Topologies and convergence structures on vector lattices of operators

Deng, Y.

Author: Deng, Y.
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Stellingen

behorend bij het proefschrift

"Topologies and convergence structures on vector lattices of operators"

I A vector lattice E admits a Hausdorff uo-Lebesgue topology whenever its order continuous dual $E_{oc}^{\sim}$ separates the points of E. It is, more generally, true that E admits such a topology whenever there exists a Dedekind complete vector lattice F that admits a Hausdorff uo-Lebesgue topology and is such that $\mathscr{L}_{oc}(E, F)$ separates the points of E. The former result can be derived by using methods in the existing literature, but the latter can not. [Ch. 2, 3]

II For a vector lattice E and a locally solid Dedekind complete vector lattice (F, τ), the absolute strong operator topology that τ generates on $\mathscr{L}_{ob}(E, F)$ is more natural than the strong operator topology that it generates. [Ch. 3]

III Results such as Corollary 3.7.3, Theorem 3.9.9, Theorem 3.9.12, and Corollary 4.6.16 in this thesis are necessary to have at one's disposal when studying bicommutants of subalgebras of $\mathscr{L}_{ob}(E)$ for a Dedekind complete vector lattice E. [Ch. 3, 4]

IV With only applications in representation theory in Banach lattices in mind, it would have been sufficient in Chapters 3 and 4 to suppose that all order bounded operators on the pertinent vector lattices are actually order continuous. [Ch. 3, 4]

V The terminology *order dense* and *order separable* as employed in, e.g., [5] is inept and should no longer be used.

VI In [3, Theorem 9.5], the condition that X have a weak unit can be replaced by the weaker condition that Y have the countable sup property.

VII In [4, Definition 6.1], the authors define when a vector lattice has the property that uo-convergence on it is sequential. This is the case if and only if the vector lattice has the countable sup property.

VIII It is a classical result that, for a finite measure space, a sequence of measurable functions that converges in measure to zero has a subsequence that converges almost everywhere to zero. This has been generalised to vector and Banach lattices in various ways, see [1, Theorem 4.19], [2, Proposition 4.1], [3, Theorem 9.5], [4, Theorem 6.7], and Theorem 2.7.6 in this thesis. It is possible to prove a single theorem that has these generalisations as special cases.

IX Doing mathematical research is like walking on thin ice.

X Intuition guides us in finding details and details guide us in building intuition.

References

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Yang Deng
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