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## Approach to Markov operators on spaces of measures by means of equicontinuity

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by Means of Equicontinuity

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*To Jos and Julian*



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## Contents

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# Notation

Here we state some conventions regarding mathematical notation that we will use throughout the thesis.

- $\mathbb{N}$  denotes the set of natural numbers  $\{1, 2, 3, \dots\}$ ,  $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$
- $\mathbb{R}_+ := \{x \in \mathbb{R} : x \geq 0\}$
- $\mathcal{M}(S)$  is the real vector space of finite signed measures on  $S$
- $\mathcal{M}^+(S)$  is the cone of positive measures in  $\mathcal{M}(S)$
- $\mathcal{P}(S)$  is the set of probability measures in  $\mathcal{M}^+(S)$
- $\|\cdot\|_{TV}$  denotes the total variation norm on  $\mathcal{M}(S)$ .  $\|\mu\|_{TV} = \mu^+(S) + \mu^-(S)$
- $\mathbb{1}_E$  is the indicator function of  $E \subset S$
- For a measurable function  $f : S \rightarrow \mathbb{R}$  and  $\mu \in \mathcal{M}(S)$  we denote

$$\langle \mu, f \rangle = \int_S f d\mu$$

- $P : \mathcal{M}(S) \rightarrow \mathcal{M}(S)$  denotes Markov operator with a dual operator  $U$
- $B(x, r)$  denotes the open ball of radius  $r$  centered at  $x$
- In a metric space  $(S, d)$ , if  $A \subset S$  is nonempty, we denote by  $A^\epsilon := \{x \in S : d(x, A) \leq \epsilon\}$  the closed  $\epsilon$ -neighbourhood of  $A$
- If  $S$  is a topological space,  $C_b(S)$  is the Banach space of bounded continuous functions from  $S$  to  $\mathbb{R}$ , endowed with the supremum norm  $\|\cdot\|_\infty$ .
- $\langle \mu, f \rangle := \int_\Omega f d\mu$
- Markov operator is a map  $P : \mathcal{M}^+(S) \rightarrow \mathcal{M}^+(S)$  such that:

(MO1)  $P$  is additive and  $\mathbb{R}_+$  homogeneous;

(MO2)  $\|P\mu\|_{TV} = \|\mu\|_{TV}$  for all  $\mu \in \mathcal{M}^+(S)$ ;

$P$  extends to a positive bounded linear operator on  $(\mathcal{M}(S), \|\cdot\|_{TV})$  by  $P\mu := P\mu^+ - P\mu^-$ .

- We say that Markov process is stationary if its moments do not depend on the time shift.

