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Geometric quadratic chabauty and other topics in number theory

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GEOMETRIC QUADRATIC CHABAUTY AND OTHER TOPICS IN NUMBER THEORY

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The images in the front cover of this thesis, from the bottom to the top, are: an illustration of the geometric quadratic Chabauty method drawn by Sachi Hashimoto, available in [51], and representations of the modular curves $X_{\text{ns}}(13)$ and $X_{\text{ns}}^+(13)$, drawn using the equations in [36].

The illustration in the back cover has been drawn by Giulia Caudullo and represents the descent algorithm in Chapter 4.

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