**Multi-objective evolutionary algorithms for optimal scheduling**

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# Chapter 7

# Conclusions

## 7.1 Summary

This thesis investigated several important aspects of multi-objective optimization and its application in the domain of scheduling optimization. In particular the thesis studied flexible job shop scheduling and dynamic prediction-based maintenance scheduling. Moreover, improvements in multi-objective optimization algorithms were proposed, such as using diversity indicators in selection; cone orders for faster convergence in many-objective optimization.

Next, the achievements and new insights discussed in each chapter will be briefly summarized.

**Chapter 2** covered the basics of multi-objective optimization and provided an in-depth discussion of different ways to define dominance orders, including Pareto dominance, trade-off based dominance, angle dominance, etc. At the end of this chapter the formal relationships between these orders were highlighted and analysed. In particular it was clarified that orders defined by trade-off constraints and coefficients can be mapped to orders defined by cones via angles (or cone-generating vectors), and vice versa. This offers alternative ways for the decision maker or algorithm designer to define orders that are formally equivalent in his/her preferred way, which could be, for example, by means of trade-off bounds or by means of angles and/or cone-generating vectors.

**Chapter 3** answered two research questions. The first question is:

*RQ1 How can a multi-objective evolutionary algorithm (MOEA) be developed that generates uniformly distributed sets on the Pareto front regardless of the shape of the Pareto front?*

To achieve a uniformly distributed Pareto front approximation regardless of the shape of the Pareto front and avoid complex algorithm structure and parameter setting, a diversity-indicator based multi-objective evolutionary algorithm (DI-MOEA) was proposed. DI-MOEA was tested on the state-of-the-art benchmark problems and showed excellent performance in terms of uniformity and convergence for Pareto fronts with convex, concave and disconnected shapes. The results also demonstrated that it is possible to use a diversity indicator (like the geometric mean gap in the proposed algorithm) that is not Pareto compliant, given that the selection gives priority to the non-dominated solutions and the diversity indicator is only used to select the most diverse subset among the non-dominated solutions. It is important to note, that a diversity indicator is here defined on a set as a whole and not on a single point, as it is the case for the crowding distance.

The complexities posed by many real-world optimization problems significantly worsen the performance of optimization algorithms, especially with the increase in the number of objectives. The following research question was therefore of our interest:

*RQ2 How can the performance of MOEAs be improved generally?*

Through analysis of Pareto dominance, it has been shown that its performance can be enhanced by utilizing geometric features of the Pareto order cone. Based on this technique, a new dominance relation, the edge-rotated cone order, was proposed to improve the performance of MOEAs. The edge-rotated cone order was implemented by rotating the edges of the standard Pareto order cone towards the outside. The edge-rotated cone order was integrated with several state-of-the-art MOEAs, and its ability to improve the performance of MOEAs has been tested on multi-objective optimization test problems. Especially on many-objective optimization problems the edge-rotated cone order led to even better performance.

**Chapters 4** investigated the preference based multi-objective evolutionary algorithms in order to answer the third research question:

*RQ3 Instead of the whole Pareto front, how can preferred solutions which are of real interest to the decision maker be obtained?*

From the algorithm perspective, the majority of real-world multi-objective optimization problems suffer from having a large objective space and extent of the Pareto front, which result in a time consuming optimization process. Only focusing on the region of interest can reduce the objective space and speed up the convergence of the optimization algorithms.

Two preference based multi-objective evolutionary algorithms were presented in Chapter 4. The first algorithm is an a priori method. The DMs can provide a point, a region, multiple points or multiple regions as their preferences, the proposed algorithm can find solutions around the reference point(s) or in the reference region(s). Considering that it is difficult for the DMs to set preferences, the second algorithm is an automatic preference based algorithm which can generate the preference region automatically after identifying a knee point, narrow down the feasible objective space progressively, and eventually obtain the preferred solutions. The underlying idea is that in many cases the knee point provides a good compromised solution where all objectives achieve a good value. In particular this holds for connected and convex Pareto fronts, whereas for other shapes of Pareto fronts a default choice for a good solution on a Pareto front is more difficult to justify.

**Chapter 5** discussed the design of novel algorithms for scheduling optimization problems. Our real-world application problem was a multi-objective vehicle fleet maintenance scheduling optimization problem (MOVFMSOP). Before solving this specific problem, one more common scheduling optimization problem in computer science and operations research, i.e., the multi-objective flexible job shop scheduling problem (MOFJSP) was investigated to answer the fourth research question:

> *RQ4 How to solve multi-objective flexible job shop scheduling problems?*

A novel multi-objective evolutionary algorithm was developed for the MOFJSP. Multiple initialization approaches, various crossover operators and local searches were used in the algorithm. In particular, an algorithm configurator was adopted to tune the parameter setting in the algorithm. Based on the study of the MOFJSP, the MOVFMSOP was formulated mathematically and the algorithm was proposed in this chapter to answer the fifth research question:

> *RQ5 How to represent and solve a real-world scheduling optimization problem in the EA world?*

To solve this problem, a three-vector chromosome was designed in the algorithm. The grouping strategy was proposed to determine the feasible grouping structure for the grouping vector based on the predicted remaining useful time of components. At the same time, the starting time vector gave maintenance times for group activities, and the workshop vector decided which workshop performs each maintenance activity. Based on the encoding, the corresponding crossover and mutation operators were designed and applied separately to the three parts of the chromosome. The problem-specific multi-objective evolutionary algorithm was incorporated with several state-of-the-art MOEAs to investigate their performance on the MOVFMSOP.

**Chapter 6** applied and compared the proposed MOEAs and preference based MOEAs on the vehicle fleet maintenance scheduling optimization problems to answer the sixth research question:

> *RQ6 How to apply and adapt the developed algorithms to the dynamic prediction-based maintenance scheduling optimization problem?*

In this chapter, the complete process of maintaining a vehicle fleet was simulated, including the simulation of the vehicle fleet and the prediction of remaining useful life for the fleet. The proposed MOEAs and preference based MOEAs were used in two ways: static and dynamic algorithms. Different updating frequency, computing budget, different scale of problems were adopted to show their performance.

In summary, following the discussion of issues on multi-objective optimization, this thesis proposed the multi-objective evolutionary algorithm, preference-based multi-objective algorithm, and the method to improve the general multi-objective optimization algorithms. Then the real-world vehicle fleet maintenance optimization problem was formulated as a scalable benchmark in an industrially relevant setting. The proposed algorithms were practically used on the application instances. To be able to optimize the maintenance schedule in a dynamic environment, the algorithms were further extended to dynamic algorithms and an additional objective, i.e., stability, was taken into consideration to retain stability of the schedule.

## 7.2 Future Work

Many potential lines of research are worthy of attention based on the achieved progress in the context of this thesis.

DI-MOEA uses the Euclidean distance based geometric mean gap as the diversity indicator, which is described in Chapter 3. The obvious feature of DI-MOEA is the uniformity of the achieved solution set. Besides the geometric mean gap, other gap indicators, such as minimal gap, arithmetic mean gap [37] or the Riesz S-Energy [39], can also be used as the quality indicator. Furthermore, other distances, such as Chebyshev distance, crowding distance, mean ideal distance can replace the Euclidean distance to measure the distance between solutions in the search process. The features of the solution sets based on different distances and gaps can be compared. By doing so, the different strategies can be chosen when solving application problems with different features.

Regarding the edge-rotated cone, the mechanism that relates the properties of the problem with the rotation angle should be researched. Further research on its ability on articulating the preference on multi-objective optimization should be done. Especially on many-objective optimization, the angle setting of the cone order may provide a practical way to guide decision makers in choosing their preferences.

The knee point is defined as the point having the shortest distance to the line (the number of objectives is 2), plane (the number of objectives is 3) or hyperplane (the number of objectives is larger than 3) formed by the extreme points on the Pareto front in Chapter 4. However, it can also be defined in other ways, for example, the point having shortest distance to the Utopia point. Variants of methods to generate the knee point can be compared. By doing so, it is also possible to define multiple knee points to handle problems with irregular shapes of the Pareto front.

When optimizing for the flexible job shop scheduling problems in Chapter 5, we use an algorithm configurator to tune the mutation probabilities in MOEA. It would be a promising topic to utilize the automated parameter configuration more and the multi-objective evolutionary optimization can benefit more from the popular field of machine learning.

In Chapter 6, the optimization is performed periodically to update the maintenance schedule in the simulator. However, a better solution would be to detect the change in the dynamic environment and update the maintenance schedule when the change reaches a threshold. Moreover, since we try to maximize the similarity between the new schedule and the previous one, the arrangement of maintenance activities in the current running schedule can be used in the initialization of the population when updating the schedule. In this way information collected from previous searches can be utilized and the PF approximation can be found more effectively.