Author: Ziemlańska, M.A.
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Summary

The subject of this thesis, ‘Approach to Markov Operators on Spaces of Measures by Means of Equicontinuity’, combines an analytical and probabilistic approach to Markov operators. We look at Markov operators coming from deterministic dynamical systems and also stochastic processes which come from a probabilistic approach.

In the study of Markov operators and Markov semigroups the central problems are to understand the behaviour of the processes and semigroups. Of particular interest is to identify the existence and uniqueness of invariant measures and long term behaviour of the process and dynamical system defined by the associated Markov operator or semigroup. Research on these questions dates back to the works of Andrey Markov, who described a Markov property for chains. A big part of theory for Markov chains can be found in the book by Meyn and Tweedie, who made a big contribution to the theory of Markov chains and gave a noteworthy description of e-chains, which was the motivation to working with equicontinuity properties for many authors. This theory is applicable when the underlying state space is locally compact. If it is not - in the generality of so-called Polish spaces - there is theory under development. Lasota and Szarek, and in recent years Worm generalized theory of Markov operators and families of Markov operators to this setting. In subsequent years, the theory was being developed starting with contractive Markov operators in the works of Lasota, through non-expansive Markov operators in Szarek’s,, and finally equicontinuous families of Markov operators in that of Szarek, Hille and Worm. We extend their results and give a new light to the existing ones by providing less restrictive conditions in cases.

A connection between weak and strong (norm) convergence of sequences of signed measures is a starting point of the thesis. The crucial thing is the extension of the results from positive measures to the spaces of signed measures. This is a very general tool which can be used not only in a theory of Markov operators, but also in general (advanced) measure theory. With the aid of this result, for which we give a self-contained proof, one gets an enlightening correspondence between the equicontinuity property as defined by Komorowski, Peszat and Szarek in [KPS10] (e-property) and the usual notion of an equicontinuous family of maps, furnished by the semigroup defined by the Markov operator on measures. With this result, we are able to prove a Lie-Trotter product formula for Markov semigroups.

The key ideas of the generalization of the Lie-Trotter product formula to Markov semigroups is to give realistic conditions and prove convergence in the relevant norms. The crucial assumptions are to drop strong continuity and boundedness of the semigroup, as

Markov semigroups are often neither strongly continuous nor consist of bounded operators on signed measures in the appropriate Dudley or Fortet-Mourier norm on signed measures. Also, the properties of the limit semigroups coming from the starting semigroups are being analysed. This gives an additional tool for dealing with complicated problems by splitting them into alternative "easier" ones.

The next part of this thesis describes the relation between equicontinuity and stability of Markov operators. In particular, it is shown that any asymptotically stable Markov operator with an invariant measure such that the interior of its support is non-empty, satisfies the e-property. These results are of importance as they extend similar, existing results on compact spaces to the theory of Polish spaces which does not assume compactness.

Finally, we show the Central Limit Theorem (CLT) for some non-stationary Markov chains on Polish spaces. Recent CLT results by Komorowski and co-workers for non-stationary Markov processes show the importance of the topic. In particular in applications, the validity of Law of Large Numbers and the CLT for non-stationary chains allows one in principle to get information on the 'shape' of the invariant measure by means of simulating sample trajectories of the chain and averaging. The CLT provides the rate of convergence of this procedure. The extension of Gordin and Lifšic that we prove is possible by the aid of the spectral gap property in the Kantorovich-Rubinstein norm. Some delicate approximation allows us to also obtain a stronger result then Komorowski's.