Computational electrocatalysis: methods and fundamental applications on CO2 reduction and formic acid oxidation
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A SEMIEMPIRICAL METHOD TO DETECT AND CORRECT DFT-BASED GAS-PHASE ERRORS AND ITS APPLICATION IN ELECTROCATALYSIS

Computational models of adsorption at metal surfaces are often based on DFT and make use of the generalized gradient approximation. This likely implies the presence of sizable errors in the gas-phase energetics. Here, we take a step closer toward chemical accuracy with a semiempirical method to correct the gas-phase energetics of PBE, PW91, RPBE, and BEEF-vdW exchange–correlation functionals. The proposed two-step method is tested on a data set of 27 gas-phase molecules belonging to the carbon cycle: first, the errors are pinpointed based on formation energies, and second, the respective corrections are sequentially applied to ensure the progressive lowering of the data set’s mean and maximum errors. We illustrate the benefits of the method in electrocatalysis by a substantial improvement of the calculated equilibrium and onset potentials for CO₂ reduction to CO on Au, Ag, and Cu electrodes. This suggests that fast and systematic gas-phase corrections can be devised to augment the predictive power of computational catalysis models.

This chapter is based on Granda-Marulanda, L. P.; Rendón-Calle, A.; Builes, S.; Illas, F.; Koper, M. T. M.; Calle-Vallejo, F. A Semiempirical Method to Detect and Correct DFT-Based Gas-Phase Errors and Its Application in Electrocatalysis. ACS Catal. 2020, 10 (12), 6900–6907
4.1 Introduction

For decades, considerable effort has been devoted to increasing the accuracy of density functional theory (DFT). This has been done by developing more accurate exchange-correlation functionals at the generalized gradient approximation (GGA) level,\textsuperscript{1–3} hybrid functionals,\textsuperscript{4–6} and range-separated functionals.\textsuperscript{7–9} In addition, different correction schemes have been developed to account for electron localization\textsuperscript{10} or dispersion interactions.\textsuperscript{11–13} Lately, machine learning schemes\textsuperscript{14} have also been proposed to bypass Kohn-Sham equations. In general, these efforts include careful computational benchmarking and comparison to experiments.\textsuperscript{15–17}

An agreement has been reached in the scientific community about the level of theory required to simulate certain materials with a good tradeoff between computational time and accuracy. For instance, hybrid functionals are advisable for molecules and solids with localized electrons, while GGAs usually suffice for bulk and surface metals.\textsuperscript{13,16} However, the choice is not trivial when dealing with systems where metals and molecules are involved and ought to be simulated at the same level of theory. In such a case, the accuracy may be improved by using GGA functionals and adding semiempirical corrections to the DFT energies of molecules, as done for thermochemical reaction energies of interest in catalysis,\textsuperscript{18,19} formation and decomposition energies of solids,\textsuperscript{20,21} and catalytic kinetic barriers.\textsuperscript{22,23}

In this Chapter, we provide a simple and fast procedure for detecting gas-phase errors based on the formation energies of reactants and products calculated with DFT. Improving the description of the gas-phase is shown to enhance catalytic predictive power by analyzing the electrocatalytic CO\textsubscript{2} reduction reaction to CO on Au, Ag, and Cu electrodes. The reduction of CO\textsubscript{2} and CO (hereafter denoted as CO\textsubscript{2}RR and CORR, respectively) are of great importance in catalysis science and technology, as they lead to valuable feedstocks and fuels such as methane, ethylene, ethanol, and formic acid while helping in balancing the carbon cycle.\textsuperscript{24–26} Although DFT has been used to predict enhanced catalysts for other electrocatalytic reactions,\textsuperscript{27–29} it has been so far challenging to elaborate robust design routines for CO\textsubscript{2}RR and CORR to hydrocarbons and oxygenates.\textsuperscript{30} Thus, the method presented in this Chapter may help boost materials design via screening for those paramount reactions.

4.2 Computational Methods

All calculations were performed using the Vienna Ab initio Simulation Package.\textsuperscript{31} Dissimilar gas-phase errors have been pointed out in previous studies for the total energy of CO(g) and CO\textsubscript{2}(g)\textsuperscript{18,19,32} using PBE and RPBE.\textsuperscript{33} In addition, others suggested a correction for the total energy of H\textsubscript{2}(g) to be applied only when using BEEF-vdW.\textsuperscript{34} Thus, we made a functional-dependent analysis including four different xc functionals habitually used in catalysis, namely PBE,\textsuperscript{35} PW91,\textsuperscript{36} RPBE,\textsuperscript{33} and BEEF-vdW.\textsuperscript{9}

The gas-phase molecules were relaxed with the conjugate gradient algorithm in boxes of ~3375 Å\textsuperscript{3}, considering only the Γ point. The effect of the cores on the valence electron density are incorporated using the projector-augmented wave
(PAW) method.\textsuperscript{37} To compute the formation energies of the molecules, graphite was represented by graphene. Approximating graphene as the standard state of carbon is based on the weak interlayer cohesive energy of graphite (0.031 - 0.064 eV/atom)\textsuperscript{38-43} (see Section C6 in Appendix C). The optimized interatomic distances of graphene are 1.43 (PBE and RPBE) and 1.42 Å (PW91 and BEEF-vdW).

The convergence criterion for the maximal forces on the atoms for all simulations was 0.01 eV Å\textsuperscript{-1}, and the plane wave cutoff was set to 400 eV. Convergence tests for the free energy of reaction of CO\textsubscript{2}(g) + H\textsubscript{2}(g) → CO(g) + H\textsubscript{2}O(g) with plane-wave cutoffs in the range of 300 – 1000 eV within PBE showed that 400 eV is enough to achieve accurate reaction energies with an average difference of ~5 meV (see Table C1). None of the species analyzed has unpaired electrons, so spin unrestricted calculations were not required. Gaussian smearing with $k_b T = 0.001$ eV was used. In all cases, the energies were extrapolated to 0 K.

The reaction free energies were obtained as $\Delta G^\theta = \Delta E_{\text{DFT}} + \Delta ZPE - T \Delta S^\theta$, where ZPE is the zero-point energy contribution calculated from the vibrational frequencies obtained using the harmonic-oscillator approximation. The standard total entropies ($S^\theta$) and the experimental standard free energies ($\Delta G^\text{exp}$) were obtained from thermodynamic tables\textsuperscript{44-46} at $T = 298.15$ K. In cases where $\Delta G^\text{exp}$ was not tabulated, it was evaluated by combining entropy and enthalpy values: $\Delta G^\text{exp}_\text{H} = \Delta H^\text{exp} - T \Delta S^\text{exp}_\text{H}$. We did not include heat capacity effects as recent studies showed that formation energies are not significantly modified by them from 0 to 298.15 K.\textsuperscript{21}

Electrocatalytic CO\textsubscript{2} reduction to CO was modeled based on the free energy scheme described in previous reports,\textsuperscript{47} making use of the computational hydrogen electrode\textsuperscript{48} for the description of proton-electron transfers. The reaction pathway proceeds via CO\textsubscript{2} hydrogenation (step 1: CO\textsubscript{2} + H\textsuperscript{+} + e\textsuperscript{-} → *COOH\textsuperscript{+}), followed by *CO formation (step 2: *COOH + H\textsuperscript{+} + e\textsuperscript{-} → *CO + H\textsubscript{2}O\textsubscript{(g)}), and desorption (step 3: *CO → * + CO). In this approach, the onset potential is numerically equivalent to the additive inverse of the largest positive reaction energy considering steps 1 and 2 only ($U_{\text{onset}} = -\max(\Delta G_1, \Delta G_2)/e\textsuperscript{-}$) as step 3 is not electrochemical. We note that alternative pathways for CO\textsubscript{2}RR to CO in the experimental literature suggest that CO\textsubscript{2} may be activated by an electron transfer prior to its adsorption, and the adsorbed species is stabilized by a hydrated cation close to the surface.\textsuperscript{49-53} Since the modeling of decoupled proton-electron transfers is challenging from a plane-wave DFT standpoint, here we limit ourselves to the standard mechanism\textsuperscript{47} using corrected gas-phase energies.

4.3 Results and Discussion

4.3.1 Pinpointing Errors

The data set used to determine the errors (data set A) consists of 27 molecules involved in the CO\textsubscript{2}RR and CORR, in which we include at least one representative molecule of the following functional groups: hydrocarbons, alcohols, carboxylic acids, esters, ethers, aldehydes, and ketones. We included
compounds with one to five carbon atoms in the structure (see the full list of compounds in Table C2). Data set A contains the DFT-calculated standard free energy of formation ($\Delta G^0_{\text{DFT}}$) of the target molecules ($\gamma$) using C(s), O$_2$(g), and H$_2$(g) as a reference:

$$aC + bO_2 + cH_2 \rightarrow \gamma$$

4.1

For instance, for acetaldehyde, Eq. 4.1 is $2C + \frac{1}{2} O_2 + 2H_2 \rightarrow C_2H_4O$. The total errors in the formation energy of each molecule in data set A ($\varepsilon_T$), represent the discrepancy between $\Delta G^0_{\text{DFT}}$ and $\Delta G^0_{\text{exp}}$:

$$\varepsilon_T = \Delta G^0_{\text{DFT}} - \Delta G^0_{\text{exp}}$$

4.2

It is worth noting that $\varepsilon_T$ can either be positive or negative (or zero, in case there is a perfect energetic description). As a first approximation, we consider a group-additivity type of scheme\textsuperscript{34} where a given molecule with different functional groups may have different errors present in its $\Delta G^0_{\text{DFT}}$. Thus, the total error ($\varepsilon_T$) can be decoupled in the separate contributions of the functional groups present in the molecule ($\varepsilon_i$). In mathematical terms this is expressed as: $\varepsilon_T \approx \sum \varepsilon_i$, so that the total error with respect to experiments for a given molecule ($\varepsilon_T$) is approximately the sum of the errors inherited from the $n$ functional groups present in the molecule ($\varepsilon_i$). As shown in Table C2, data set A is formed by CO, CO$_2$, and molecules containing CH$_x$, hydroxyl, carbonyl, carboxyl, ether, and ester functional groups.

A second data set (data set B) consists of calculated free energies of reaction for the CO$_2$RR and CORR to produce the molecules in data set A (see Tables C3-C4). We use data set B to verify whether the corrections implemented in data set A are appropriate. This is the case when there is a decrease in the mean absolute error (MAE) and maximum absolute error (MAX) in data set B as the corrections are successively applied. The free energies of reaction in data set B are grouped in two: firstly, reactions with CO as a reactant and $\gamma$ as a product, as shown in Eq. 4.3 (see Table C4).

$$gCO + kH_2 \rightarrow \gamma + mH_2O$$

4.3

For instance, for acetaldehyde, Eq. 4.3 is: $2CO + 3H_2 \rightarrow C_2H_4O$. Particular cases are the formation of CO$_2$ and HCOOH from CO, which follow Eq. 4.4.

$$CO + H_2O \rightarrow \gamma + rH_2$$

4.4

Secondly, data set B contains reactions with CO$_2$ as a reactant and $\gamma$ as a product, as shown in Eq. 4.5 (see Table C3).
Eq. 4.5 applied to acetaldehyde is: \( 2CO_2 + 5H_2 \rightarrow C_2H_4O + 3H_2O \). In these equations, water is considered to be in gas phase (\( H_2O(g) \)); see Section C3 in the Appendix C. We categorized the errors for each functional based on organic functional groups (-CH, hydroxyl, carbonyl, carboxyl, ether, and ester functional groups) and molecules (in particular, CO and CO2), as shown in Table 4.1. For example, acetaldehyde has one -CH, (-CH3) group and one carbonyl (-CHO) group. Table C5 contains the corrections added per exchange-correlation functional and organic functional group.

The errors in the standard free energies (hereafter referred to simply as errors) in Table 4.1 are xc-functional-dependent, so that the signs and magnitude change in each case, in line with previous studies. This dependence can be expected because exchange-correlation functionals are fitted for certain applications using different data sets. In the following, we will explain how the errors in Table 4.1 were determined, taking PBE as an example. Note in passing that the analysis is similar for the other functionals included in this study, and all values are tabulated in Section C4 of the Appendix C.

To pinpoint the errors, we first determined all deviations (\( \varepsilon_r \)) in the calculated free energies of formation of the molecules in data set A relative to the experimental ones using Eq. 4.2. We paid special attention to CO2 and CO as they are the reactants of CO2-RR and CORR, respectively (all reactions in data set B). For PBE, the error in CO2 is \( \varepsilon_r^{CO2} = -0.19 \text{ eV} \), whereas that of CO is \( \varepsilon_r^{CO} = 0.24 \text{ eV} \). Thus, the magnitudes of the two errors are comparable but the signs are opposite. The CO2 error appears in similar molecules such as HCOOH (\( \varepsilon_r^{HCOOH} = -0.19 \text{ eV} \)) and CH3COOH (\( \varepsilon_r^{CH3COOH} = -0.15 \text{ eV} \)) and is commonly referred to as the OCO backbone error in the literature. Previous studies reported corrections of -0.45 eV for RPBE and -0.59 eV for BEEF-vdW, which agree well with our values of -0.46 and -0.56 eV, respectively. The small correction of -0.07 eV for CO(g) in RPBE is likely a reflection of RPBE’s original fit against CO adsorption energies. We note in passing that simultaneous OCO/H2 corrections are also available in the literature for BEEF-vdW of 0.33/0.09, 0.41/0.09, and 0.29/0.10 eV.

We continued the correction procedure, summarized in scheme 4.1, with the simplest molecules in the list, namely, alcanes (only C-H and single C-C bonds) and observed an increasingly positive error depending on the number of hydrocarbon units (-CHx) (see Table C6). For PBE, that error is on average \( \varepsilon_r^{CH_x} \approx 0.03 \text{ eV/CH}_x \). Although small, such an error is cumulative, and therefore, for a molecule with 5 -CHx units it becomes \( \varepsilon_r^{CH_x} = 0.03 \times 5 \text{CH}_x \approx 0.15 \text{ eV} \). Note that we obtained \( \varepsilon_r^{CH_x} \) by dividing the error in the formation energy of each alkane by the number of -CHx units in it and averaging the results for all alcanes in data set A.

Beyond alcanes, one can increase the complexity of the molecules with additional functional groups. For example, we noted that the error for aldehydes and ketones decreased proportionally to the length of the chain. Therefore, to decouple the error associated to carbonyl groups from that of -CHx groups, we subtracted from the total error of the molecules the error provided by their -CHx.
units (see for instance Table C7). In mathematical terms, for a molecule with the formula $R_1\text{C}=\text{OR}_2$ (where $R_1$ and $R_2$ are either -H or -CH$_x$ units):

$$\varepsilon_i \approx n_c \cdot \varepsilon_{\text{CH}_x} + \varepsilon_{\text{C}=\text{O}}$$

where $n_c$ is the number of -CH$_x$ units.

To illustrate the use of the formula, consider the total error ($\varepsilon_{\text{total}}^{\text{C}=\text{O}}$) for acetaldehyde of -0.09 eV, and the -CH$_x$ error ($\varepsilon_{\text{CH}_x}$) of 0.03 eV. The carbonyl-associated error is

$$\varepsilon_{\text{total}}^{\text{C}=\text{O}} \approx \varepsilon_{\text{C}=\text{O}} - n_c \cdot \varepsilon_{\text{CH}_x} = -0.11 \text{ eV}.$$ 

Averaging over all the aldehydes and ketones in this study, we obtained $\varepsilon_{\text{C}=\text{O}} = -0.10 \text{ eV}$ for PBE. Table 4.1 shows the CO and CO$_2$ errors as well as the average errors determined for the following organic functional groups: -C=O- (aldehydes and ketones), -CH$_x$ (alkanes), -(C=O)O- (carboxylic acids and esters) and -OH (alcohols). Note that the error for -(C=O)O- in PBE is identical to that of CO$_2$, whereas for PW91, RPBE, and BEEF-vdW, that is not the case, as the errors have the same signs but sizably different magnitudes.

**Scheme 4.1**
Workflow to determine functional-dependent errors in energies related to functional groups.
The error in the -OH group for PBE and PW91 is not large enough to warrant correction for simple alcohols. However, this correction may be needed for polyalcohols and in studies focused specifically on methanol and ethanol (see Appendix C, Section C4.1.4 for more details). Before closing this subsection, we stress that a detailed description of the assessment of all errors for every xc-functional can be found in Section C4 in Appendix C. We note that ethylene, acetylene, ethylene oxide and dimethyl ether are present in data set A. Since a larger sample of molecules would be necessary to determine the errors corresponding to their respective functional groups (alkenes, alkyynes, and (cyclic) ethers), here the corrections for those molecules are limited to the corrections in the reactants only (CO and CO₂).

### Table 4.1

Gas-Phase error corrections for the standard free energy of CO₂, CO, and molecules containing -C=O- (Carbonyl Groups in Aldehydes and Ketones), -CHx (Alkanes), and -(C=O)O- (carboxyl groups in carboxylic acids and esters) as per xc functional.

<table>
<thead>
<tr>
<th>Error</th>
<th>PBE</th>
<th>PW91</th>
<th>RPBE</th>
<th>BEEF-vdW</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO₂</td>
<td>-0.19</td>
<td>-0.15</td>
<td>-0.46</td>
<td>-0.56</td>
</tr>
<tr>
<td>CO</td>
<td>0.24</td>
<td>0.25</td>
<td>-0.07</td>
<td>-0.18</td>
</tr>
<tr>
<td>-C=O-</td>
<td>-0.10</td>
<td>-0.10</td>
<td>-0.21</td>
<td>-0.27</td>
</tr>
<tr>
<td>-CHx</td>
<td>0.03</td>
<td>-0.01</td>
<td>0.08</td>
<td>0.21</td>
</tr>
<tr>
<td>-(C=O)O-</td>
<td>-0.19</td>
<td>-0.19</td>
<td>-0.27</td>
<td>-0.34 (-0.44)</td>
</tr>
<tr>
<td>-OH</td>
<td>-0.04</td>
<td>-0.04</td>
<td>-0.01</td>
<td>-0.14</td>
</tr>
</tbody>
</table>

*The two values reported for -(C=O)O- when using BEEF-vdW are for carboxylic acids and esters (the latter in parentheses). All values are in eV.*

### 4.3.2 Implementing Energy Corrections

Data set A was used not only to determine total errors in the formation energies of molecules (εᵣ), but also to assess the organic group contributions to such errors (εᵣ). In principle, one can use those errors to correct the formation energies of molecules, the combination of which should lead to accurate reaction energies. In this order of ideas, corrected reaction energies (Δ⁰_G_{DFT,corr}) can be calculated as:

$$Δ^0_G_{DFT,corr} = Δ^0_G_{DFT} - (\sum ε^pᵣ - \sum ε^rᵣ)$$

where the sums collect all the errors associated to the reactants (εᵣ) and products (εᵣ), taking into account the stoichiometric coefficients. For example, consider the reduction of CO₂ to acetic acid: $2CO_2 + 4H_2 \rightarrow CH_3COOH + 2H_2O$. We find with
RPBE that \( \Delta G_{\text{DFT}}^0 = 0.32 \) eV, whereas \( \Delta G_{\text{exp}}^0 = -0.44 \) eV, which corresponds to a large total error \( \epsilon_p^{\text{CH\_COOH}} = 0.76 \) eV. According to Table 4.1, RPBE has errors associated to the description of \( \text{CO}_2 \) the -COOH group and the -CH$_3$ moiety in CH$_2$COOH. If the errors pinpointed using data set A are contributing to the large total error, suitably correcting \( \text{CO}_2 \) and CH$_2$COOH should lead to a sizable reduction of the total error. This is what we find, as \( \sum \epsilon_{\text{exp}}^p = \epsilon_{\text{CH}_2} + \epsilon_{\text{C\_COOH}} = -0.92 \) eV and \( \sum \epsilon_{\text{exp}}^p = \epsilon_{\text{CH}_2} + \epsilon_{\text{C\_COOH}} = -0.19 \) eV, so that \( \Delta G_{\text{DFT, corr}}^0 = -0.41 \) eV, which differs from the experimental value (\( \Delta G_{\text{exp}}^0 = -0.44 \) eV) by 0.03 eV only.

To verify that the errors in the reaction energies of data set B are systematically reduced upon applying the corrections in Table 4.1, we followed a stepwise procedure. First, we applied corrections to data set B only related to reactants (namely \( \text{CO}_2 \) and \( \text{CO} \)). Next, we applied corrections related to products. Figure 4.1 and Figure 4.2 show the calculated free energies of reaction versus the experimental free energies for the four functionals studied (PBE, PW91, RPBE, BEEF-vdW). Figure 4.1 provides parity plots for CO-based reactions (Eqs. 4.3 and 4.4) and Figure 4.2 does so for \( \text{CO}_2 \)-based reactions (Eq. 4.5). From the three columns in each figure, the first one corresponds to the non-corrected DFT data, the plots in the second column contain the data upon correcting for reactant-related errors (namely \( \text{CO} \) or \( \text{CO}_2 \)), and the third column contains the data upon correcting for reactant- and product-related errors altogether.

More molecules can be added to data set A so as to include more organic functional groups and molecules with several groups in their structure. Molecules with alkene, alkynes, epoxy, and ether functional groups as well as aromatic compounds are necessary in data set A to determine their corresponding errors. Here, the free energies of production from \( \text{CO} \) or \( \text{CO}_2 \) of ethylene, acetylene, dimethyl ether, and ethylene oxide were corrected for the errors in the reactants only, and no product-related corrections were made (see Table C5).

The gray-shaded areas in Figure 4.1 and Figure 4.2 cover an area around the parity line of ± MAE, and the purple-shaded area extends over ± 0.15 eV around the parity line. For CO reduction reactions and PBE calculations, the MAE is initially 0.61 eV (left column) and is lowered to 0.10 eV after applying the CO correction (central column) and to 0.04 eV after applying both CO and product-related corrections (right column). Similarly, the MAXs go from 1.04 to 0.20 and then to 0.17 eV. For the \( \text{CO}_2 \) reduction reactions and PBE, the MAE is successively reduced from 0.43 to 0.10 and then to 0.04 eV. Likewise, the MAXs decrease from 1.10 to 0.24 and finally to 0.17 eV. Further details can be found in Table C24, where the MAEs after the first and second correction for all the xc-functionals are provided. We conclude from those values that the errors in data set B are lowered by one order of magnitude once the correction scheme is applied to the species in data set A.

We have included an alternative analysis where data set A is divided into a training set and an extrapolation set in Section C7 of Appendix C. From this analysis we find approximately the same functional-related errors as in Table 4.1 (within ± 0.01 eV on average). The MAEs in the extrapolation set after the corrections are comparable to those in Figure 4.1 and Figure 4.2, illustrating the predictive power of the method and its statistical reliability.
Figure 4.1
Parity plots for the experimental and DFT-calculated free energies of production of 27 different compounds from CO and H₂ using PBE, PW91, RPBE and BEEF-vdW. The left column shows the data calculated with DFT without any correction. The center column shows the data upon the first correction (errors in CO), and the right column shows the data after correcting for errors in CO and the products. The mean and maximum absolute errors (MAE and MAX) are shown in each case. The shaded gray area is ± MAE in each case. The blue shaded area around the parity line covers an area of ± 0.15 eV.
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**Figure 4.2**
Parity plot for the experimental and DFT-calculated free energies of production of 27 different products from CO$_2$ and H$_2$ using PBE, PW91, RPBE and BEEF-vdW. The left column shows the data calculated with DFT without any correction. The center column shows the data upon the first correction (errors in CO$_2$), and the right column shows the data after correcting for errors in CO$_2$ and the products. The mean and maximum absolute errors (MAE and MAX) are shown in each case. The shaded gray area is ± MAE in each case. The blue shaded area around the parity line covers an area of ± 0.15 eV.
4.3.3 Applications in Electrocatalysis

Table 4.2 reveals an important commonality among the xc functionals under study: although the CO and CO$_2$ errors change from one functional to the next, their difference is nearly constant and equal to ~0.4 eV, on average. This constant energetic separation poses a fundamental limitation for the modeling of catalytic reactions wherein those two compounds are involved, one as a reactant and the other as a product. To show the reaches of this finding, let us consider the example of CO$_2$ electrocatalytic reduction (CO$_2$RR) to CO

$$CO_2 + 2(H^+ + e^-) \rightarrow CO + H_2O(l)$$  \hspace{1cm} 4.7

The backward reaction is known as CO oxidation and is also an important electrocatalytic reaction involved in direct ethanol and methanol fuel cells. Moreover, Eq. 4.7 can also be catalyzed in the gas phase using H$_2$ in a process called reverse water-gas shift, and the backward reaction is the industrial process known as the water-gas shift. In brief, DFT-based models of this seemingly simple process with numerous applications in electrocatalysis and heterogeneous catalysis may have large gas-phase associated errors.

**Table 4.2**
CO$_2$ and CO errors and their nearly constant difference ($\varepsilon_r^{CO} - \varepsilon_r^{CO_2}$) across xc-functionals.$^a$

<table>
<thead>
<tr>
<th>Error</th>
<th>PBE</th>
<th>PW91</th>
<th>RPBE</th>
<th>BEEF-vdW</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO</td>
<td>0.24</td>
<td>0.25</td>
<td>-0.07</td>
<td>-0.18</td>
</tr>
<tr>
<td>$\varepsilon_r^{CO_2}$</td>
<td>-0.19</td>
<td>-0.15</td>
<td>-0.46</td>
<td>-0.56</td>
</tr>
<tr>
<td>$\varepsilon_r^{CO} - \varepsilon_r^{CO_2}$</td>
<td>0.43</td>
<td>0.40</td>
<td>0.39</td>
<td>0.38</td>
</tr>
<tr>
<td>Average</td>
<td>0.40</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.02</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$^a$All values are in eV.

Figure 4.3 compares CO$_2$RR to CO on Au(111) single-crystal electrodes using PBE with (Figure 4.3b) and without (Figure 4.3a) gas-phase corrections applied to CO$_2$ and CO. Likewise, Figures C5 and C6 in Appendix C, section C5, provide data for Au(100) and Au(110). In Figure 4.3a, where DFT data appear as is, the reaction energy of Eq. 4.7 is 0.63 eV. Conversely, it is 0.20 eV in Figure 4.3b, where the energies of CO$_2$ and CO have been corrected. For comparison, such difference is 0.20 eV in experiments$^{25}$ (it is 0.30 eV in Table C3. The difference stems from the liquid state of water in Eq. 4.7). In terms of the equilibrium potential of the reaction, this all means that PBE predicts it to be at -0.32 V vs RHE, whereas both the correction method and experiments set it at -0.10 V vs RHE. The difference is substantial and amounts to ~220 mV. Note in passing that there are no changes in the energy differences between $^*$COOH and $^*$CO, as the corrections are only
applied to the gas phase. Corrections for adsorbates have been proposed before\textsuperscript{18} but have escaped the subject and scope of this study.

Within the context of CO\textsubscript{2}RR modeling with the computational hydrogen electrode,\textsuperscript{47,48} the onset potential is given by the largest-positive consecutive difference in Figure 4.3 ($U_{\text{onset}} = - \max(\Delta G_i, \Delta G_j) / e^{-}$; see the Computational Methods section). In Figure 4.3a, such difference is 0.90 eV, whereas in Figure 4.3b it is 0.71 eV, so that the predicted onset potentials are -0.90 and -0.71 V vs RHE, respectively. As the experimental value of the onset potential is -0.66 V vs RHE,\textsuperscript{58} the deviations from experiments are $\sim$0.24 (as is) and 0.05 V (corrected).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4_3.png}
\caption{Free energy diagrams for CO\textsubscript{2} reduction to CO using Au(111) single-crystal electrodes. (a) Using DFT-PBE data as is, and (b) correcting CO\textsubscript{2} and CO for their gas-phase errors. The black dashed line at 0.66 eV marks the free energy corresponding to the experimental onset potential of -0.66 V vs RHE.\textsuperscript{58}}
\end{figure}

We note that the sizable lowering of the error from 0.24 to 0.05 V is a direct result of correcting gas-phase energetics. To assess whether this is a particularity of Au(111) electrodes or part of a more general trend, we also compared the calculated and experimental onset potentials for Au(100), Au(110), Au\textsubscript{poly}, Ag(111), Ag\textsubscript{poly}, and Cu\textsubscript{poly}. The results in Figure 4.4a show that DFT data are systematically deviated from the parity line, which results in a MAE of 0.20 V and a MAX of 0.27 V. Conversely, the CO\textsubscript{2}-and-CO corrected data in Figure 4.4b are located around the parity line with MAE = 0.06 V and MAX = 0.09 V. Substantial improvements are also observed for Au(111) and Au(100) using gas-phase corrections with RPBE, see Figures C7-C8. Thus, we conclude that models
for CO$_2$RR to CO may in general benefit from the gas-phase corrections found in this work.

**Figure 4.4**
Parity plots comparing the onset potentials for electrochemical CO$_2$ reduction to CO using different metals. (a) Using DFT-PBE data as is, and (b) correcting CO$_2$ and CO for their gas-phase errors. The mean and maximum absolute errors (MAE and MAX) are shown in each case. The gray areas around the parity line cover an area of ± 0.15 V around it. The experimental data were taken from references 58-61 (see details in Table C25).
4.4 Conclusions

When interfaces between metals and fluids are simulated at the GGA level, sizable errors may appear in the description of the gas-phase molecules energetics. Here, we proposed a two-step semiempirical method to determine gas-phase errors, based on the formation energies of 27 different molecules. Furthermore, implementing the corresponding corrections allows for predictions in the analyzed data set of CO$_2$RR and CORR reaction energies that lower by 1 order of magnitude the average and maximum errors with respect to experiments.

The method also shows that the errors for CO$_2$ and CO differ by $\sim$0.4 eV for all the examined exchange-correlation functionals. Thus, an intrinsic limitation of DFT exists for the accurate description of reaction energies containing these two molecules, as is the case for CO$_2$ reduction to CO or HCOOH, CO oxidation to CO$_2$, etc. Such limited description leads to inaccurate predictions of equilibrium and onset potentials, which may hinder the rational catalyst design.

Conversely, using our correction scheme on various Au, Ag and Cu electrodes decreased the average error in the predicted onset potentials from 0.21 to 0.06 V with respect to experiments. Therefore, in addition to pinpointing and lowering gas-phase errors, the method also helps in providing more accurate electrocatalytic models.

While the present corrections have been applied for electrochemical reactions, the procedure is general enough to be applied to correct the thermochemistry of heterogeneously catalyzed reactions where reactants and products are in the gas phase but the intermediate steps take place at the catalyst surface. Finally, the correction protocol can be enriched by adding more gas-phase molecules to the data set and using machine learning algorithms to detect and predict errors in structurally more complex substances.

4.5 References


A semiempirical method to detect and correct DFT-based gas-phase errors and its application in electrocatalysis


