Torsion points on elliptic curves over number fields of small degree

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Title: Torsion points on elliptic curves over number fields of small degree
Issue Date: 2016-09-21

STELLINGEN

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Torsion points on elliptic curves over number fields of small degree

van Maarten Derickx

1. For an integer d let $S(d)$ denote the set of primes p for which there exists an elliptic curve over a number field K of degree d with a K-rational point of order p and let $\mathrm{Primes}(d)$ denote the set of primes $\leq d$. Then we have the following equalities of sets:

$$S(4) = \mathrm{Primes}(17),$$
$$S(5) = \mathrm{Primes}(19), \text{ and}$$
$$S(6) = \mathrm{Primes}(19) \cup \{37\}.$$

2. The $\mathbb{Q}$-gonality of $X_1(37)$ is 18.

3. $X_1(N)$ has infinitely many places of degree $d = 5$ resp. $d = 6$ over $\mathbb{Q}$ iff

   - for $d = 5$: $N \leqslant 25$ and $N \neq 23$.
   - for $d = 6$: $N \leqslant 30$ and $N \neq 23, 25, 29$.

4. There exist 12 distinct $\mathbb{Q}$-rational functions $f_1, \ldots, f_{12} : X_1(17) \to \mathbb{P}^1$ of degree 4 such that

$$\bigcup_{i=1}^{12} f_i^{-1}(\mathbb{P}^1(\mathbb{Q}))$$

   contains all the point in $X_1(17)(\overline{\mathbb{Q}})$ whose field of definition is of degree 4 over $\mathbb{Q}$.

5. From the 12 functions above, exactly 4 factor via $X_1(17)/\langle 13 \rangle$.

   For these 4 functions the elliptic curves corresponding to the points in $f^{-1}(\mathbb{P}^1(\mathbb{Q}))$ all have even Mordell-Weil rank.

6. Let $N \geq 4$ be an integer and $\mathcal{Y}_1(N) : \mathrm{Sch}_{\mathbb{Z}[1/N]} \to \mathrm{Sets}^{op}$, be the functor which sends a $\mathbb{Z}[1/N]$ scheme S to the set of all isomorphism classes of pairs (E, P) of elliptic curve with a point P of order *exactly* N.

   The functor $\mathcal{Y}_1(N)$ has a right adjoint.

7. Let $K \subseteq L$ be a purely inseparable field extension in characteristic p and of degree p^e with $e > 2$. Then the set of subgroups of S_{p^e} for which L/K has a G-closure[1] contains two subgroups which are minimal with respect to inclusion and in addition are not isomorphic and hence not conjugate.

8. The ring $R := \mathbb{C}[x, y]/(5x^4 + 2xy^2, 2x^2y + 4y^3, xy^3, y^5)$ is local and finite dimensional as $\mathbb{C}$ vector space. The dimension of the kernel of $d : R \to \Omega^1_{R/\mathbb{C}}$ is greater than 1.

9. Let $S^1 := \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$ be the unit circle considered as a Riemaniann manifold, whose Riemann metric is the induced metric form $\mathbb{R}^2$. Let $b \in \mathbb{R}_{>0}$ and let $\iota_1 : S^1 \to \mathbb{R}^2$ and $\iota_2 : [-b, b] \to \mathbb{R}$ be the inclusion maps.

   If $b < \frac{1}{2}\pi$, then there exists a homotopy $h : \big(S^1 \times [-b, b]\big) \times [0, 1] \to \mathbb{R}^3$ from (ι_1, ι_2) to $(\iota_1, -\iota_2)$ such that for each $t \in [0, 1]$ the map $h(\_, t)$ is a an immersion of Riemannian manifolds with boundary, and if $b \geq \pi$, then such a homotopy does not exist.

10. The distances that can be constructed with 3-dimensional rigid origami are exactly the positive real algebraic numbers.

11. Sign mistakes do not only occur in mathematics, they also happen in real life.

[1] The notion of G-closure is as in Owen Biesel's Ph.D. thesis "Galois Closures for Rings".