

Geometric quadratic chabauty and other topics in number theory $% \left(\mathbf{r}\right) =\left(\mathbf{r}\right)$

Lido, G.M.

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Stellingen

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Geometric quadratic Chabauty and other topics in number theory

- I. The projective hyperelliptic curve defined by the equation $y^2+y=x^6-3x^5+x^4+3x^3-x^2-x$ has exactly 14 rational points. This can be shown using the geometric quadratic Chabauty method.
- II. In characteristic zero, all formal commutative biextensions are isomorphic to an additive biextension.
- III. If p^e be a prime power greater than 12 and different from $3^3, 2^4, 2^5, 2^6$, then all the automorphisms of a Cartan modular curve of level p^e are modular. The same statement is false when $p^e \le 11$, while the cases $3^3, 2^4, 2^5, 2^6$ are open, in the author's knowledge.
- IV. If p is a prime number and e > p is an integer, then the discrete logarithm problem in the group $\mathbb{F}_{p^e}^{\times}$ can be solved in $(\log(p^e))^{O(\log\log(p^e))}$ operations.
- V. The article [1] is one of the most important articles about automorphisms of modular curves. Yet, as shown in [2], there are some mistakes in the statements of Lemma 1.6, Lemma 2.15 there contained (the main theorem of the article is true, up to excluding the case N = 108).
- VI. The geometric quadratic Chabauty method described in chapter 1 can be generalised to arbitrary number fields. See the ongoing project [3] by Pavel Čoupek, David Lilienfeldt, Luciena Xiao and Zijian Yao.
- VII. Interestingly, the implementation details of discrete logarithm algorithms based on elliptic basis have been already studied in [4]
- VIII. Chapter 1 uses geometric methods, that can and should be visualised. A cartoon guide to it is available at [5]

Guido Maria Lido, Leiden ??-??-???

References

- [1] M. A. Kenku, F. Momose, Automorphism groups of the modular curves $X_0(N)$. Compositio Mathematica, 65 (1988)
- [2] M. C. Harrison, A New Automorphism Of X₀(108). ArXiv preprint 1108.5595 (2011)
- [3] https://www.math.mcgill.ca/lilien/Chabauty-Part1.pdf
- [4] A. Joux, C. Pierrot, Algorithmic aspects of elliptic bases in finite field discrete logarithm algorithms. arXiv preprint 1907.02689 (2019).
- [5] S. Hashimoto, Cartoon guide to finding Q-points with geometric quadratic Chabauty. https://github.com/sachihashimoto/cartoon-guide-gqc