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**Title:** Torsion points on elliptic curves over number fields of small degree
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1. For an integer $d$ let $S(d)$ denote the set of primes $p$ for which there exists an elliptic curve over a number field $K$ of degree $d$ with a $K$-rational point of order $p$ and let Primes($d$) denote the set of primes $\leq d$. Then we have the following equalities of sets:

\[ S(4) = \text{Primes}(17), \]
\[ S(5) = \text{Primes}(19), \text{ and} \]
\[ S(6) = \text{Primes}(19) \cup \{37\}. \]

2. The $\mathbb{Q}$-gonality of $X_1(37)$ is 18.

3. $X_1(N)$ has infinitely many places of degree $d = 5$ resp. $d = 6$ over $\mathbb{Q}$ iff

- for $d = 5$: $N \leq 25$ and $N \neq 23$.
- for $d = 6$: $N \leq 30$ and $N \neq 23, 25, 29$.

4. There exist 12 distinct $\mathbb{Q}$-rational functions $f_1, \ldots, f_{12} : X_1(17) \to \mathbb{P}^1$ of degree 4 such that

\[ \bigcup_{i=1}^{12} f_i^{-1}(\mathbb{P}^1(\mathbb{Q})) \]

contains all the points in $X_1(17)(\overline{\mathbb{Q}})$ whose field of definition is of degree 4 over $\mathbb{Q}$.

5. From the 12 functions above, exactly 4 factor via $X_1(17)/\langle 13 \rangle$.

For these 4 functions the elliptic curves corresponding to the points in $f^{-1}(\mathbb{P}^1(\mathbb{Q}))$ all have even Mordell-Weil rank.
6. Let \( N \geq 4 \) be an integer and \( \mathcal{Y}_1(N) : \text{Sch}_{\mathbb{Z}[1/N]} \to \text{Sets}^{op} \), be the functor which sends a \( \mathbb{Z}[1/N] \) scheme \( S \) to the set of all isomorphism classes of pairs \((E,P)\) of elliptic curve with a point \( P \) of order \( exactly \) \( N \).

The functor \( \mathcal{Y}_1(N) \) has a right adjoint.

7. Let \( K \subseteq L \) be a purely inseparable field extension in characteristic \( p \) and of degree \( p^e \) with \( e > 2 \). Then the set of subgroups of \( S_{p^e} \) for which \( L/K \) has a \( G \)-closure\(^1\) contains two subgroups which are minimal with respect to inclusion and in addition are not isomorphic and hence not conjugate.

8. The ring \( R := \mathbb{C}[x,y]/(5x^4 + 2xy^2, 2x^2y + 4y^3, xy^3, y^5) \) is local and finite dimensional as \( \mathbb{C} \) vector space. The dimension of the kernel of \( d : R \to \Omega^1_{R/\mathbb{C}} \) is greater than 1.

9. Let \( S^1 := \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1 \} \) be the unit circle considered as a Riemannian manifold, whose Riemann metric is the induced metric form \( \mathbb{R}^2 \). Let \( b \in \mathbb{R}_{>0} \) and let \( \iota_1 : S^1 \to \mathbb{R}^2 \) and \( \iota_2 : [-b, b] \to \mathbb{R} \) be the inclusion maps.

If \( b < \frac{1}{2} \pi \), then there exists a homotopy \( h : (S^1 \times [-b, b]) \times [0, 1] \to \mathbb{R}^3 \) from \((\iota_1, \iota_2)\) to \((\iota_1, -\iota_2)\) such that for each \( t \in [0, 1] \) the map \( h(\_ , t) \) is a an immersion of Riemannian manifolds with boundary, and if \( b \geq \pi \), then such a homotopy does not exist.

10. The distances that can be constructed with 3-dimensional rigid origami are exactly the positive real algebraic numbers.

11. Sign mistakes do not only occur in mathematics, they also happen in real life.

\(^1\)The notion of \( G \)-closure is as in Owen Biesel’s Ph.D. thesis “Galois Closures for Rings”.