More than people and pots: identity and regionalization in Ancient Egypt during the second intermediate period, ca. 1775-1550 BC

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NETWORK ANALYSIS

This chapter elaborates on network analysis, which is the methodology used in the present work. What is its theoretical background? How is it conducted? Considering that, in the present work, this methodology is applied to archaeological material, more questions arise: how is network analysis applicable in archaeological research? Which issues are connected to it and how can they be tackled? In this chapter, the basic theory and terminology of network analysis, especially the elements most relevant to the present work, are discussed, as well its application to archaeological research. The main issues present in this methodology and how researchers have dealt with them are also addressed. Lastly, the use of network analysis in the present work is illustrated, by detailing how the material is going to be analysed in the present work.

WHAT IS NETWORK ANALYSIS?

Network analysis started from sociometry, which studies social atoms, namely the individual and his/her social, economic, or cultural ties. It also studies how the social atoms link into groups and how these groups connect into a society.[1] Network analysis is based on the belief that interpersonal relations, as well as relations between organizations and countries, are important because they are means of transmission of behaviours, information and goods.[2] As a consequence, in order to understand the role and behaviour of entities, or actors, it is important to study how they interact and the relations that they establish in the network to which they belong: this is the main goal of network analysis.[3] An entity, or actor, is any person, organization, or land participating in a relation.[4]

1 De Nooy, Mrvar, and Batagelj 2005, 3; Scott 2012.
2 De Nooy, Mrvar, and Batagelj 2005, 3; Scott 2017, 2–3.
3 Brughmans, Isaksen, and Earl 2012, 360; Collar 2014, 99; Collar et al. 2015, 6; De Nooy, Mrvar, and Batagelj 2005, 5; Mills, Clark, et al. 2013, 5875; Sindbæk 2013, 72–73.
4 De Nooy, Mrvar, and Batagelj 2005, 5.

Furthermore, in the same way that to understand an entity or actor it is necessary to study the universe of its connections, also the reverse is true, because the actors and their actions define each other.[5] Thus, the interpretation of different processes and phenomena is based on the relations that entities establish and on the role that they have.[6] This is an exploratory approach, which, based on the belief that a pattern detectable in a network is significant to the actors of the networks and therefore to the researcher, means that the researcher investigates a network for meaningful patterns, instead of using the network to test a specific hypothesis.[7]

This way of proceeding has the advantage that, apart from the definition of which entities are being analysed and what are the elements linking them, other analytical constructs, such as the definition of a core and a periphery, are avoided.[8] Moreover, the analysis can be conducted on multiple scales, which can be synthetically visualized in a graph.[9] It is important though, to clearly define the boundaries of the network, namely what is the extent of the entities or actors analysed, because this can affect the outcome of the analysis.[10] In archaeology, for example, the entities are often sites, contexts such as tombs or particular parts of the sites, or objects, but it is up to the scholar to define the range of sites or contexts or objects analysed.[11]

NETWORK ANALYSIS IN ARCHAEOLOGY

Network analysis has been used in archaeology to study relations between persons, places, objects, or even decorative motifs. It has shed new light on old data and has given new potential to archaeological research, by giving the possibility to focus on the human relations and on the social groupings witnessed by the objects,[12] because these relations are seen as means that allow material and non-material resources to flow between groups.[13] In an archaeological two-mode network, where two groups of entities are examined, one group is often constituted by the contexts analysed, while the other group is often formed by their attributes, mostly objects such as pottery; this means

5 Brughmans 2013, 632–33; Brughmans, Isaksen, and Earl 2012, 360.
6 Brughmans 2013, 632–33; Golitko and Feinman 2015, 212–13; Mills, Clark, et al. 2013, 5875.
7 De Nooy, Mrvar, and Batagelj 2005, 5.
8 Knappett 2013, 4.
9 Golitko and Feinman 2015, 212–13; Knappett 2013, 4–6.
10 De Nooy, Mrvar, and Batagelj 2005, 6; Scott 2017, 46–48.
11 As shown in: Brughmans 2010; Östborn and Gerding 2014.
12 Collar et al. 2015, 6; Mills, Roberts Jr., et al. 2013, 181–82; Östborn and Gerding 2014, 76.
13 Brughmans 2013, 632–33.

that the contexts are linked to their attributes. As an example, in Figure 1 in Chapter 7, each site is linked to the types of beads excavated there. In an archaeological one-mode network, the entities can be either the contexts or the attributes; this means that each context is linked to another context if they share a particular attribute, or that each attribute is linked to another attribute if they are found in the same context.[14] As an example, in Figure 2 in Chapter 7 each site is linked to the other sites with which it has types of beads in common.

However, when using network analysis for archaeological research it is important to distinguish it from social network analysis.[15] While the latter implies studying relations between persons without the intermediation of the objects used by them, in archaeology the focus is on the objects that people from the past used and have left. Though these objects can be used also to reconstruct social relations, they show these social relations in an indirect way and their study cannot be limited to that.[16]

In other words, in social network analysis the studies start from the relations detectable in a group and then examine its effect, while in archaeology the starting point is the effect of the relations, namely the objects exchanged. From these objects, the connections between entities that made the exchange possible are reconstructed. Because of this, in archaeology it is more correct to use the definition network synthesis instead of network analysis.[17] Moreover, in archaeological research, the connections detected through the objects often take into consideration the geographical location of the elements studied, so that geography and the use of a software for geographical information system are an integral part of the analysis.[18]

Another difference concerns the fact that, when used in archaeology, network analysis has fewer and less complex equations than when used in other fields like sociology or physics. There are, though, other elements making network analysis difficult in archaeology, first of all the nature of the data set.[19] In archaeology, the objects are often the main constituents of the data set, but the links connecting them are absent, like having a black box where the elements of the circuit are present but not connected.[20] For example, in the present research, the data about the objects and the sites are available, but it is not clear how the sites were connected or how the objects arrived at the sites.

14 Brughmans 2010; Östborn and Gerding 2014, 76.
15 Brughmans 2010, 282.
16 Brughmans 2010, 282; Knappett 2013, 7–8.
17 For the discussion about this point: Sindbæk 2013, 76.
18 Mills, Roberts Jr., et al. 2013, 182.
19 For a discussion about this point: Knappett 2013, 7–8.
20 Sindbæk 2013, 72.

Therefore, the present work uses only undirected graphs, where directions from one site to another are not considered, as explained later.

In addition, the nature of the data collected in archaeology is often incomplete and subjected to disturbing factors, which make the array of data uneven. These factors include the extent and methods with which sites have been excavated, as well as the methods followed to collect, study, and publish the material.[21] This produces the so-called archaeological bias, which means that sites that happen to have been more extensively excavated, or more extensively or more accurately published, could appear more important and be over-estimated in the analysis because they proved more data, while sites less excavated or published could appear less important and be under-estimated in the analysis because they provide fewer data.[22] To reduce the risk of archaeological bias, only the presence/absence of objects at a site is taking into consideration, without taking into account the amount of contexts or the abundance, as explained later.

Therefore, it should be kept in mind that the data examined are not complete, but a sample, which could be unrepresentative, and in its turn this could affect the analysis and its results.[23] This makes it necessary to recur to statistical tests, mathematical models, and the setting of thresholds, namely minimum values that the nodes need to have to stay in the network, as decided by the researcher, in order to understand the strength and value of the data and reconstruct a realistic picture.[24] Examples of this are particularly found in Brughmans' research on the distribution of Roman table wares in the Eastern Mediterranean,[25] and in a research conducted on pre-Hispanic U.S. Southwest.[26]

Furthermore, the dataset in archaeological research has a complex nature. In other words, the entities have many attributes and can connect to each other in different ways on the basis of the attributes examined.[27] For example, in Östborn and Gerding's analysis of the fired bricks in pre-Hellenistic times, the bricks registered in the data set could be related on the basis of their contexts of use, or of the marks found on them, or of their shape.[28] In the present work, the links between the sites examined have been created based on types, defined as objects of specific shape and specific material.

21 Knappett 2013, 7–8.
22 For a discussion about this point: Knappett 2013, 7–8.
23 Östborn and Gerding 2014, 81–83; Peeples and Roberts Jr. 2013, 3002.
24 Brughmans 2013; Brughmans, Isaksen, and Earl 2012; Knappett 2013; Östborn and Gerding 2014, 81–83; Peeples and Roberts Jr. 2013, 3002.
25 Brughmans 2010.
26 Peeples and Roberts Jr. 2013.
27 Brughmans 2010, 285; Sindbæk 2013, 73.
28 Östborn and Gerding 2015.

It should also be added that in archaeology there is a background knowledge that is relevant to understand the material, such as the context and position where it was found, the assemblage, or group of objects, of which it was part, and its use. This complexity means that the attributes to insert in a matrix are more or less arbitrarily chosen by each scholar.[29] This shows also that it is difficult to render in a matrix and in quantitative terms all the aspects connected to each entity and each attribute.[30] That is why even archaeologists using network analysis caution against expecting statistical exactness and against over-interpreting the network graphs.[31]

To tackle this issue, it is useful to consider the difference between visualization and representation. While the former is the visualization of the data as a network graph, the latter is the process that precedes it and that includes the choices and parameters set by the scholar to establish what entities need to be represented and what attributes need to be taken into account.[32]

Lastly, another problem when using network analysis in archaeology concerns visualizing the diachronic aspect of historical processes. The archaeological record allows us to have a glimpse at specific moments of these processes,[33] like looking at single frames from different scenes of a film. Can network analysis be used to reconstruct the processes, or, to use again the metaphor, to reconstruct entire scenes or even the entire film from the single frames? From research conducted so far it seems that it is feasible. For example, studying the co-presence of specific objects, namely their presence in the same sites at the same time, over different periods can inform about how the distribution of these objects developed over time, helping to further understand the processes leading to this distribution, as demonstrated for example in Brughmans' research.[34] Moreover, examining how the position and role of entities in a network eventually changes through time can show the underlying processes.[35] In the present work, the diachronic aspect has been achieved by dividing the sites in the three main chronological phases examined (see Chapter 2): Late Middle Kingdom, Early Second Intermediate Period, Late Second Intermediate Period.

On the same topic, Östborn and Gerding[36] have used the concept of 'complex evolution', according to which network analysis is useful in reconstructing spatial-temporal processes because it compares pairs of contexts and creates

29 Collar et al. 2015, 12.
30 Sindbæk 2013, 77.
31 Östborn and Gerding 2014, 83.
32 Collar et al. 2015, 12.
33 Brughmans 2010, 283.
34 Brughmans 2010, 288.
35 Golitko and Feinman 2015, 217; Mills, Roberts Jr., et al. 2013, 182.
36 Östborn and Gerding 2014, 80–81.

branches like the ones used in biological evolution, with one main difference: in biological evolution a branch can give origins to further branches, but new branches can never recombine to form a new one, creating a tree-shaped diagram. In cultural developments, on the contrary, traits can recombine to form a new one, so that the final diagram can contain also loops.

HOW TO BUILD NETWORKS

The starting points of network analysis are the dataset and the matrix. In the dataset, all the features relating to each entity are reported. These features can be attributes, namely features intrinsic to the entities, or relational data, namely elements such as organizations or events in which the entities participate, or 'ideational' data, namely elements such as opinions and motives shared by the entities.[37] Furthermore, a dataset can have a hierarchical or flat structure. In the first case, the value of a given feature of an entity determines the value of another feature of the same entity, while in the second case the features vary independently.[38]

The matrix is a table where each row and each column correspond to an entity, while the intersection of a row and a column is called a cell.[39] Each cell reports the number of connections, or similarities, associating the entity of that row with the entity of that column, namely how many similar features the entity of that row shares with the entity of that column.[40] If the entities of the rows and the entities of the columns are the same, then there is only one group of entities and the network is called one-mode, because it examines how each entity is connected to the others in the same group.[41] This type of matrix is also called an adjacency matrix, because it shows clearly which entities are neighbours or adjacent, namely connected, in the network.[42] If the entities of the rows are different from the entities of the columns, then there are two groups of entities and the network is called bimodal (or bipartite or two-mode), because it examines how the entities of one group are linked to the entities of another group.[43] This type of matrix is also known as incidence matrix.[44]

37 Scott 2017, 3–6.
38 Östborn and Gerding 2014, 76.
39 De Nooy, Mrvar, and Batagelj 2005, 260.
40 Scott 2017, 59–60.
41 Brughmans 2013, 626–28; De Nooy, Mrvar, and Batagelj 2005, 103; Östborn and Gerding 2014, 76; Scott 2017, 61–62.
42 De Nooy, Mrvar, and Batagelj 2005, 260; Scott 2017, 62.
43 Brughmans 2013, 626–28; De Nooy, Mrvar, and Batagelj 2005, 103; Easley and Kleinberg 2010, 94; Östborn and Gerding 2014, 76; Scott 2017, 59–63.
44 De Nooy, Mrvar, and Batagelj 2005, 261; Scott 2017, 62–63.

An alternative to the matrix is the edge list, which reports which entities share a connection. In a few words, it is a table with two columns where each row reports a specific entity, in the first column, and the entity that is contact with it, in the second column; a further column can also report the strength of the contacts, namely the number of similarities or connection that the entities of each row share.[45] From the matrix, or from the edge list, software programs specialized in network analysis, such as ORA, VISONE, Gephi, UCInet, NodeXL, and Pajek produce a graph,[46] which visualizes and specifies the relations among the entities through dots and through the lines connecting them.[47] Because of the fact that network analysis uses graphs to visualize and analyse the network, it also uses terminology from graph analysis.[48] Thus, like in graph analysis, the dots are called vertices or nodes and correspond to the entities chosen for the analysis, while the lines or links connecting them are called edges and correspond to the connections or similarities between the entities.[49] The nodes are called adjacent, or neighbours, if they share an edge.[50] Thus, a pair of nodes and the link between them form a dyad.[51] For example, in Figure 3 in Chapter 7, Lisht and Harageh are a dyad, because they have types of beads in common, which create the link between them.

Networks can be undirected or directed. Undirected networks are the ones where the relation between each pair of entities is symmetrical, that is to say it is always reciprocal and functions both ways, implying that the entities share the same number of connections or similarities.[52] All the networks produced in the present work are undirected, as for example in Figure 3 in Chapter 7. On the contrary, in directed networks the relations are not always symmetrical and they involve a flow in a pair of entities, which start from a sending entity, or sender, and ends at a receiving entity, or a receiver.[53] In a graph, a link is called tie in undirected networks and arc in directed networks.[54] An arc is

45 Cline and Cline 2015, 21–24.
46 These programs are mentioned, for example, in: Brughmans 2013, 624; Cline and Cline 2015, 21; Dulíková and Mařík 2017, 63–64; Scott 2017, 69–71.
47 Easley and Kleinberg 2010, 23.
48 Brughmans 2013, 623–24; Scott 2012; Scott 2017, 69.
49 Brughmans 2010, 277; Brughmans 2013, 626–28; Cline and Cline 2015, 26; De Nooy, Mrvar, and Batagelj 2005, 6; Easley and Kleinberg 2010, 23; Östborn and Gerding 2014, 76; Scott 2017, 74–76.
50 De Nooy, Mrvar, and Batagelj 2005, 64; Easley and Kleinberg 2010, 23; Scott 2017, 78.
51 De Nooy, Mrvar, and Batagelj 2005, 205–6.
52 Brughmans 2013, 627; Collar et al. 2015, 14; Coward 2013, 248; De Nooy, Mrvar, and Batagelj 2005, 7; Easley and Kleinberg 2010, 23; Peeples and Roberts Jr. 2013, 3002; Scott 2017, 76–78.
53 Brughmans 2013, 627; De Nooy, Mrvar, and Batagelj 2005, 7; Easley and Kleinberg 2010, 23.
54 De Nooy, Mrvar, and Batagelj 2005, 6–7; Peeples and Roberts Jr. 2013, 3002.

represented as an arrow, with the sender node at its tail and the receiver node at its head.[55] A special case is the loops, which are circular edge that connect a vertex to itself.[56]

In a graph with a directed network it is useful to examine not dyads, but triads, namely groups of three nodes and their links: these triads form the shape of a triangle and the edges connecting them can assume several possible combinations.[57] When the nodes of a triad are all connected to each other, it becomes a triadic closure,[58] to which also the clustering coefficient is connected. The clustering coefficient is the measure based on the probability that two entities are also linked to each other if they are both linked to a third entity.[59] Its calculation is based on the quantity of triads in the network,[60] and is given by the proportion between the neighbours of the examined node and the maximum number of edges possible between these neighbours.[61]

A network can be also binary or weighted. In a binary network the connections between entities are defined as either present (they have a value of 1) or absent (they have a value of 0), without considering the number of shared similarities that form their connections or links.[62] All the two-mode networks produced in the present work are binary, as e.g. Figure 1 in Chapter 7. There, all the edges between the sites and the types of beads have the same size, because they all have equal value (1); the number of contexts where each type of bead is found, or how many beads, is not taken into account, to diminish the risk of archaeological bias. In a weighted network, the connections or links are differentiated on the basis of how many similarities form each link.[63] A special case is the weight of line multiplicities, which are the lines created when the multiple lines of a bimodal graph are substituted by the single lines of a one-mode graph. In other words, the multiple lines that in the two-mode graph connect each pair of nodes of the same set through the nodes of the other set are replaced in a one-mode graph by a single line, whose value correspond to the number of those multiple lines.[64] This is the case with the first kind of one-mode networks produced in the present work, where the links between the sites are the sum of how many links they have to the same types of

55 De Nooy, Mrvar, and Batagelj 2005, 7.
56 De Nooy, Mrvar, and Batagelj 2005, 6–7.
57 De Nooy, Mrvar, and Batagelj 2005, 206–7; Scott 2017, 121.
58 Easley and Kleinberg 2010, 48–49.
59 Easley and Kleinberg 2010, 49; Newman 2001a; Newman 2010, 262–66.
60 Cline and Cline 2015, 36; Newman 2001a.
61 Brughmans 2013, 634; Newman 2001a; Newman 2010, 262–66.
62 Peeples and Roberts Jr. 2013, 3002; Peeples et al. 2016, 65–66; Scott 2017, 76–78.
63 Collar et al. 2015, 14; Newman 2004; Peeples and Roberts Jr. 2013, 3003; Peeples et al. 2016, 65–66.
64 De Nooy, Mrvar, and Batagelj 2005, 105.

objects in the bimodal graph. For example, in Figures 29-32 in Chapter 8, the size of the link between Edfu and Harageh is given by the sum of the types of stone vessels to which they are both linked in Figure 28 in the same chapter (Types 1, 6, 7, 23, 27, 30). The line multiplicity is also at the base of m-slices, which is a group of nodes whose edges have at least a determined value, as decided by the researcher: for example, an m-slice with a value of 3 includes all the nodes who have an edge of value 3 or higher.[65]

In archaeological research, binary networks can be founded on the presence/absence of particular types of objects or features, while weighted networks can be based, depending on the research questions and on the available data, on the number of sites or contexts in which a particular object or feature is retrieved, or on abundance, namely how many specimens are found in each context.[66] In archaeology, weighted networks can generally be preferable because they are more likely to give a nuanced picture that captures the complexity of the examined process.[67]

Network Analysis: A Step Further

The basic idea, common to all the fields where network analysis is applied, is that things, be that information, goods, technology, or anything else, travel across entities or nodes.[68] In detail, in the network visualized in a graph, the path is the sequence of edges followed to travel from a node to another, that is to say the itinerary used to travel from one node to another.[69] Alternatively, a path can also be defined as a sequence of nodes in which each pair is connected by an edge.[70] The length of a path is the number of steps, namely the sequence of edges, between two nodes and indicates also the strength of the relations between these nodes.[71] For example, in Figures 29-32 in Chapter 8, the connection between Qau el-Kebir and Ballas, created through types of stone vessels in common, is indirect and possible through two paths. One path goes from Qau el Kebir first to Matmar, then to Esna, and then to Ballas. The second path goes from Qau el Kebir directly to Esna, and then to Ballas. It is visible that the problem here is that no direct links, hence no types of stone vessels in common, are between Matmar and Ballas.

65 De Nooy, Mrvar, and Batagelj 2005, 109–10; Scott 2017, 125–26.
66 Peeples et al. 2016, 65–66.
67 Peeples and Roberts Jr. 2013.
68 Easley and Kleinberg 2010, 26.
69 De Nooy, Mrvar, and Batagelj 2005, 67; Scott 2017, 79–80.
70 Easley and Kleinberg 2010, 26.
71 Brughmans 2010, 289; De Nooy, Mrvar, and Batagelj 2005, 14; Easley and Kleinberg 2010, 32–33; Peeples and Roberts Jr. 2013, 3003.

The shortest path, that is to say the quickest itinerary or the quickest sequence of edges between pair of nodes, is called geodesic.[72] Going back to the previous example, in Figures 29-32 in Chapter 8, the geodesic between Qau el-Kebir and Ballas is the one that passes directly through Esna, skipping Matmar. To this is connected the average path length or average geodesic distance, which is the length of the average path between the entities in a network. This measure is useful because it helps understand how much connected a network is and how efficiently its entities communicate.[73] Furthermore, in a network it is possible to calculate the maximum geodesic distance, or diameter of the network, which is the length of the shortest path between the two entities that are the farthest from each other in that network.[74]

The network represented in a graph is said to be connected if for each pair of nodes there is a path connecting them.[75] Furthermore, in a graph also connected components can be distinguished. A connected component is a group of nodes where each node has a path to the other nodes of the group, but not to a larger group of nodes.[76] This is useful to analyse the internal structure of a network.

In graphs with directed networks, a path and a semi-path are also distinguished. While in paths the direction of the arcs is taken into consideration, and the all the arcs have to point in the same direction, so that each node is at head of an arc and at the tail of another, this does not happen for the semi-path.[77] Hence, a group of nodes is said to be weakly connected if its connections are all made of semi-paths, while it is said to be strongly connected if its connections are all made of paths.[78]

Moreover, in a graph with a weighted network, the strength of an edge, based on the quantity of similarities shared by the two nodes that it connects, can be indicated also by the thickness of the same edge.[79] This strength can also be visualized as a number near each edge.[80] To return to Figures 29-32 in Chapter 8, the thickness of the edges is given by how many types of stone vessels they have in common. Thus, the software programs for network analysis allow to adjust the thickness, or even the colour, of the edges on the basis

72 De Nooy, Mrvar, and Batagelj 2005, 126–27; Newman 2001b; Scott 2017, 79.
73 Cline and Cline 2015, 34; Newman 2010, 55–56.
74 Cline and Cline 2015, 34; De Nooy, Mrvar, and Batagelj 2005, 127; Newman 2010, 136–40.
75 Easley and Kleinberg 2010, 28–29.
76 Easley and Kleinberg 2010, 29–30.
77 De Nooy, Mrvar, and Batagelj 2005, 67; Scott 2017, 79–80.
78 De Nooy, Mrvar, and Batagelj 2005, 68.
79 Brughmans 2010, 291; Brughmans 2013, 626–28.
80 De Nooy, Mrvar, and Batagelj 2005, 7.

of the number of similarities forming the links and on how weak or strong they are.

The set of links and paths detected create different types of network. One of them is the so-called lattice network, where all entities are equally coupled to each other: this is rare in real life but can be used to model networks in which sites have relations only with their immediate neighbours. In these networks, the distance of the links is short, and all entities have the same importance, thus it is an egalitarian network where relations are short-distance and limited to immediate neighbours.[81]

Another possible type of network is the so-called small-world network, which is like the lattice network, but less egalitarian. It is formed by groups of entities densely linked at an intra-group level, but weakly linked at an inter-group level.[82] This means that inside each group and on a short distance the entities are well connected, while on a long distance each group is connected to other groups only by a few bridging entities.[83]

The features defining a small-world network are a short average geodesic distance and a high clustering coefficient.[84] This means that relations are mostly short-distance and that all the entities, regardless of the number of relations they establish, have a similar, though not equal, importance in the network.[85] Moreover, there is a redundancy of path, meaning that pairs of entities have more paths allowing them to reach each-other.[86] Lastly, small-world networks are especially susceptible to the setting of thresholds – in other words, to the setting of minimum values that the nodes are required to have to remain in the network – because a too low threshold can make the network look sensibly more connected that what it actually is, while a too high threshold can make it look too disconnected and only made of separate small groups.[87]

The small-world network and its characteristic are related to what is known as small-world phenomenon, meaning that an entity can reach through a short path even entities far in the network, through common links.[88] In this context, an interesting role is covered by bridges and cut-vertices. A bridge is an edge and a cut-vertex is a node whose removal creates new, isolated groups

81 Östborn and Gerding 2015, 311–12.
82 Cline and Cline 2015, 32–37; Collar 2014, 99-100; Östborn and Gerding 2015, 311–12; Scott 2017, 160–61.
83 Brughmans 2010, 277; Collar 2014, 99-100; Östborn and Gerding 2015, 311–12; Sindbæk 2007b, 61.
84 Cline and Cline 2015, 34; Newman 2010, 55–56; Östborn and Gerding 2015, 311–12.
85 Östborn and Gerding 2015, 311–12.
86 Scott 2017, 160.
87 Scott 2017, 160–61.
88 Easley and Kleinberg 2010, 35–37.

in the network, thus increasing their number.[89] Therefore, a bridge is an entity that act as an intermediary between two groups, without being really part of any of them.[90]

A special type of bridge is the so-called local bridge, which connects two entities that are not connected through any other entity.[91] The neighbourhood overlap is useful to detect local bridges, because this measure is 0 for pure local bridges, so that the nearer this measure is to 0, the more probably the edge is a local bridge.[92]

A further type of network is the so-called scale-free, where only a few entities are highly connected to the others, establishing many and strong links, while the remaining ones are peripheral; the relations can cover short or long distances.[93] This means that the highly connected entities are also the probable driving powers in the spreading of innovations, because they have the links to transmit it to the poorly-connected entities,[94] as explained by the power law distribution. The power law distribution can be detected by comparing the degree centrality – a mathematical algorithm which will be explained in detail later – of the nodes of a network. If the nodes follow a power law distribution, the network features few nodes with very high degree centrality, while the majority of the nodes have a very low degree centrality. Moreover, scale-free networks are robust against failure. In other words, if one of the entities were excluded or ceased to be part the network, the structure of the network would not change, and the network would not collapse.[95]

One more type of network is the so-called random network, where entities are joined through random links or through probability.[96]

Lastly, in a graph it is possible to also examine the ego-network of a node, which includes the examined node, its adjacent nodes and all the edges connecting them.[97] Ego-networks offer the possibility of a multi-scalar analysis of site assemblages, by allowing to zoom in and focus on single elements, such as single artefact types.[98] As an example, Sindbæk has examined the ego-networks of steatite vessels in his study of Northern Europe in the Viking era.[99]

89 De Nooy, Mrvar, and Batagelj 2005, 140; Easley and Kleinberg 2010, 51–53.
90 Scott 2017, 120.
91 Easley and Kleinberg 2010, 51–53.
92 Easley and Kleinberg 2010, 57–58.
93 Brughmans 2010, 277; Collar 2014, 100-1; Sindbæk 2007b, 61–62.
94 Collar 2014, 100-101; Östborn and Gerding 2015, 332.
95 Sindbæk 2007b, 62.
96 Östborn and Gerding 2015, 311–12.
97 Brughmans 2013, 634; De Nooy, Mrvar, and Batagelj 2005, 145–46.
98 Collar et al. 2015, 10.
99 Sindbæk 2013, 78–81.

Affiliation and diffusion networks, and their application

In archaeological research, two-mode networks often belong to the so-called affiliation networks. In social network analysis, one set of the entities of the affiliation networks is usually formed by people, while the other set is formed by their shared membership in groups or participation in common events.[100] In archaeology, one set of entities can be composed of the sites or contexts, or even categories of people,[101] sharing objects with similar features such as fabric, technology, shape, decoration, while the other set can be composed of the shared objects.[102] The problem is that these affiliation networks, like other networks analysed in archaeological research, do not offer precise and unequivocal parameters for the analysis.[103] In other words, it is not possible to connect the entities, namely the sites, through simple directional links, where artefacts originate in a place and from there are transported somewhere else. It is possible though, to detect currents or trends of comparable material.[104] This is the case with the networks produced in the present work, where the types of objects are not considered to originate at a site and be brought from there to another site. To go back to an older example, in Figures 29-32 in Chapter 8, Qau el-Kebir and Esna are linked, or affiliated, because they share two types of stone vessels (Types 6 and 12), but it is not considered if the objects originated at one of the sites and ended up at the other one.

Similar to the affiliation networks are the so-called diffusion networks, where entities are linked when their connection could eventually lead to the spread of an innovation from one of them to the other.[105] Thus, in archaeology the entities are the sites were a particular innovation is found, while the links in the graphs represent the contacts through which the innovation is transmitted.[106] These contacts change with time, as they can appear, disappear or become weaker or stronger.

Nevertheless, it should be kept in mind that, because of the nature of network analysis in archaeology, in affiliation and diffusion networks the connections shared between sites does not necessarily mean direct contact, nor one-to one transfer of knowledge or material: the similarities detected could have reached the sites in a more indirect way, which cannot be knows because

100 Brughmans 2013, 627; De Nooy, Mrvar, and Batagelj 2005, 101–20; Easley and Kleinberg 2010, 93–95.
101 Knappett 2011.
102 Brughmans 2013, 638–39; Sindbæk 2013, 74–76.
103 Brughmans 2013, 638–39; Sindbæk 2013, 76.
104 Brughmans 2013, 638–39; Sindbæk 2007b, 66; Sindbæk 2013, 82.
105 De Nooy, Mrvar, and Batagelj 2005, 161–84.
106 Östborn and Gerding 2015, 309–10.

of lack of data.[107] The connections mostly indicate that sites having many similarities share a cultural affinity and are, therefore, more likely to have been in any sort of relation than sites having few or no similarities.[108] Furthermore, when in the graph of a weighted networks the links are thick, namely have a large number of similarities forming them, the connections could be considered significant.[109] For example, in Figures 29-32 in Chapter 8, Edfu and Harageh have six types of stone vessels in common. Even though we cannot say how the types reached the sites, this tells us that these sites have a stronger connection than between Qau el-Kebir and Esna, which share only two types of stone vessels. The difference is rendered visually: the link joining Edfu and Harageh is thicker than the one joining Qau el-Kebir and Esna.

There are several processes that can be involved in affiliation and diffusion networks and, therefore, explain exchanges of goods and ideas between sites; they include trade, movements of people, local imitations, and similar parallel developments.[110] Nevertheless, the fundamental elements of these processes are communication and contacts between sites, which can be expected to be closer between sites of the same region, because these are entities of the same group, than between sites belonging to different regions, because these are entities belonging to different groups.[111] This is one of the reasons why looking for traces of this communication can help understand which places had closer interaction and, thus, were part of the same group, namely of the same region.[112] Furthermore, conducting network analysis on a larger scale, that is to say on a regional scale, gives a wider image and can help understanding which processes are at play and how the sites interacted.[113]

This is the reason why network analysis has been used also to study regionalization, such as in Knappett's research on Greece,[114] and in Blake's research on pre-Roman Italy.[115] In detail, Knappett has used material culture to examine relations on different scales, starting from the face-to-face ones happening inside a community, and ending in the regional one. Blake has detected the origins of two regions, the Etruscan and the Latin ones, on the

107 Östborn and Gerding 2015, 309–10.
108 Blake 2013, 211; Golitko and Feinman 2015, 216; Mills, Roberts Jr., et al. 2013, 187; Östborn and Gerding 2014, 75, 81; Peeples and Roberts Jr. 2013, 3003; Peeples et al. 2016, 61, 66; Sindbæk 2007b, 66; Sindbæk 2013, 74, 82.
109 Golitko and Feinman 2015, 216; Peeples and Roberts Jr. 2013, 3003.
110 Peeples and Roberts Jr. 2013, 3003; Peeples et al. 2016, 61.
111 Peeples and Roberts Jr. 2013, 3003; Peeples et al. 2016, 61; Sindbæk 2007b, 66; Sindbæk 2013, 73.
112 Blake 2013, 205.
113 Golitko and Feinman 2015, 237; Peeples and Roberts Jr. 2013, 3003; Peeples et al. 2016, 61; Sindbæk 2007b, 66; Sindbæk 2013, 73.
114 Knappett 2011.
115 Blake 2013.

basis of the two different kinds of network created by the sites of each region, showing how the use of similar objects can signify tighter communications between sites and, as a consequence, their belonging to the same regional group. Moreover, regions are defined on the basis of detected networks also in Coward's research on regional groups in the Near East during the Neolithic and Epipalaeolithic periods,[116] while the exchange of obsidian has been used to study the network in Mesoamerica between 900 BC and AD 1250.[117] Furthermore, Collar has studied epigraphic data through network analysis to examine the Jewish diaspora after the destruction of the temple of Jerusalem, showing both how the need to affirm the Jewish identity was felt after the loss of the temple, and how network analysis can be useful in studying ethnicity.[118] Lastly, Sindbæk has used network analysis to study the regional interactions and the role of towns in Northern Europe during the Viking era.[119]

In Egyptology, network analysis has been applied in very few studies. One of these is the research conducted on the network of kings and vassals in the Near East in the Late Bronze Age, as re-constructible from the data retrieved from the Amarna letters.[120] A second study has detected, from written documents, the network of the members of the royal family and court in the Old Kingdom, to study both how the ties in this network affected the career of its members and the distribution of power, and eventual cases of 'nepotism'.[121] Another study has analysed the network connected to a bishop, to better understand the development of the Coptic church based in the Theban region.[122] Lastly, a project has collected all the data related to the personal names found on papyri, in several languages, from Greco-Roman Egypt. This project makes them available on an online platform and has applied the methodologies of network analysis to several of them.[123]

Measuring the network

Measures that can be calculated to study and interpret the graphs are various, but the most used are the so-called centrality measures, which analyse the role and value of each entity in a network;[124] in most cases they do not seem

116 Coward 2010; Coward 2013.
117 Golitko and Feinman 2015.
118 Collar 2013; Collar 2014.
119 Sindbæk 2007a; Sindbæk 2007b; Sindbæk 2013.
120 Diane Harris Cline 2015; Cline and Cline 2015.
121 Dulíková and Mařík 2017.
122 Dekker 2016; Dekker 2018.
123 Broux 2017.
124 Cline and Cline 2015, 29.

to depend on the size of the network.[125] These measures can also be visualized in the graph: in the same way that is possible to adjust the thickness and colour of the edges, it is also possible to give different colours, sizes and even shapes to the nodes, on the basis of how they score in the measure taken into consideration.[126]

One of the centrality measures is the closeness centrality, which indicates how easily an entity reaches the others and can be reached by them.[127] It is based on the total distance between the examined entity and all the other entities in the network; it is calculated by dividing the number of entities in the network by the sum of all distances between the examined entity and the all the other entities.[128] From the closeness centrality it is possible to derive the closeness centralization, by dividing the closeness centrality scores of all the entities of a network by the maximum variation in closeness centrality scores possible in the same network.[129]

A further centrality measure is the betweenness centrality, which shows how important the examined entity is as intermediary between two other entities, as well as to what extent it is needed as a linking element in the chains of contacts in a network[130] and how much flow passes through it and its links.[131] The betweenness centrality of an entity is based on its position in the network and on how short or long the geodesics are between pairs of entities whose connection passes through the examined entity, as well as on the geodesics both between each pair of entities and between the two ends of the network.[132]

In detail, the betweenness centrality of an entity measures how often the examined entity is on a geodesic between other entities[133] and is calculated by making a proportion of all the geodesics that include the examined entity,[134] or by first calculating how much flow arrives to the examined node from each of his neighbours, summing it up and adding one, then dividing the result by the edges leaving the examined node.[135] In a weighted network, this measure

125 Sindbæk 2007b, 67.
126 Cline and Cline 2015, 24.
127 Brughmans 2010, 296; Brughmans 2013, 636–38; Mills, Roberts Jr., et al. 2013, 186.
128 Brughmans 2013, 636–38; De Nooy, Mrvar, and Batagelj 2005, 127; Mills, Roberts Jr., et al. 2013, 186.
129 De Nooy, Mrvar, and Batagelj 2005, 127.
130 Brughmans 2010, 296; Brughmans 2013, 636–38; De Nooy, Mrvar, and Batagelj 2005, 127; Mills, Roberts Jr., et al. 2013, 186; Peeples and Roberts Jr. 2013, 3005; Scott 2017, 99–100.
131 Easley and Kleinberg 2010, 73–76; Newman 2001b.
132 Brughmans 2010, 296; Brughmans 2013, 636–38; Mills, Roberts Jr., et al. 2013, 186; Newman 2001b; Peeples and Roberts Jr. 2013, 3005; Scott 2017, 100.
133 Cline and Cline 2015, 32–33; Newman 2001b; Newman 2010, 185.
134 De Nooy, Mrvar, and Batagelj 2005, 127; Scott 2017, 100.
135 Easley and Kleinberg 2010, 81–82; Newman 2001b; Newman 2004, 4–5.

is mostly based on the weight of the links,[136] following the assumption that more similarities forming a link mean more contacts and, thus, a lower cost to maintain them.[137] Furthermore, betweenness centralization can be calculated by dividing the betweenness centrality scores of all the entities of a network by the maximum variation in betweenness centrality scores possible in the same network.[138]

Another centrality measure here introduced is the degree centrality. It indicates how important an entity is, on the basis of the number of its connections, and it is calculated by counting how many links the examined entity has.[139] Though the importance of an entity in a network is not always revealed by the degree centrality,[140] this measure can actually be informative: for example, in a scale-free network it can show a group of important sites that could have been better linked to others and, thus, more influential in the spreading of innovations.[141] This can be seen in Östborn and Gerding's study of the diffusion of fired bricks in Hellenistic Europe,[142] as well as in the study of networks in pre-Hispanic US Southwest,[143] which is based on similarities in specific types of ware and obsidian objects.

In a binary network, degree centrality is simply formed by the number of links established by each entity, while in a weighted network this measure is formed by the sum of the weights of the links established by the examined entity.[144] In a directed network, it is possible to calculate also the indegree measure and the outdegree measure of an entity. The indegree measure of an entity is the number of arcs whose receiver is the examined entity, namely the number of arrows in the graph that point towards the examined node, while the outdegree is the number of arcs whose sender is the examined entity, namely the number of arrows in the graph that start from the examined node.[145] Furthermore, from degree centrality it is possible to calculate the degree centralization of a network, which is the proportion between the degree

136 Newman 2004, 4–5; Peeples and Roberts Jr. 2013, 3005.
137 Peeples and Roberts Jr. 2013, 3005.
138 De Nooy, Mrvar, and Batagelj 2005, 131.
139 Brughmans 2010, 296; Brughmans 2013, 636–38; Cline and Cline 2015, 29; De Nooy, Mrvar, and Batagelj 2005, 63–64; Mills, Roberts Jr., et al. 2013, 186; Newman 2010, 168; Peeples and Roberts Jr. 2013, 3005; Peeples et al. 2016, 63.
140 Cline and Cline 2015, 30.
141 Östborn and Gerding 2015, 332.
142 Östborn and Gerding 2015.
143 Mills, Clark, et al. 2013; Mills, Roberts Jr., et al. 2013; Peeples et al. 2016.
144 De Nooy, Mrvar, and Batagelj 2005, 63–64; Peeples and Roberts Jr. 2013, 3005; Peeples et al. 2016, 63.
145 De Nooy, Mrvar, and Batagelj 2005, 64; Scott 2017, 79–80.

centrality scores of the entities in a network and the maximum variation of degree centrality scores that the same network could contain.[146]

The degree measure is used in network analysis also to the detect k-cores, the structural cohesion of a network, and the power law distribution. A k-core is a group of nodes, or sub-network, associated by a least degree measure, so that in a network it is possible to determine several k-core groups based on the degree scores found.[147] Thus, k-cores are based on the number of links, because the degree measure is based on that:[148] they are useful to understand if nodes with a high degree are clustered or more sparse in the network.[149] The structural cohesion, or density, of a network represented in a graph is derived from the average degree of all the entities of the same network.[150] It does not depend on the size of the network, so that it can be compared between networks of different sizes.[151] The power law distribution is measured by comparing the degree centrality scores of the nodes included in a network and by detecting if this is more or less evenly distributed or if some nodes have more ties and can be more influential in a network.[152]

The last of the most used centrality measures is the eigenvector centrality, which indicates the influence of an entity in a network. It is based on the principle that entities have their importance increased by the connection to other entities that are themselves important in the network.[153] In other words, the importance of an entity is not based on the quantity of its connections, as in the degree centrality, but on the quality of these connections and on the importance of the entities with which it is linked.[154] Furthermore, in a weighted network, also the strength of each link is included in calculating the eigenvector centrality.[155] This measure is useful in larger graphs because it is calculated taking the entire network into consideration.[156] Also the eigenvector centrality can be very informative in examining the role of entities. For example, in a diffusion network where the sites all have an equal role in spreading the innovation, the eigenvector centrality is the measure that actually reveals

146 De Nooy, Mrvar, and Batagelj 2005, 126.
147 Brughmans 2010, 291; De Nooy, Mrvar, and Batagelj 2005, 70–72; Scott 2017, 127–30.
148 Brughmans 2010, 291.
149 De Nooy, Mrvar, and Batagelj 2005, 70–72.
150 De Nooy, Mrvar, and Batagelj 2005, 63–64; Newman 2010, 134.
151 De Nooy, Mrvar, and Batagelj 2005, 63–64.
152 Cline and Cline 2015, 35; Collar 2013.
153 Brughmans 2013, 636–38; Mills, Roberts Jr., et al. 2013, 187–88; Newman 2010, 169–172; Peeples and Roberts Jr. 2013, 3005; Peeples et al. 2016, 63.
154 Cline and Cline 2015, 30–31; Newman 2010, 169–72.
155 Mills, Roberts Jr., et al. 2013, 187–88; Peeples and Roberts Jr. 2013, 3005; Peeples et al. 2016, 63.
156 Peeples et al. 2016, 63.

how the sites have all the same importance, as also addressed in the study on networks in pre-Hispanic US Southwest.[157]

Furthermore, it is possible to measure also the density of a network, which is the proportion between the links actually present in the graph visualizing the network and the maximum number of links that the same graph could contain;[158] when the density is at its maximum, the network visualized in the graph is called complete,[159] meaning that each entity is connected to all the other entities of the network.[160] This measure indicates the general connectedness of the network visualized in a graph,[161] and depends on the dimension of the network, so that to compare different networks is better to use the structural cohesion.[162]

Another measure, which is applied to edges, is the neighbourhood overlap. It is calculated by dividing the number of nodes neighbouring both the nodes that the examined edge connects by the number of nodes neighbouring at least one of those nodes.[163] Related to this measure is the embeddedness of an edge, which is made of the number of nodes adjacent to both the nodes linked by the examined edge.[164]

Lastly, it is possible to apply partition to the graph. Partition is achieved by grouping the entities in the network, allotting each node to a group on the basis of a specific property resulting from measures calculated in the network analysis, or on the basis of an attribute registered in the database and independent from these measures.[165]

METHODOLOGY OF THE PRESENT RESEARCH

In the present work, the entities are the sites where one or more contexts have been dated to the Late Middle Kingdom and the Second Intermediate Period, as well as objects found in these contexts, which include beads, stone vessels, scarabs, and weapons. Pottery is too extensive a material to insert in the research at the present stage, thus only very distinctive types, such as the Tell el-Yahudiyah ware and imports and imitation of Cypriot pottery are included in the analysis.

157 Peeples et al. 2016, 66–67.
158 De Nooy, Mrvar, and Batagelj 2005, 63; Easley and Kleinberg 2010, 120; Scott 2017, 81–84.
159 De Nooy, Mrvar, and Batagelj 2005, 63; Easley and Kleinberg 2010, 120; Scott 2017, 84.
160 Easley and Kleinberg 2010, 120.
161 Scott 2017, 81–84.
162 De Nooy, Mrvar, and Batagelj 2005, 64; Scott 2017, 87–89.
163 Easley and Kleinberg 2010, 57–58.
164 Easley and Kleinberg 2010, 65–66.
165 De Nooy, Mrvar, and Batagelj 2005, 31–32.

The databases

Every object, or group of objects, of the same shape and material is considered a type, and is an entry in the database, where each column report one of its attributes. Each class of objects has its own database, and the attributes reported in the columns vary depending on the category of objects. The only attributes that remain constant in these databases are: the material that each type is made of, in the column 'Material'; the site and context where each type has been found, respectively reported in the columns 'Place' and 'Context'; the chronological phase to which the archaeological context/contexts, where each type is found, is/are dated, in the column 'Dating'; the publications where the objects are mentioned, in the column 'Bibliography'.

Where applicable, other columns are added to the database. One of these columns is 'Object', which reports the use of the objects: in the analysis of the beads it mentions if the objects are classified as beads, amulets, or pendants in the publications. While the use of the objects is not taken into consideration in the analysis, because it is not always clear and the publications are not sufficient to determine it, when clearly stated in the publications it has been included in the database for sake of completeness. When an already existing typology has been followed in the present work, such as for instance in the case of the scarabs, the Tell el-Yahudiyah ware, and the Cypriot pottery, another column added to the database is 'Type', which reports the number or denomination of the type under which the object can be classified according to the existing typology. The database of the Cypriot pottery has also a column where is specified if the specimens are locally made in Egypt or imported, because the two groups give different information and, therefore, have been considered separately in the analysis. The columns 'Object' and 'Type' are present also in the database of the weapons. They respectively specify the use of the weapons, which is taken into consideration in the analysis because it makes a significant difference between the weapons, and the types to which their belong: these types follow the classification specifically constructed for the present research.

Finally, other columns are specific to the classes of objects. For instance, the database of the beads also contains the columns 'Shape' and 'Colour', were the shape and the colour of each type are reported: the colour is actually mentioned only for sake of completeness, but is not taken into consideration in the analysis, because it is not always recognizable and does not seem to have any particular significance for the purpose of the present work. The database of the stone vessels contains the columns 'Body', 'Rim', and 'Base', where the main parts of each type of vessel are described, as explained in the relevant chapter. The database of the scarabs contains the column 'Head, back', where, when possible, the shape of the objects is described, according to an existing typology, as explained in the relevant chapter: however, this attribute is not taken into account in the analysis, because the available data are not sufficient. The database of the Tell el-Yahudiyah ware contains the column 'Fabric', where the fabric of the vessels is reported. Lastly, the database of the weapons contains the columns 'Attachment' and 'Blade', which respectively describe how the weapons were attached to their haft or shaft and the shape (and possible decoration) of their blade.

The matrices

From each database, three types of matrices have been derived, to generate the three different types of networks and graphs needed to answer the research questions of the present works. Each type of matrix has always been divided into the three phases studied in the present work, namely the Late Middle Kingdom, the Early Second Intermediate Period, and the Late Second Intermediate Period. Furthermore, on the contrary of the database, the structure of each type of matrix is the same for all the categories of objects.

The first type of matrix is a binary two-mode matrix, where each row corresponds to one of the sites examined and each column corresponds to one of the objects found at the sites. In this matrix, each cell reports the presence or absence of the objects corresponding to the column at the site corresponding to the row. Considering the quantity of contexts or the abundance in the analysis has not been preferred in the present work, because of the incomplete data available at present. While these data can be sufficient to consider the simple presence of absence of a type of object at a site, that is not the case when considering the number of contexts or the number of specimens inside each context.

The second type of matrix is weighted and one-mode. In this matrix, each row and each column both correspond to a site, and each cell reports how many types of objects are shared by the site corresponding to the row and the site corresponding to the column. Another option would have been reporting in an unweighted matrix only if similarities are present or absent, but this has been tried and has not given insightful results.

The third type of matrix is again one-mode but is based on the similarity index. In other words, the structure of this matrix is similar to the previous ones but, instead of reporting the number of types of objects shared, the cells report the similarity, numerically expressed, between the types of objects of the site of each row and of the site of each column. This also implies that, while the second matrix only considers part of the material culture, this third matrix considers the full range of the material culture. To obtain a similarity index, the two-mode matrices have been subjected to similarity analysis in the PAST program. The similarity index is a statistical method that measures and ranks how similar the entities, corresponding to the rows of the matrix, are to each other on the bases of their attributes, corresponding to the column of the matrix: the scores range from 0, which is when absolutely no similarity is detected, to 1, which is the similarity that each site has to itself.

In our analysis, the sites are the entities to rank, and the objects are the attributes on which to base the ranking. Thus, the two-mode matrices used for the network analysis have been used also to calculate the similarity index, because the sites are reported in the rows and the objects are reported in the columns. There are several algorithms available to calculate the index similarity. Among these, in the present work the so-called Jaccard[166] has been used, which ranks the entities on the basis of the number of similar attributes shared, without taking into consideration their abundance, namely how many times each attribute is found for each entity. This statistical method, which was first used in botanical studies to examine the floral distribution in the Alps, has been chosen because its binary (i.e. presence/absence) nature is more fitting to the binary nature of the two-mode matrix and to the nature of the data available. It has also been used in archaeology, as for instance in the analysis of Neolithic networks in western Anatolia, the Balkans, and the Aegean,[167] where the number of sites is small, as in the present research.

The networks and the graphs

From the described matrices, ORA has been chosen as program to visualize the graphs of the networks, which are all undirected, because at the present stage it is not yet possible to recognize sending and receiving entities. The four centrality measures have been applied to all the graphs, so that each of the two one-mode graph has four versions, and in each one of them the size of the nodes is calibrated on one of the measures. In the graphs in the present work, the thickness of the links has also been calibrated on the weights, namely the number of shared similarities between the entities, or on the similarity index reported in the matrices.

From the first one-mode matrix, graphs based on one-mode weighted networks have been created. In the networks visualized in these graphs, the entities are the sites, and the links are based on the number of objects shared. This allows to focus merely on the connections between the sites and allows us to test the connections detected in the two-mode graphs.

From the third type of matrix, graphs based on one-mode unweighted networks have been again elaborated, where the entities are again the sites, but in this case the links are based on the similarity index between each pair

166 Jaccard 1912.
167 De Groot 2019.

of sites. This helps to understand which sites could be more related based on a similar material culture, because the similarity index is based on the full range of material culture and is useful to test the results of the previous graphs.

THE ANALYSIS OF THE MEASURES

In the following subsections, different aspects of the analysis are discussed: the measures in the first one-mode graph, the measures in the second one-mode graph, and the ranks.

The measures in the first one-mode graph

The four centrality measures analysed for the first one-mode graph include the degree centrality, the betweenness centrality, the eigenvector centrality, and the closeness centrality. The degree centrality is based on the number and strength of the links established by each site.[168] This means that this measure takes into consideration both the number of sites with which the examined site has more types of objects in common, and the number of the types of objects shared. Thus, this measure shows which sites have more objects in common with the larger number of sites and, as a consequence, have the stronger connections and could be considered the ending or starting point of the flow of communications and of the trend observed in the material culture. In Figure 2 and in Table 25 in Appendix II, for example, the degree centrality of Abydos calculates with how many other sites of the Late Middle Kingdom it has types of beads in common, and how many types are shared, and the size of its icon in the graph depends on this calculation.

The betweenness centrality focuses on how important each entity is as an intermediary in the relation between two other entities, thus how important each site is in bringing two other sites together.[169] This measure, then, shows important centres that could be passageways or (re)distribution centres.[170] In the first one-mode graph, this path is determined by the types of objects in common between the sites, with the idea that if two related sites, with similar types of objects, were passing by a third site to communicate with each other, or were in any way connected through a third site, there would be a trail in the material culture of this third site: the trail would be made of part of the types of objects in common between the first two sites. Therefore, the betweenness

168 Brughmans 2010, 296; Brughmans 2013, 636–38; De Nooy, Mrvar, and Batagelj 2005, 63–64; Newman 2010, 168; Peeples and Roberts Jr. 2013, 3005; Peeples et al. 2016, 63.
169 Brughmans 2010, 296; Brughmans 2013, 636–38; De Nooy, Mrvar, and Batagelj 2005, 127; Peeples and Roberts Jr. 2013, 3005; Scott 2017, 99–100.
170 Gjesfjeld 2015; Rivers, Knappett, and Evans 2013.

centrality informs on which sites could be the most likely joining points in the communication between other sites. For example, in Figure 3 and Table 38 in Appendix II, the betweenness centrality of Abydos is based on how many sites of the Late Middle Kingdom are linked through it in the network of beads.

The eigenvector centrality is based on how connected the sites are to the sites more important to the network.[171] In detail, this measure informs here about which sites established the connections of better quality, as based on the types of objects shared among them. For instance, in Figure 4 and Table 51 in Appendix II, the eigenvector centrality of Abydos shows with how many sites, important in the network of beads during the Late Middle Kingdom, it was connected.

The closeness centrality shows the sites that could be reached more easily through the connections established in the network.[172] In detail, closeness centrality focuses on how reachable the sites were on the basis of the connections are as detected through the types of objects shared among them. For instance, in Figure 5 and Table 61 in Appendix II, the closeness centrality of Abydos informs about how easy it was for the other sites of the Late Middle Kingdom to reach it in the network of beads.

The measures in the second one-mode graph

The same measures analysed for the first one-ode graph are analysed also for the second one-mode graph, based on the Jaccard similarity. The first centrality measure analysed is the degree centrality, which is based on the strength of the similarity in material culture between the sites. In other words, this measure considers how the full range of objects found at each site is similar to the found at the other sites, hence showing which sites display the higher degree of similarity in material culture with the higher number of sites. This means that the higher the score, the higher the number of sites with which it is similar. For example, in Figure 6 and Table 77 in Appendix III, the degree centrality of Dahshur calculates with how many sites of the Late Middle Kingdom it has a connection, while each connection is based on the similarity in the overall range of beads between Dahshur and the sites connected to it.

The betweenness centrality is based on the path that connects the sites through the similarity in their material culture, and it shows which sites can be detected as major intermediaries in these paths. That is to say that this measure determines which sites display such a similarity in their full range

171 Brughmans 2013, 636–38; Newman 2010, 169–72; Peeples and Roberts Jr. 2013, 3005; Peeples et al. 2016, 63.

172 Brughmans 2010, 296; Brughmans 2013, 636–38.

of objects with other sites, that they could be the more important sites in bringing together the other ones. For instance, in Figure 7 and Table 90 in Appendix III, the betweenness centrality of Dahshur is based on how many sites of the Late Middle Kingdom are linked through it, while each link is based on the similarity in the full range of beads between the sites that it connects.

The eigenvector centrality focuses on how connected the sites are to the sites more important in the network, based on the similarity in their material culture. Therefore, this measure informs us about which sites established the connections of better quality, based on the similarities in their full range of objects. For instance, in Figure 8 and Table 103 in Appendix III, the eigenvector centrality of Dahshur shows with how many sites of the Late Middle Kingdom important in the network it is linked, and each link shows the similarity in the full range of beads between Dahshur and these important sites.

The closeness centrality shows the sites that were more reachable through the links established in the network, as detected through the similarity in their material culture. In detail, this measure focuses on how reachable the sites were through the connections, established based on the similarities in their full range of objects. For example, in Figure 9 and Table 116 in Appendix III, the closeness centrality of Dahshur informs about how easy it is for the other sites of the Late Middle Kingdom to reach it, while each link is based on the similarity in the full range of beads between the sites that it joins.

The ranks

To better analyse the difference between the sites, the scores obtained by the sites for each measure are divided into five ranks, as conceived by the author of the present work: VH (=Very High), H (=High), M (=Middle), L (=Low), and VL (=Very Low). To create the ranks, at first the scale has been calculated, by dividing by five the difference between the highest and the lowest values scored by the sites. Then, the scale has been added five times to the lowest value scored by the sites, to calculate the lowest and highest score, namely the range, of each rank. For example, in Table 25 in Appendix II, 277 is the highest score detected for the degree centrality of the sites of the Late Middle Kingdom in the network of beads, while 28 is the lowest. At first, 28 has been subtracted from 277, giving 249. Successively, the number 249 has been divided by five, giving 49.8, which is the scale. Then, 49.8 has been added to 28 five times, giving the results 77.8, 127.6, 177.4, 227.2 and 277. These five results are the upper borders of the five ranks.

THE CORRESPONDENCE ANALYSIS

Given that the data in the analysis are only a sample, is it important to understand if the data available from each site can influence the results of the analysis. With this aim, the correspondence analysis is used in the present work. Correspondence analysis is an analytical tool provided by PAST, the same software used to calculate the Jaccard similarity for the second one-mode graphs in the present work. It examines the pattern between two sets of variables, namely between the data in the rows and the data in the columns of a table.

In the present work, the correspondence analysis examines the relations between the variety of types retrieved at the sites and how the same sites score for the different measures. Given that the measures for the two types of one-mode graphs sometimes differ in a remarkable way, the correspondence analysis is conducted for each one of them. More specifically, the columns report the ranks detected for the measures, so that each column corresponds to one of these ranks. The following columns are included in the table: VHD (Very High Degree centrality), HD (High Degree centrality), MD (Middle Degree centrality), LD (Low Degree centrality), VLD (Very Low Degree centrality), VHB (Very High Betweenness centrality), HB (High Betweenness centrality), MB (Middle Betweenness centrality), LB (Low Betweenness centrality), VLB (Very Low Betweenness centrality), VHE (Very High Eigenvector centrality), HE (High Eigenvector centrality), ME (Middle Eigenvector centrality), LE (Low Eigenvector centrality), VLE (Very Low Eigenvector centrality). The ranks of the closeness centrality have not been included in the table, because this measure is mostly similar for all the sites and has not proven to be very informative so far.

The rows of the table number five in total; they group the sites based on the number of types found at each site. In detail, the smallest number reported has been subtracted from the largest one. Then, the remaining number has been divided into five, to determine the scale. This scale has then been added five times to the smallest number of types, to calculate the lowest and highest number of each group. Five groups have been assigned, starting from the one with the sites having the lowest number: very low variability (VLV), low variability (LV), middle variability (MV), high variability (HV), very high variability (VHV).

For example, for the Late Middle Kingdom the largest number of types included in the analysis is 103, from Harageh, while the smallest is three, from Ain Asil. Thus, three has been subtracted from 103, and the remaining 100 has been divided into five: the result is of course twenty. Then, twenty has been added five times to three. In short, each cell reports how many of the sites belonging to the group of each row belong to the rank represented by the column. For each phase, one table has been created with the results of the first one-mode graph, and one table has been created with the results of the second one-mode graph. Then, each table has been imported in PAST and examined with the tool of the correspondence analysis.