Approach to Markov operators on spaces of measures by means of equicontinuity

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Notation

Here we state some conventions regarding mathematical notation that we will use throughout the thesis.

- $\mathbb{N}$ denotes the set of natural numbers$\{1, 2, 3, \cdots\}$, $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$
- $\mathbb{R}_+ := \{x \in \mathbb{R} : x \geq 0\}$
- $\mathcal{M}(S)$ is the real vector space of finite signed measures on S
- $\mathcal{M}^+(S)$ is the cone of positive measures in $\mathcal{M}(S)$
- $\mathcal{P}(S)$ is the set of probability measures in $\mathcal{M}^+(S)$
- $\|\cdot\|_T V$ denotes the total variation norm on $\mathcal{M}(S)$. $\|\mu\|_{TV} = \mu^+(S) + \mu^-(S)$
- $\mathbb{1}_E$ is the indicator function of $E \subset S$
- For a measurable function $f : S \to \mathbb{R}$ and $\mu \in \mathcal{M}(S)$ we denote

$$\langle \mu, f \rangle = \int_S f d\mu$$

- $P : \mathcal{M}(S) \to \mathcal{M}(S)$ denotes Markov operator with a dual operator U
- $B(x, r)$ denotes the open ball of radius r centered at x
- In a metric space (S, d), if $A \subset S$ is nonempty, we denote by $A^\epsilon := \{x \in S : d(x, A) \leq \epsilon\}$ the closed ϵ-neighbourhood of A
- If S is a topological space, $C_b(S)$ is the Banach space of bounded continuous functions from S to $\mathbb{R}$, endowed with the supremum norm $\|\cdot\|_\infty$.
- $\langle \mu, f \rangle := \int_\Omega f d\mu$
- Markov operator is a map $P : \mathcal{M}^+(S) \to \mathcal{M}^+(S)$ such that:

(MO1) P is additive and $\mathbb{R}_+$ homogeneous;

(MO2) $\|P\mu\|_{TV} = \|\mu\|_{TV}$ for all $\mu \in \mathcal{M}^+(S)$;

P extends to a positive bounded linear operator on $(\mathcal{M}(S), \|\cdot\|_{TV})$ by $P\mu := P\mu^+ - P\mu^-$.

- We say that Markov process is stationary if its moments do not depend on the time shift.