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The many faces of online learning

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The Many Faces of Online Learning

Dirk van der Hoeven

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The Many Faces of Online Learning

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