



Universiteit
Leiden
The Netherlands

Scaling limits in algebra, geometry, and probability

Arzhakova, E.

Citation

Arzhakova, E. (2022, February 23). *Scaling limits in algebra, geometry, and probability*. Retrieved from <https://hdl.handle.net/1887/3276037>

Version: Publisher's Version

License: [Licence agreement concerning inclusion of doctoral thesis in the Institutional Repository of the University of Leiden](#)

Downloaded from: <https://hdl.handle.net/1887/3276037>

Note: To cite this publication please use the final published version (if applicable).

Scaling limits in algebra, geometry, and probability

Proefschrift

ter verkrijging van
de graad van doctor aan de Universiteit Leiden,
op gezag van rector magnificus Prof. dr. ir. H. Bijl,
volgens besluit van het college voor promoties
te verdedigen op woensdag 23 februari 2022
klokke 10:00 uur

door

Elizaveta Arzhakova
geboren te Moskou, Rusland
in 1993

Promotores:

Prof. dr. E. A. Verbitskiy (Universiteit Leiden, Universiteit Groningen)
Prof. dr. W. Th. F. den Hollander

Co-promotor:

Dr. D. E. Terhesiu

Promotiecommissie:

Prof. dr. Ronald van Luijk
Prof. dr. Frans de Haas
Prof. dr. Beatrice de Tilière (University Paris Dauphine)
Prof. dr. Hanfeng Li (University at Buffalo)
Prof. dr. Sergey Shadrin (University of Amsterdam)

Contents

1	Introduction	5
1.1	Dimer configurations and decimations	7
1.2	Spanning trees and determinantal point processes	9
1.3	Central Limit Theorem for dynamical systems	11
1.4	Dynamics on moduli space	12
2	Tropical limits of decimated polynomials	15
2.1	Introduction	15
2.2	Decimation of polynomials	18
2.3	The scaling limit	19
3	Decimation limits of principal algebraic \mathbb{Z}^d-actions	27
3.1	Introduction	27
3.2	Examples	31
3.3	Convex functions and Legendre duals	37
3.4	Amoebas and Ronkin functions	38
3.5	Decimation limits of polynomials	39
3.6	Decimations of principal actions and contracted ideals	43
3.7	Absolutely irreducible factorizations and Gauss's Lemma	47
3.8	Decimated polynomials and decimated actions	49

3.9	Remarks and questions	55
3.10	Example of computing the decimation limit	59
4	On the determinantal process associated to spanning trees	65
4.1	Discrete determinantal processes	65
4.2	Projection operator	71
4.3	Graphs with abelian symmetries	75
4.4	Explicit expressions of correlation kernels	82
5	Rates of convergence in CLT for ergodic toral automorphisms	97
5.1	Introduction and main result	97
5.2	A brief survey of the methods of proof of CLT for dynamical systems	103
5.3	Stein's method for establishing CLT with rates of convergence .	109
5.4	Proof of CLT with rates of convergence for ergodic toral automorphisms	114
6	Connectivity of real isoperiodic sets on a torus with 3 poles	125
6.1	Introduction	125
6.2	Rigid forms	130
6.3	Proof of the connectivity of real isoperiodic sets in $\Omega\mathcal{S}_{1,3}$	145
6.4	Appendix: connectivity of real isoperiodic sets in $\Omega\mathcal{S}_{1,2}$	149
	Bibliography	155
	Samenvatting	164
	Acknowledgements	166
	Curriculum vitae	167