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Scaling limits in algebra, geometry, and probability

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Scaling limits in algebra, geometry, and probability

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