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## Topologies and convergence structures on vector lattices of operators

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# **Topologies and convergence structures on vector lattices of operators**

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