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## Scaling limits in algebra, geometry, and probability

Arzhakova, E.

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# **Scaling limits in algebra, geometry, and probability**

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Elizaveta Arzhakova  
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**Promotores:**

Prof. dr. E. A. Verbitskiy (Universiteit Leiden, Universiteit Groningen)  
Prof. dr. W. Th. F. den Hollander

**Co-promotor:**

Dr. D. E. Terhesiu

**Promotiecommissie:**

Prof. dr. Ronald van Luijk  
Prof. dr. Frans de Haas  
Prof. dr. Beatrice de Tilière (University Paris Dauphine)  
Prof. dr. Hanfeng Li (University at Buffalo)  
Prof. dr. Sergey Shadrin (University of Amsterdam)

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