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Measures and matching for number systems

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*“Sarebbe questa la libertà?
Esistere, lentamente, dolcemente,
come questi alberi,
come una pozza d’acqua,
come il sedile rosso del tram.”*
Jean-Paul Sartre, *La nausea*

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