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Bayesian learning: Challenges, limitations and pragmatics

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Origin of the material

This dissertation is based on the following papers. The author of this dissertation contributed substantially to each of these papers.

Chapter 2 is based on the paper that is under review as

Tom Sterkenburg and Rianne de Heide. On the truth-convergence of open-minded Bayesianism.

Chapter 3 is accepted for publication in *Psychonomic Bulletin & Review*, and is available as the technical report

Rianne de Heide and Peter Grünwald. Why optional stopping can be a problem for Bayesians. *arXiv 1708.08278*. August 2017.

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Allard Hendriksen, Rianne de Heide and Peter Grünwald. Optional Stopping with Bayes Factors: a categorization and extension of folklore results, with an application to invariant situations. *Bayesian Analysis*, advance publication, 18 August 2020. doi:10.1214/20-BA1234.

Chapter 5 is based on the technical report

Peter Grünwald, Rianne de Heide and Wouter Koolen. Safe Testing. *arXiv 1906.07801*. June 2019.

Chapter 6 is published as

Rianne de Heide, Alisa Kirichenko, Nishant Mehta and Peter Grünwald. Safe-Bayesian Generalized Linear Regression. AISTATS 2020, PMLR 108:2623-2633.

The software for this chapter is partly available as

Rianne de Heide (2016). SafeBayes: Generalized and Safe-Bayesian Ridge and Lasso Regression. R package version 1.1. <https://cran.r-project.org/src/contrib/Archive/SafeBayes/>

Chapter 7 is published as

Xuedong Shang, Rianne de Heide, Emilie Kaufman, Pierre Ménard and Michal Valko. Fixed-Confidence Guarantees for Bayesian Best-Arm Identification. AISTATS 2020, PMLR 108:1823-1832.

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