Statistical physics and information theory for systems with local constraints

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Propositions

accompanying the thesis

Statistical physics and information theory for systems with local constraints

I. The asymptotic equivalence between canonical and microcanonical ensembles is a central concept in statistical physics. It does not apply under all conditions.

Chapter 1 of this thesis

II. In the presence of extensively many local constraints, the vanishing of the relative fluctuations of the constraints in the canonical ensemble no longer guarantees the equivalence with the microcanonical ensemble.

Chapter 2 of this thesis

III. Ensemble nonequivalence in systems with local constraints can be as strong as in the presence of phase transitions, yet unrestricted to a specific region of parameter space.

Chapter 3 of this thesis

IV. The breakdown of ensemble equivalence can imply new data compression bounds if the information sources are subject to local constraints.

Chapter 4 of this thesis

V. The traditional results of Shannon information theory can be generalized to sources with hard constraints and dependent symbols, through the calculation of the relative entropy between canonical and microcanonical ensembles.

Chapter 5 of this thesis

VI. The work by Touchette shows that the vanishing of the relative entropy density between canonical and microcanonical ensembles in the thermodynamic limit rigorously captures the fundamental property of ensemble equivalence. It turns out that the relative entropy density does not vanish in more situations than traditionally expected.

H. Touchette, J Stat Phys. **159**, 987–1016 (2015).

VII. The traditional textbook by Cover and Thomas emphasizes that information-theoretic quantities such as entropy and relative entropy can provide key answers to fundamental questions in communication and statistics. They can be generalized even further to include more complicated constraints than those considered by the authors.

T. M. Cover and J.A. Thomas, Elements of information theory (2005).

VIII. Squartini *et al.* show that the presence of an extensive number of local constraints in networks provides an alternative mechanism for ensemble nonequivalence, without phase transitions. That mechanism can be made even stronger by considering more general systems with both spatial and temporal constraints.

T. Squartini *et al*., Phys. Rev. Lett. **115**, 268701 (2015).

IX. Jaynes clarified that statistical physics can be formulated by taking entropy as a primitive concept, more fundamental even than energy. When systems are subject to local conservation rules, there are many separately conserved energies, yet entropy remains the primitive concept that can be adapted to the presence of multiple constraints.

E. T. Jaynes, Physical Review **106**(4), 620 (1957)

X. Information theory can be generalized significantly by incorporating results from current research in the statistical physics of systems with local constraints.

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December 1, 2021