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Global fields and their L-functions

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Citation

Solomatin, P. (2021, March 2). *Global fields and their L-functions*. Retrieved from <https://hdl.handle.net/1887/3147167>

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Title: Global field and their L-functions

Issue Date: 2021-03-02

Acknowledgements

The present thesis has been written with the kind support of a huge number of unique people. The total amount of members in this group is so big that it would not be possible to name each and every one here, but nevertheless, I am going to make a desperate attempt to thank people who contributed the most to the existence of the present text.

First and foremost I would like to say thank you to my principal advisor, professor Bart de Smit. Bart's contribution to the text cannot be overestimated: he was always open to guide me by lighting the way in the dark wood of my mathematical adventure towards the final goal: to become an independent researcher. I agree that it is by no means an easy job to do, but I think we enjoyed every moment of our collaboration when we were working together on various mathematical puzzles that appeared attractive to us.

Secondly, I would like to express my gratitude to my co-advisor professor Karim Belabas and the reading committee members, namely professors Elisa Lorenzo García, Marc Hindry and Michael Tsfasman. I appreciate the brave decision you made to read this manuscript and I want to emphasise that it was a big pleasure for me to receive your valuable comments.

Another round of applause goes to all the faculty members of the Mathematical Institute, who surrounded and supported me during my studies at Leiden University. In particular I would like to mention professors of the research group "Algebra and Geometry", namely Bas Edixhoven, Peter Stevenhagen, Marco Streng, Hendrik Lenstra, Peter Bruin, Ronald van Luijk and professor of the probability research group Evgeny Verbitskiy from whom I learned a lot about various kinds of impressive mathematical results both directly and indirectly.

Besides the long list of professors mentioned above, the mathematical community formed around Leiden University brought to my life many exciting connections. Along the way some of these professional connections actually transformed to the form of close friendship. Just to name a few of them: Alexey Beshenov, Abtien Javanpeykar, Carlo Pagano, Maxim Mornev, Dima Shvetsov, Rosa Winter, Richard Griffon, Evgeny Goncharov, Garnet Akeyr, Liza Arzhakova and many many others. Thank you all guys for being around all this time.

Last but not least, I need to emphasise the role of the non-mathematical community whose love and care supported me during my studies. Of course, the biggest gratitude goes to my family and especially to my mother Elena who devoted a huge amount of efforts to educate me, despite all the difficulties that we were experiencing in our life. It is also hard to estimate how much I gained from the friends of mine even if sometimes hundreds or even thousands of kilometres were between us. I am immensely grateful to you: Alexey, Nikolay, Shayekh, Gerben, Tatiana, Michael, Ivan and Flera.

Curriculum Vitae

Pavel Solomatin was born on May 14, 1991 in Moscow, Russian Federation. Started from 2004 he attended experimental division of the mathematical high school No. 179, with the curriculum focusing on both computer science and mathematics.

Upon graduating in 2008 he went on to obtain a bachelor diploma in pure mathematics from the National Research University Higher School of Economics (HSE). There he fell in love with number theory and defended his bachelor thesis titled “*On Arithmetic Properties of Abelian Varieties over Finite Fields*” under scientific direction of professor Alexey Zykin.

In 2012 Pavel enrolled in the master program at HSE math department, graduating in 2014 with the thesis titled “*Curves with Many Points over Finite Fields: The Class Field Theory Approach*”, which was also written under kind Alexey’s guidance. During his master studies Pavel’s research received support from the grant “*Arithmetic Properties of Abelian Varieties over Finite and Function Fields*” issued by the Russian Academy of Science. Directly after graduation in 2014 he received an ALGANT scholarship which allowed him to continue his mathematical studies as a Ph.D. candidate under scientific direction of professor Bart de Smit (Leiden University) and professor Karim Belabas (University of Bordeaux). Besides the scientific research Pavel was also devoting efforts to teaching various kinds of mathematical disciplines for graduate and undergraduate students. For this job he received the “*Best teacher 2013–2014*” award according to the choice of students from the HSE-NES joint bachelor program in economics. While staying in the Netherlands his teaching duties were related to assistance for multiple courses of the Dutch “Master in Mathematics” program, such as “Number Theory” and “Elliptic Curves”.

Starting from February 2018 he is working full-time as a software engineer at a Dutch IT-company called Ortec. There he is applying his passion for mathematics in order to improve our world.

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