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Statistical physics and information theory for systems with local constraints

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Statistical physics and information theory for systems with local constraints

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The cover shows a growing of constraint's number forming ensemble nonequivalence.

To My Family

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