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## On the coexistence of Landau levels and superconductivity

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# On the coexistence of Landau levels and superconductivity

PROEFSCHRIFT

TER VERKRIJGING VAN  
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Cover: a watercolor painting of northern lights over an arctic landscape by Yevheniia Cheipesh. The northern lights, produced by charged particles swirling in the Earth's magnetic field, are depicted to form a striped pattern, resembling the density of a Landau level produced by electrons in the magnetic field in a two-dimensional topological superconductor (Fig. 5.4).

*To my Mom, my Grandparents, and all my Family.*



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