corresponding sequence. If the local minimum is not deep enough, we evaluate which side of the energy well has the lowest wall. If the left or right wall is lowest, we set $c_{-5} \rightarrow$ $c_{-5}-0.1$ or $c_{5} \rightarrow c_{5}-0.1$, respectively, and move to the next iteration. If there is no local minimum, we perform one of the two distinct schemes: 'regular decrement' and 'neighbor decrement', introduced below. We perform a 'regular decrement' at iterations 1-20, 41-60, etc., and a 'neighbor decrement' at all other iterations.

The regular decrement is defined as follows: if the position with the lowest energy is $p_{\min }+i$ instead of the intended position $p_{\min }$, we perform $c_{i} \rightarrow c_{i}-0.1$. Differently stated, we give our algorithm an incentive to raise the energy at positions where the energy is lower than at $p_{\text {min }}$. The main problem of the regular decrement is that the lowest energy position often alternates between $p_{\min }+1$ and $p_{\min }-1$. Making the decrements smaller turned out to be ineffective in solving this problem, so instead we define the 'neighbour decrement'.

The neighbour decrement is the same as the regular decrement, with one dif-


Figure B.3: For each possible iteration, the percentage of positions solved (i.e. with a deep enough minimum found) is depicted. All positions on genes from yeast $S$. cerevisiae (ignoring genes with introns) where evaluated. The bulk of the positions were completed at the first iteration.
ference: if the position with the lowest energy is $p_{\min } \pm 1$ instead of the intended position $p_{\min }$, we perform $c_{i \pm 2} \rightarrow c_{i \pm 2}-0.1$.

It is possible that, after 160 iterations, no deep enough minimum is found. Then we take the deepest minimum we encountered (if any exists) as our result. The percentage of positions resolved at which iteration is depicted by Fig. B.3. It shows that the bulk of the positions were completed at the first iteration.

## Appendix C

## Multiplexing mechanical and translational cues on genes

## C. 1 Graph to obtain highest and lowest possible nucleosome energy

In Chapter 4 we use a graph representation of all possible sequences that code for the same protein. To understand the new method we use in this chapter, we first shortly summarize the method we used in Chapter $3^{1}$, where we were able to obtain the highest and lowest possible nucleosome energies on all positions of a gene. To obtain these energies we use a graph containing all synonymous codons of the gene section corresponding to one nucleosome position.

The DNA on a nucleosome consists of 147 base pairs, which corresponds to either 49 or 50 codons. Suppose we have a sequence of 50 codons. These codons encode a sequence of amino acids $p_{0}, p_{1}, p_{2}, \ldots, p_{49}$. The number of different codons coding for the same amino acid is 6 at most. Therefore, the most general representation of all possible ways to code for the same protein at one nucleosome position is given by figure C. 1 (we use the most general representation to make it easier to understand the graphs related to three layers of information).

In this figure, under each amino acid $p_{n}$, six numbers are shown representing the (at most) six possible codons, which we will refer to in the following as $p_{n}(1), p_{n}(2), \ldots, p_{n}(6)$. The actual base pairs of the codons depend on the amino acid in question. To obtain this graph we draw the following weighted edges: from start to $p_{0}(i)$ with weight zero for any $i$, from $p_{49}(i)$ to end with weight $w_{\text {end }}\left(p_{49}(i)\right)$ for any $i$, and from $p_{n}(i)$ to $p_{n+1}(j)$ with weight $w_{n}\left(p_{n}(i), p_{n+1}(j)\right)$ for any $i, j$ and $n=0,1, \ldots, 48$. The weight $w_{i}$ is given by

$$
\begin{equation*}
w_{i}(C, D)=E_{3 i-2}\left(C_{1}, C_{2}, C_{3}\right)+E_{3 i-1}\left(C_{2}, C_{3}, D_{1}\right)+E_{3 i}\left(C_{3}, D_{1}, D_{2}\right) \tag{C.1}
\end{equation*}
$$

[^0]

Figure C.1: Graph $\mathcal{G}_{\mathrm{E}}$ shows all synonymous ways to encode a given amino acid sequence $p_{0}, p_{1}, \ldots, p_{49}$ in the most general case. For each amino acid six options are shown, representing the at most six possible ways to code for the same amino acid. The actual bases depend on the amino acid in question. When there are less than six options, one can simply leave out the surplus of nodes. Weights are assigned such that each path from start to end has a length equal to the total energy of the corresponding codon sequence.
and the weight $w_{\text {end }}$ by

$$
\begin{equation*}
w_{\text {end }}(D)=E_{145}\left(D_{1}, D_{2}, D_{3}\right) \tag{C.2}
\end{equation*}
$$

where $C_{k}$ and $D_{k}$ denote the kth base of codons C and D . Now the length of a path from start to end in the graph equals the energy of a corresponding sequence. The lowest and highest energy can be found using a shortest path algorithm.

## C. 2 Obtaining the highest and lowest possible nucleosome energy, incorporating translation speed

Here we describe the method used to obtain the highest and lowest possible nucleosome energy, incorporating a restriction on the translation speed. The method uses a graph $\mathcal{G}_{\text {T\&E }}$, which is similar to graph $\mathcal{G}_{\mathrm{E}}$. Since we study in chapter 4 five-codon averages of the translation speed, $\mathcal{G}_{\text {T\&E }}$ incorporates translation speed by using nodes consisting of five codons, see figure C.2. These nodes are connected such that any node

$$
p_{n}\left(x_{n}\right) p_{n+1}\left(x_{n+1}\right) p_{n+2}\left(x_{n+2}\right) p_{n+3}\left(x_{n+3}\right) p_{n+4}\left(x_{n+4}\right)
$$

can only be connected to nodes

$$
p_{n+1}\left(x_{n+1}\right) p_{n+2}\left(x_{n+2}\right) p_{n+3}\left(x_{n+3}\right) p_{n+4}\left(x_{n+4}\right) p_{n+5}\left(x_{n+5}\right)
$$

for any $x_{i} \in\{1,2, \ldots, 6\}$, with weight $w_{n+4}\left(p_{n+4}\left(x_{n+4}\right), p_{n+5}\left(x_{n+5}\right)\right)$. All other edges have zero weight. Now, to ensure that one does not alter the translation speed landscape too much when changing the nucleosome energy, one can, for each node, calculate the difference between the translation speed of that node and the original speed. When the difference exceeds a certain threshold, the node needs to be pruned, such that each path through the graph corresponds to a sequence that does not change the underlying amino acid sequence and the translation speed landscape remains the same up to the threshold. Again, the lowest and highest energy can be found using a shortest path algorithm.


Figure C.2: Graph $\mathcal{G}_{\text {T\&E }}$ is similar to graph $\mathcal{G}_{\mathrm{E}}$ from figure C.1. It incorporates translation speed by using nodes consisting of five codons. These nodes are connected such that any node $p_{n}\left(x_{n}\right) p_{n+1}\left(x_{n+1}\right) p_{n+2}\left(x_{n+2}\right) p_{n+3}\left(x_{n+3}\right) p_{n+4}\left(x_{n+4}\right)$ can only be connected to nodes $p_{n+1}\left(x_{n+1}\right) p_{n+2}\left(x_{n+2}\right) p_{n+3}\left(x_{n+3}\right) p_{n+4}\left(x_{n+4}\right) p_{n+5}\left(x_{n+5}\right)$ for any $x_{i} \in\{1,2, \ldots, 6\}$. When the translation speed of five codons (a node) is too different from the original speed, it is pruned. The weights of the graph are again chosen such that any path length corresponds to the nucleosome energy of the corresponding sequence.

## C. 3 Recovering the original nucleosome energy and translation speed landscapes in host organisms

To create the closest possible translation speed landscape in a different organism, we modify graph $\mathcal{G}_{\mathrm{T} \& \mathrm{E}}$ to become gene-wide and obtain graph $\mathcal{G}_{\text {gene }}$ see figure C.3.

We also change the weights. We denote the weights corresponding to the closest possible translation speed landscape by $w^{\mathrm{T}}$. Again these nodes are connected such that any node $p_{n}\left(x_{n}\right) p_{n+1}\left(x_{n+1}\right) p_{n+2}\left(x_{n+2}\right) p_{n+3}\left(x_{n+3}\right) p_{n+4}\left(x_{n+4}\right)$ can only be connected to nodes $p_{n+1}\left(x_{n+1}\right) p_{n+2}\left(x_{n+2}\right) p_{n+3}\left(x_{n+3}\right) p_{n+4}\left(x_{n+4}\right) p_{n+5}\left(x_{n+5}\right)$ for any $x_{i} \in\{1,2, \ldots, 6\}$, but now with weight $w^{\mathrm{T}}$ given by

$$
\begin{equation*}
w^{\mathrm{T}}=\left|\sum_{i=1}^{i=5} T_{\text {original }}\left(p_{n+i}\left(s_{n+i}\right)\right)-T_{\text {host }}\left(p_{n+i}\left(x_{n+i}\right)\right)\right| \tag{C.3}
\end{equation*}
$$

where $s_{i}$ denote the original codon choices in the original organism. Now this weight denotes the linear difference between five original codon choices in the organism human, and five (possibly different) choices in host organism yeast. Note that the translation speed functions $T$ now explicitely denote for which organism they are calculated, the original or host.

To find the sequence $G^{\prime \prime}$ where both the translation speed landscape and the nucleosome energy landscape in a host organism are close to their original counterparts, we only need to change the weights of $\mathcal{G}_{\text {gene }}$. The weight $w^{\mathrm{T} \mathrm{\& E}}$ of edges between

$$
p_{n}\left(x_{n}\right) p_{n+1}\left(x_{n+1}\right) p_{n+2}\left(x_{n+2}\right) p_{n+3}\left(x_{n+3}\right) p_{n+4}\left(x_{n+4}\right)
$$

and

$$
p_{n+1}\left(x_{n+1}\right) p_{n+2}\left(x_{n+2}\right) p_{n+3}\left(x_{n+3}\right) p_{n+4}\left(x_{n+4}\right) p_{n+5}\left(x_{n+5}\right)
$$



Figure C.3: Graph $\mathcal{G}_{\text {gene }}$ is similar to graph $\mathcal{G}_{\mathrm{E}}$, see figure C.2. The graph includes the entire gene with $N$ the number of codons on the gene. The weights of the graph depend on its purpose: the weights can be defined such that the closest possible translation speed landscape is found for a gene in a host organism, or a combination of the closest translation speed and nucleosome energy landscapes.
for any $x_{i} \in\{1,2, \ldots, 6\}$ are now given by

$$
\begin{equation*}
w^{\mathrm{T} \& \mathrm{E}}=c_{T} w^{\mathrm{T}}+c_{E} w^{\mathrm{E}} \tag{C.4}
\end{equation*}
$$

with

$$
\begin{equation*}
w^{\mathrm{E}}=\sum_{j=-7}^{147+7-2}\left|\sum_{i=-7}^{i=7-2} E_{i+j}\left(S_{p+2+i}, S_{p+1+i}, S_{p+i}\right)-E_{i+j}\left(X_{p+2+i}, X_{p+1+i}, X_{p+i}\right)\right| \tag{C.5}
\end{equation*}
$$

where $X$ is a sequence of 15 base pairs, the sequence corresponding to

$$
p_{n+1}\left(x_{n+1}\right) p_{n+2}\left(x_{n+2}\right) p_{n+3}\left(x_{n+3}\right) p_{n+4}\left(x_{n+4}\right) p_{n+5}\left(x_{n+5}\right)
$$

and $S$ denotes the 15 base pairs in the original organism corresponding to

$$
p_{n+1}\left(s_{n+1}\right) p_{n+2}\left(s_{n+2}\right) p_{n+3}\left(s_{n+3}\right) p_{n+4}\left(s_{n+4}\right) p_{n+5}\left(s_{n+5}\right)
$$

## C. 4 Genetically modified organisms: many genes

In section 4.5, we introduced a method to, when one puts a gene in a different organism, this all three layers of information on the gene would be close to the original. Fig. 4.5 showed the results for one exon of the gene TNF. To remove possible bias from our results, we use the same method on a variety of human genes, randomly selected with a few non-biasing features: the exons of each transcript are fully translated and the first exon has a length $\geq 500$ (the latter condition ensures a nucleosome landscape of significant size). The results are depicted in Figs. C.4-C.14.


$$
\left[\begin{array}{ll}
-\cdots & \text { original sequence in human } \\
\cdots & \text { gene converted to yeast } \\
\text { gene converted to yeast, } \\
--- & \text { closest possible translation } \\
\text { speed landscape } \\
\text { } & \text { gene in yeast, speed } \\
-\cdot & \text { and energy landscapes } \\
\text { close to original }
\end{array}\right.
$$




Figure C.4: Same as Fig. 4.5 but for the first exon of transcript OR6P1-001 of gene OR6P1 from human. This transcript was randomly selected with a few non-biasing features: the exons of each transcript are fully translated and the first exon has a length $\geq 500$. As in Fig. 4.5, (a) depicts the translation speed landscape of this exon in three organisms: the original (human) and two possible host organisms: yeast and rice. Again, (b) shows the original landscape as well as the highest and lowest possible translation speed values in the hosts.


| $-\cdots$ | original sequence in human |
| :--- | :--- |
| gene converted to yeast |  |
| gene converted to yeast, |  |
| --- | cosest possible translation |
| speed landscape |  |
| gene in yeast, speed |  |
| $-\cdots$ | and energy landscapes <br> close to original |



$$
\begin{aligned}
& \text { original sequence in human, same as } \\
& \text { gene converted to yeast } \\
& \text { gene converted to yeast, } \\
& \text {--- closest possible translation } \\
& \text { speed landscape } \\
& \text { gene in yeast, speed } \\
& \text {--- and energy landscapes } \\
& \text { close to original }
\end{aligned}
$$

Figure C.5: Same as Fig. C. 4 but for the first exon of transcript OR10J3-201 of gene OR10J3 from human.


$|$| $-\quad$ original sequence in human |  |
| :--- | :--- |
| $\cdots$ | gene converted to yeast |
| gene converted to yeast, |  |
| --- | closest possible translation <br>  <br> speed landscape <br>  <br> gene in yeast, speed <br> $-\cdots$ <br> and energy landscapes <br> close to original |



$$
\begin{aligned}
& \text { original sequence in human, same as } \\
& \text { gene converted to yeast } \\
& \text { gene converted to yeast, } \\
& \text {--- closest possible translation } \\
& \text { speed landscape } \\
& \quad \text { gene in yeast, speed } \\
& \text {--- and energy landscapes }
\end{aligned}
$$

Figure C.6: Same as Fig. C. 4 but for the first exon of transcript OR10T2-201 of gene OR10T2 from human.


| $-\cdots$ | original sequence in human |
| :--- | :--- |
| gene converted to yeast |  |
| gene converted to yeast, |  |
| --- | closest possible translation |
| speed landscape |  |
| gene in yeast, speed |  |
| $-\cdots$ | and energy landscapes |
| close to original |  |



- original sequence in human, same as
gene converted to yeast
--- closest possible translation
speed landscape
gene in yeast, speed
--- and energy landscapes

Figure C.7: Same as Fig. C. 4 but for the first exon of transcript OR2T6-201 of gene OR2T6 from human.


Figure C.8: Same as Fig. C. 4 but for the first exon of transcript OR2M4-201 of gene OR2M4 from human.


| $-\cdots$ | original sequence in human |
| :--- | :--- |
| gene converted to yeast |  |
| gene converted to yeast, |  |
| $--\quad$ closest possible translation |  |
| speed landscape |  |
| gene in yeast, speed |  |
| $-\cdots$ | and energy landscapes <br> close to original |




Figure C.9: Same as Fig. C. 4 but for the first exon of transcript OR14K1-201 of gene OR14K1 from human.


| - original sequence in human <br> gene converted to yeast gene converted to yeast, <br> --- closest possible translation speed landscape gene in yeast, speed <br> --- and energy landscapes close to original |  |
| :---: | :---: |
|  |  |
|  |  |



$$
\begin{aligned}
& \text { original sequence in human, same as } \\
& \text { gene converted to yeast } \\
& \text { gene converted to yeast, } \\
& \text {--- closest possible translation } \\
& \text { speed landscape } \\
& \text { gene in yeast, speed } \\
& \text {--- and energy landscapes } \\
& \text { close to original }
\end{aligned}
$$

Figure C.10: Same as Fig. C. 4 but for the first exon of transcript OR10K2 of gene OR10K2-201 from human.




$$
\begin{aligned}
& \text { original sequence in human, same as } \\
& \text { gene converted to yeast } \\
& \text { gene converted to yeast, } \\
& \text {--- closest possible translation } \\
& \text { speed landscape } \\
& \text { gene in yeast, speed } \\
& \text {--- and energy landscapes } \\
& \text { close to oriainal }
\end{aligned}
$$

dyad position w.r.t. exon start [bp]
of Figure C.11: Same as Fig.
gene OR2T35 from human.


| $-\cdots$ | original sequence in human |
| :--- | :--- |
| gene converted to yeast |  |
| gene converted to yeast, |  |
| $-\quad$ closest possible translation |  |
| speed landscape |  |
| gene in yeast, speed |  |
| $-\cdots$ | and energy landscapes <br> close to original |

(a)



Figure C.12: Same as Fig. C. 4 but for the first exon of transcript OR2M7-201 of gene OR2M7 from human.


| $-\cdots$ | original sequence in human |
| :--- | :--- |
| gene converted to yeast |  |
| gene converted to yeast, |  |
| --- | closest possible translation |
|  | speed landscape |
| gene in yeast, speed |  |
| $-\cdots$ | and energy landscapes |
| close to original |  |




Figure C.13: Same as Fig. C. 4 but for the first exon of transcript OR6Y1 of gene OR6Y1-201 from human.


| $-\cdots$ | original sequence in human |
| :--- | :--- |
| gene converted to yeast |  |
| gene converted to yeast, |  |
| $--=$ | cosest possible translation |
| speed landscape |  |
| -.- | gene in yeast, speed |
| and engy landscapes |  |
| close to original |  |




Figure C.14: Same as Fig. C. 4 but for the first exon of transcript OR14C36 of gene OR14C36-201 from human.

## Appendix D

## How mechanical information is multiplexed on the transcribed regions of protein-coding genes

## D. 1 Data acquisition using Biomart

Here we provide a manual of sorts to obtain genome data the same way we did. We have acquired the genome data from the Ensembl Project website (www.ensembl.org), using their web-based tool Biomart. From Biomart, we always used the database Ensembl Genes 101. From the database we could pick an organism (such as Human genes). Under Filters, we expand the Gene menu and set the transcript type to protein-coding. Under Attributes, we chose Sequences, and under the SEQUENCES menu we chose Unspliced (Transcript), set Upstream flank to 1000 and Downstream flank to 1000. Under Attributes, in the HEADER INFORMATION menu, we chose, in this exact order: Transcript stable ID, Strand, Transcription start site (TSS), Genomic coding start, Genomic coding end, Exon region start (bp), Exon region end (bp). Using this header information we could determine the positions of the exons, introns and UTRs on any gene. By pressing the results button, and subsequently the Go button, one obtains the data in a plain text file.

Alternatively, one can download the data more efficiently. By pressing the XML button one can show the query in XML Web Service Format, which can then be used to download the data using a software package such as wget. We downloaded the required genomes by simply replacing the name of the organism in the XML string by the name of any available organism. An example of a command line to download the data for human is
wget -0 human.txt 'http://www.ensembl.org/biomart/martservice?query=<?xml
version="1.0" encoding="UTF-8"?><!DOCTYPE Query><Query virtualSchemaName = "default" formatter = "FASTA" header = "0" uniqueRows = "0" count = "" datasetConfigVersion = "0.6" ><Dataset name = "hsapiens_gene_ensembl" interface = "default" ><Filter name = "downstream_flank" value = "1000"/><Filter name = "upstream_flank" value = "1000"/><Filter name = "transcript_biotype" value = "protein_coding"/><Attribute name = "ensembl_transcript_id" /><Attribute name = "transcript_exon_intron" /><Attribute name = "strand" /><Attribute name = "transcription_start_site" /><Attribute name = "genomic_coding_start" /><Attribute name = "genomic_coding_end" /><Attribute name = "exon_chrom_start" /><Attribute name = "exon_chrom_end" /></Dataset></Query>'

For plants, e.g. for Oryza sativa, the command line can be given by

```
wget -0 osativa.txt
    'http://plants.ensembl.org/biomart/martservice?query=<?xml
    version="1.0" encoding="UTF-8"?><!DOCTYPE Query><Query
    virtualSchemaName = "plants_mart" formatter = "FASTA" header = "0"
    uniqueRows = "0" count = "" datasetConfigVersion = "0.6" ><Dataset
    name = "osativa_eg_gene" interface = "default" ><Filter name =
    "downstream_flank" value = "1000"/><Filter name = "upstream_flank"
    value = "1000"/><Filter name = "transcript_biotype" value =
    "protein_coding"/><Attribute name = "ensembl_transcript_id"
    /><Attribute name = "transcript_exon_intron" /><Attribute name =
    "strand" /><Attribute name = "transcription_start_site" /><Attribute
    name = "genomic_coding_start" /><Attribute name =
    "genomic_coding_end" /><Attribute name = "exon_chrom_start"
    /><Attribute name = "exon_chrom_end" /></Dataset></Query>'
```

The Python code we use to turn the raw data into usable data is depicted below. An early version of this code was provided by Rhys Bird.
\#this function requires a data file from the Biomart webtool. It provides three lists: cds2 is a list containing header information such as the name of the transcript, seq2 is a list containing 2000 bases corresponding to any transcipt, starting 1000 bp before the TSS. The list codingseq2 contains the same, but some of the base pairs have been replaced: all intronic bp are replaced by ";", 5'UTRs are replaced by "<", 3'UTR by ">".
def GetGenesShort(inputfile_string,upstream=1000, downstream=1000):
cds = []
seq $=[]$
tempseq = ',
$\mathrm{x}=0$
\#here seq will be a list containing ALL bases corresponding to any transcipt
with open(inputfile_string) as inputfile:

```
for line in inputfile:
if line[0] == '>':
x+=1
if line.strip() != '>':
cds.append(line.replace('>','').split('|'))
if tempseq != '':
seq.append(tempseq)
tempseq = ',
else:
tempseq += line.replace('\n','')
for i in range(len(cds)):
cds[i][3] = cds[i][3].split(';')
cds[i][4] = cds[i][4].split(';')
cds[i][5] = cds[i][5].split(';')
cds[i][6] = cds[i][6].split(';')
codingseq = ['']*len(seq)
for i in range(len(seq)):
Xseq = [';']*len(seq[i])
#cds[i][1] tells us whether the raw data is 5' to 3', or 3' to 5'. In the
    latter scenario, the data is flipped such that everything is 5' to 3'.
#all UTRs are first replaced by ">", later we substitute it by "<" for
    5'UTRs.
if int(cds[i][1]) == 1:
for j in range(len(cds[i][3])):
start = int(cds[i][3][j]) - int(cds[i][2])+upstream
end = int(cds[i][4][j]) - (int(cds[i][2])-1)+upstream
Xseq[start:end] = [">" for _ in range(abs(end-start))]
list(seq[i][start:end])
for j in range(len(cds[i][5])):
start = int(cds[i][5][j]) - int(cds[i][2])+upstream
end = int(cds[i][6][j]) - (int(cds[i][2])-1)+upstream
Xseq[start:end] = list(seq[i][start:end])
elif int(cds[i][1]) == -1:
for j in range(len(cds[i][3])):
start = int(cds[i][2]) - int(cds[i][4][j])+upstream
end = int(cds[i][2]) - (int(cds[i][3][j])-1)+upstream
Xseq[start:end] = [">" for _ in range(abs(end-start))]
for j in range(len(cds[i][5])):
start = int(cds[i][2]) - int(cds[i][6][j])+upstream
end = int(cds[i][2]) - (int(cds[i][5][j])-1)+upstream
Xseq[start:end] = list(seq[i][start:end])
codingseq[i] += ''.join(Xseq)
for i in range(len(codingseq)):
codingseq[i]=upstream*", "+codingseq[i][upstream:len(codingseq[i])-downstream]
    + downstream*","
seq2 = []
codingseq2 = []
cds2 = []
x=0
#here ">" is substituted by "<" for 5'UTRs. Also, all sequences are cut
```

off after length 2000.
for i in range(len(seq)):
if len(seq[i])==len(codingseq[i]):
seq2.append (seq[i] [0:2000])
cds2.append (cds[i] [0:2000])
codingseq2.append([])
codingseq2[x]=codingseq[i] [0:1000]
for $j$ in range (1000,2000):
if codingseq[i][j] in ["A","T","C","G"]:
codingseq2 [x]+=codingseq[i] [j:2000]
break
if codingseq[i][j]==">":
codingseq2[x]+="<"
else:
codingseq2[x]+=codingseq[i] [j]
$\mathrm{x}+=1$
return cds2,seq2,codingseq2

## D. 2 List of animals used to obtain data

This section serves to provide a table with the list of animals of which data was obtained from biomart.

| 1 | dmelanogaster | Drosophila melanogaster | A fruitfly |
| :--- | :--- | :--- | :--- |
| 2 | celegans | Caenorhabditis elegans | A nematode |
| 3 | cintestinales | Ciona intestinalis | Vase tunicate |
| 4 | csavignyi | Ciona savignyi | Solitary sea squirt |
| 5 | pmarinus | Petromyzon marinus | Sea lamprey |
| 6 | loculatus | Lepisosteus oculatus | Spotted gar |
| 7 | amexicanus | Astyanax mexicanus | Mexican tetra |
| 8 | trubripes | Takifugu rubripes | Japanese puffer |
| 9 | tnigrovirides | Tetraodon nigroviridis | Green spotted puffer |
| 10 | oniloticus | Oreochromis niloticus | Nile tilapia |
| 11 | gaculeatus | Gasterosteus aculeatus | Three-spined stickleback |
| 12 | olhni | Oryzias latipes | Japanese rice fish |
| 13 | pformosa | Poecilia formosa | Amazon molly |
| 14 | xmaculatus | Xiphophorus maculatus | Southern platyfish |
| 15 | lchalumnae | Latimeria chalumnae | West Indian Ocean coelacanth |
| 16 | xtropicalis | Xenopus tropicalis | Western clawed frog |
| 17 | acarolinensis | Anolis carolinensis | Green anole |
| 18 | psinensis | Pelodiscus sinensis | Chinese softshell turtle |
| 19 | falbicollis | Ficedula albicollis | Collared flycatcher |
| 20 | aplatyrhynchos | Anas platyrhynchos | Mallard |
| 21 | ggallus | Gallus gallus | Red junglefowl |
| 22 | mgallopavo | Meleagris gallopavo | Wild turkey |
| 23 | oanatinus | Ornithorhynchus anatinus | Platypus |
| 24 | mdomestica | Monodelphis domestica | Gray short-tailed opossum |
| 25 | sharrisii | Sarcophilus harrisii | Tasmanian devil |
| 26 | sscrofa | Sus scrofa | Wild boar |
| 27 | btaurus | Bos taurus | Cow |
| 28 | oaries | Ovis aries | Sheep |
| 29 | mlucifugus | ecallabus | Myotis lucifugus |
| 31 | fcatus | Equus caballus | Little brown bat |
| 32 | mpfuro | Felis catus | Horse |
| 33 | ocuniculus | Mustela putorius furo | Cat |
| 34 | cporcellus | Frrret |  |
| 35 | itridecemlineatus | Oryctolagus cuniculus | European rabbit |
| 36 | dordii | Ictidomys tridecemlineatus | Guinea pig |
| 37 | mmusculus | Dipodomys ordii | Ord's kangaroo rat |
| 38 | rnorvegicus squirrel |  |  |
| 39 | mmurinus | Mus musculus | House mouse |
| 41 | ogarnettii | Rattus norvegicus | Brown rat |
| 42 | csyrichta | Microcebus murinus | Gray mouse lemur |
| 43 | cjacchus | Otolemur garnettii | Northern greater galago |
| 44 | csabaeus | panubis | Carlito syrichta |
| Philippine tarsier |  |  |  |
|  | Callithrix jacchus | Common marmoset |  |
| Chlorocebus sabaeus | Green monkey |  |  |
| Papio anubis | Olive baboon |  |  |

[^1]| 46 | mmulatta | Macaca mulatta | Rhesus macaque |
| :--- | :--- | :--- | :--- |
| 47 | nleucogenys | Nomascus leucogenys | Northern white-cheeked gibbon |
| 48 | pabelii | Pongo abelii | Sumatran orangutan |
| 49 | ggorilla | Gorilla gorilla gorilla | Western lowland gorilla |
| 50 | hsapiens | Homo sapiens | Human |
| 51 | ptroglodytes | Pan troglodytes | Chimpanzee |

Table D.1: This table depicts the list of animals used to obtain data. It depicts the names of the organisms as they appear in Biomart, as well as their Latin names and a short description of the animal.

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[^0]:    ${ }^{1}$ There are two main advantages to summarizing this method again, as opposed to simply referring to the previous chapter/appendix. It makes Chapter 4, in combination with its appendix, readable (and hopefully comprehensable) as a single unit. Secondly: the notation we use here is quite different, such that we can more easily incorporate translation speed in the graph (see appendix C.2).

[^1]:    continues on the next page

