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Approach to Markov operators on spaces of measures by means of equicontinuity

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Citation

Ziemlańska, M. A. (2021, February 10). *Approach to Markov operators on spaces of measures by means of equicontinuity*. Retrieved from <https://hdl.handle.net/1887/3135034>

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Author: Ziemiańska, M.A.

Title: Approach to Markov Operators on spaces of measures by means of equicontinuity

Issue Date: 2021-02-10

Approach to Markov Operators on Spaces of Measures
by Means of Equicontinuity

Proefschrift

ter verkrijging van
de graad van Doctor aan de Universiteit Leiden,
op gezag van Rector Magnificus Prof.dr.ir. H. Bijl,
volgens besluit van het College voor Promoties
te verdedigen op woensdag 10 februari 2021
klokke 11:15 uur

door

Maria Aleksandra Ziemiańska

geboren te Gdańsk, Polen
in 1985

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Printed in the Netherlands

To Jos and Julian

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Contents

Notation

Here we state some conventions regarding mathematical notation that we will use throughout the thesis.

- $\mathbb{N}$ denotes the set of natural numbers $\{1, 2, 3, \dots\}$, $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$
- $\mathbb{R}_+ := \{x \in \mathbb{R} : x \geq 0\}$
- $\mathcal{M}(S)$ is the real vector space of finite signed measures on S
- $\mathcal{M}^+(S)$ is the cone of positive measures in $\mathcal{M}(S)$
- $\mathcal{P}(S)$ is the set of probability measures in $\mathcal{M}^+(S)$
- $\|\cdot\|_{TV}$ denotes the total variation norm on $\mathcal{M}(S)$. $\|\mu\|_{TV} = \mu^+(S) + \mu^-(S)$
- $\mathbb{1}_E$ is the indicator function of $E \subset S$
- For a measurable function $f : S \rightarrow \mathbb{R}$ and $\mu \in \mathcal{M}(S)$ we denote

$$\langle \mu, f \rangle = \int_S f d\mu$$

- $P : \mathcal{M}(S) \rightarrow \mathcal{M}(S)$ denotes Markov operator with a dual operator U
- $B(x, r)$ denotes the open ball of radius r centered at x
- In a metric space (S, d) , if $A \subset S$ is nonempty, we denote by $A^\epsilon := \{x \in S : d(x, A) \leq \epsilon\}$ the closed ϵ -neighbourhood of A
- If S is a topological space, $C_b(S)$ is the Banach space of bounded continuous functions from S to $\mathbb{R}$, endowed with the supremum norm $\|\cdot\|_\infty$.
- $\langle \mu, f \rangle := \int_\Omega f d\mu$
- Markov operator is a map $P : \mathcal{M}^+(S) \rightarrow \mathcal{M}^+(S)$ such that:

(MO1) P is additive and $\mathbb{R}_+$ homogeneous;

(MO2) $\|P\mu\|_{TV} = \|\mu\|_{TV}$ for all $\mu \in \mathcal{M}^+(S)$;

P extends to a positive bounded linear operator on $(\mathcal{M}(S), \|\cdot\|_{TV})$ by $P\mu := P\mu^+ - P\mu^-$.

- We say that Markov process is stationary if its moments do not depend on the time shift.

