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## Statistical methods for frailty models: studies on old-age mortality and recurrent events

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**Statistical Methods for Frailty Models:  
Studies on Old-Age Mortality and Recurrent Events**

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# **Statistical Methods for Frailty Models: Studies on Old-Age Mortality and Recurrent Events**

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