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On the coexistence of Landau levels and superconductivity

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On the coexistence of Landau levels and superconductivity

PROEFSCHRIFT

TER VERKRIJGING VAN
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Cover: a watercolor painting of northern lights over an arctic landscape by Yevheniia Cheipesh. The northern lights, produced by charged particles swirling in the Earth's magnetic field, are depicted to form a striped pattern, resembling the density of a Landau level produced by electrons in the magnetic field in a two-dimensional topological superconductor (Fig. 5.4).

To my Mom, my Grandparents, and all my Family.

Contents

1	Introduction	1
1.1	Preface	1
1.2	Landau levels	2
1.2.1	Particles in a magnetic field	2
1.2.2	Weyl semimetal	4
1.2.3	Zeroth Landau level	6
1.2.4	Chiral anomaly	7
1.2.5	Chiral magnetic effect	10
1.3	Superconductivity	11
1.3.1	Bogoliubov-de Gennes formalism	11
1.3.2	Weyl superconductor	12
1.3.3	Vortex lattice	14
1.4	Outline of this thesis	15
2	Topologically protected Landau level in the vortex lattice of a Weyl superconductor	19
2.1	Introduction	19
2.2	Weyl superconductor in the mixed phase	21
2.3	Calculation of the zero-modes	24
2.4	Comparison with numerics	26
2.5	Thermal conductance	26
2.6	Conclusion	28
2.A	Boundary condition at the vortex core	31
2.B	Details of the tight-binding calculations	33
2.B.1	Weyl superconductor	33
2.B.2	Superconducting phase	34
2.B.3	d -wave superconductor	36
2.C	Quasiparticle density profile near the vortex core	38
2.D	Arbitrary angle between internal magnetization and external magnetic field	39
2.E	Tilting of the Weyl cones	39
2.E.1	Hamiltonian of a type-I Weyl superconductor	40
2.E.2	Generalized chiral symmetry protects the zeroth Landau level	41

Contents

2.E.3	Chiral dispersion along the magnetic field	43
3	Effect of charge renormalization on electric and thermo-electric transport along the vortex lattice of a Weyl superconductor	45
3.1	Introduction	45
3.2	Landau level Hamiltonian in the vortex lattice	46
3.2.1	Dispersion relation	47
3.2.2	Effective Hamiltonian	49
3.2.3	Zeroth Landau level wave functions	50
3.3	Transmission through the NSN junction	50
3.3.1	Renormalized charge transfer	51
3.3.2	Transmission matrix	52
3.4	Transport properties	53
3.4.1	Thermal conductance	54
3.4.2	Electrical conductance	54
3.4.3	Shot noise	55
3.4.4	Thermo-electricity	56
3.5	Numerical simulations	56
3.5.1	Tight-binding Hamiltonian	56
3.5.2	Results	59
3.5.3	Test for isotropy of the charge renormalization	60
3.6	Conclusion	60
3.A	Calculation of transport properties from the continuum limit of the tight-binding model	63
3.A.1	Matching condition	63
3.A.2	Evanescence modes	64
3.A.3	Transmitted wave	66
3.A.4	Charge transfer	68
3.A.5	Transport properties	68
4	Universal chiral magnetic effect in the vortex lattice of a Weyl superconductor	71
4.1	Introduction	71
4.2	Formulation of the problem	72
4.3	Chirality confinement in a vortex lattice	75
4.3.1	Landau bands	76
4.3.2	Vortex core bands	78
4.4	Chiral magnetic effect	80
4.4.1	Charge renormalization	80
4.4.2	On-shell contributions	82

4.4.3	Off-shell contributions	83
4.5	Conclusion	83
4.A	Details of the numerical calculation	87
5	Deconfinement of Majorana vortex modes produces a superconducting Landau level	91
5.1	Introduction	91
5.2	Confined phase	92
5.3	Deconfined phase	95
5.4	Numerical simulation	98
5.5	Striped local density of states	100
5.6	Conclusion	101
5.A	Details of the numerical simulation	103
5.A.1	Tight-binding model	103
5.A.2	Additional numerical results	104
5.A.3	Effect of overlap of top and bottom surface states .	104
5.B	Solution of the Helmholtz equation for the Majorana Landau level	105
5.C	Chain of vortices	111
5.D	Renormalized charge in the Majorana Landau level	112
5.E	Comparison of numerics and analytics	113
6	Generalized eigenproblem without fermion doubling for Dirac fermions on a lattice	117
6.1	Introduction	117
6.2	Construction of the generalized eigenproblem	119
6.2.1	Staggered discretization	119
6.2.2	Translationally invariant system	120
6.2.3	Including a disorder potential	121
6.3	Symmetrization of the generalized eigenproblem	123
6.4	Locally conserved particle current	125
6.5	Spectral statistics	126
6.6	Conclusion	129
6.A	Susskind discretization breaks symplectic symmetry	131
6.B	Derivation of the local conservation law for the particle current	132
6.C	Gauge invariant vector potential	133
Bibliography		135
Samenvatting		147

Contents

Summary	149
Curriculum Vitæ	151
List of publications	153