Scaling limits in algebra, geometry, and probability

Arzhakova, E.

Stellingen

behorende bij het proefschrift

Scaling Limits in Algebra, Geometry, and Probability

van

Elizaveta Arzhakova

1. The convex hull of the decimated coefficients of a Laurent polynomial always has a scaling limit. Taking the convex hull is essential, because it is not clear how to identify polynomials (even in two variables) whose decimated coefficients themselves have a scaling limit. (Chapters 2 and 3)

2. Decimation was a form of ancient Roman military punishment, where every tenth soldier was executed. Mathematical decimation is more cruel: a common example is that on a $\mathbb{Z}^d$-lattice with values at nodes, the only values that survive have each coordinate divisible by 10. In the one-dimensional setting of soldiers standing in a line, this implies that only every tenth soldier survives. (Chapters 2 and 3)

3. Instead of using the Fourier transform approach to obtain an explicit expression for the correlations (as done in Chapter 3), one can instead use the cohomological approach. The two approaches offer similar proofs, but the advantage of the latter is that it is coordinate-free. (Chapter 4)

4. Spectral properties of non-hyperbolic ergodic toral automorphisms are weaker than those of hyperbolic toral automorphisms. In this setting, the advantage of Stein's method to prove the Central Limit Theorem lies in the fact that it does not appeal to the spectral properties of the automorphisms. (Chapter 5)

5. A zero of a rigid form on a torus with 2 poles has a unique combinatorial type. In case of 3 poles, the number of combinatorial types (up to orientation) is two: butterflies and octopodes. It is reasonable to suspect that the zoo becomes larger when the number of poles increases. (Chapter 6)

6. Stochastic differential equations describing random motion on Lie groups feature a drift term. The presence of this term might appear counterintuitive to a novice,

but is essential in order for the motion to stay in the Lie group. [Stochastic differential equations for Lie group valued moment maps, Anton Alekseev, Elizaveta Arzhakova, Daria Smirnova, Electron. Commun. Probab. 26: 1-9, 2021]

7. Degree plays an important role in the study of holomorphic differentials. In particular, the statements of connectivity of isoperiodic sets of holomorphic differentials hold when the degree is at least 3. The degree restriction is not relevant in the case of meromorphic differentials, because their volume is infinite. [A transfer principle: from periods to isoperiodic foliations, Gabriel Calsamiglia, Bertrand Deroin, Stefano Francaviglia, arXiv:1511.07635, 2021]

8. Interval exchange transformations (IETs) arise as first-return maps in billiards. IETs are well-studied, as opposed to their close relatives – interval exchange transformations with flips (amusingly abbreviated as fIETs, which means "bicycle" in Dutch). The identification of transitive components of the Rauzy graph associated to fIETs, analogously to the case of linear involutions, would have numerous ergodic implications. [Dynamics and geometry of the Rauzy–Veech induction for quadratic differentials, Corentin Boissy, Erwan Lanneau, Ergodic Theory and Dynamical Systems, 29(3), 767-816, 2009]

9. In his "Discourse on the Method of Rightly Conducting the Reason", René Descartes suggests to ascend from the simplest to the most complex arguments. As mathematical knowledge develops and the gap between simple and complex increases, it is important to follow René Descartes' advice and carefully review all steps of reasoning.

10. It is exciting to discover hidden connections between different mathematical topics and analyse their implications. While the Universe is expanding, we establish links between seemingly distant mathematical topics, thereby, acting on mathematical knowledge as contracting operators.