Anomalous Integer Quantum Hall Effect in the Ballistic Regime with Quantum Point Contacts

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The Hall conductance of a wide two-dimensional electron gas has been measured in a geometry in which two quantum point contacts form controllable current and voltage probes, separated by less than the transport mean free path. Adjustable barriers in the point contacts allow selective population and detection of Landau levels. The quantization of the Hall conductance is determined by the number of quantum channels in the point contacts and is independent of the number of occupied bulk Landau levels. A theoretical description based on the Landauer-Büttiker formalism is given.

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Ever since its discovery[1] in 1980, the quantum Hall effect (QHE) has presented a major theoretical challenge. In macroscopic samples the QHE is usually explained[2] by invoking the presence of localized states. However, the observation of the QHE in high-mobility samples with submicron dimensions,[3] in which only a few impurities are present, indicates that localization is not a prerequisite for the appearance of the QHE in microscopic systems.

Recently alternative approaches to the QHE have been proposed,[4-6] based on the Landauer formalism for electron transport.[7] In these descriptions the current in high magnetic fields is carried by edge channels, which consist of the current-carrying states of each Landau level and are located at the boundaries of the two-dimensional electron gas (2D EG). It can be shown[8] that, in the absence of spin splitting, each edge channel carries a current $\Delta I = (2e/h)\Delta\mu$, in which $\Delta\mu$ is the difference in electrochemical potential between edge states on opposite sides of the 2D EG. If N_L edge channels (or Landau levels) are occupied, one obtains quantization of the Hall conductance $G_H = eI/\Delta\mu = (2e^2/h)N_L$.

However, it has been pointed out by Büttiker[5] that, in the absence of inelastic scattering, this result only applies if the current contacts fully populate all N_L edge channels up to their electrochemical potentials. In the case of incomplete population of one or more edge channels, G_H can depend on the details of the coupling of both current and voltage contacts to different edge channels, and deviations from ideal quantization might be expected.

In this Letter we study the QHE in the ballistic regime, in a geometry in which two quantum point contacts, defined by a split-gate technique, form a voltage and a current probe, separated by less than both the elastic and the inelastic mean free paths. In contrast to other experiments,[9] the electron density of the 2D EG of which the Hall conductance is measured is fixed, and only the properties of the contacts which are attached to it are varied. The number of quantum channels (or magnetoelectric subbands) in these point contacts can be controlled by the applied gate voltage.[10,11] As explained below, edge channels can be *selectively* populated and detected by these point contacts. An anomalous integer quantum Hall effect is observed, in which the quantization of G_H is determined by the number of quantum channels in the point contacts, *independent* of the number of Landau levels in the wide 2D EG region.

A schematic layout of the sample is given in Fig. 1(a). It is similar to those employed for the study of coherent electron focusing[12] and conductance quantization[10] in zero and nonzero magnetic fields. The ohmic contacts 1 to 4 are attached to a wide, Hall-bar-shaped, GaAs/$Al_{0.33}Ga_{0.67}As$ heterostructure, which contains a 2D EG. The electron density is $3.5\times10^{15}/m^2$, and the elastic mean free path (at 1.3 K) is 8 μm. Gates are fabricated on top of the heterostructure, employing electron-beam lithography and lift-off techniques. Application of a negative bias depletes the electron gas underneath the gates, resulting in the formation at -0.7 V of two point contacts with a lithographic width of 250 nm and a separation of 1.5 μm. A more negative gate voltage enhances the electric field present around the gates, thus widening the depletion regions. At the point contacts the depletion regions of the adjoining gates overlap. As a result, the width of the contacts is reduced, and also the bottom of the conduction band in the contacts is raised. A saddle-shaped potential is formed, with a barrier height V_{bar} that can be controlled separately by V_B (voltage contact) or by both V_A and V_B (current contact) (see Fig. 1).

In high perpendicular magnetic fields the location of

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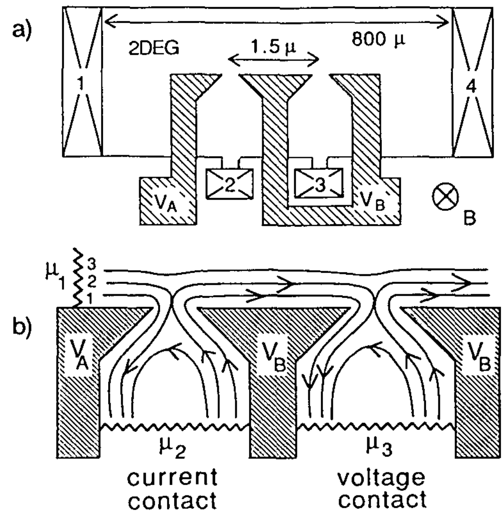

FIG. 1. (a) Schematic layout of the device (not to scale). The gates (with separation 250 nm) define two closely spaced point contacts, which can be controlled separately by applying different voltages V_A and V_B to the gates. (b) Current flow in high magnetic fields. The positions of the quantum channels are indicated. The arrows indicate the electron flow in the channels which are populated when current is injected through the current point contact.

the wave functions of the electrons with the Fermi energy E_F, which constitute the quantum channels, is determined by their guiding-center energy[2] E_G:

$$E_G = E_F - (n - \tfrac{1}{2})\hbar\omega_c, \quad \omega_c = eB/m\,. \tag{1}$$

The electrons injected by the current contact move along lines of constant potential $V(x,y) = E_G$, determined by the quantum number n associated with the quantization of the cyclotron motion, which labels the channels [Fig. 1(b)]. Spin splitting is not taken into account, since it is not resolved in the experiments. The number of quantum channels (or magnetoelectric subbands) in the point contacts is determined by both the height of the potential barriers $V_{\rm bar}$ and the magnetic field, and is equal to or smaller than the number of edge channels (Landau levels) N_L in the wide regions on either side of the point contacts.[10] Only those channels for which $E_G > V_{\rm bar}$ are transmitted through the point contacts (tunneling is disregarded). Figure 1(b) illustrates the case $N_L = 3$. The arrows indicate the direction of the electron flow in the channels which are populated when a current I is injected through the current contact. In this example the $n=1$ channel is fully transmitted, whereas the $n=3$ channel is fully reflected. The wave functions of the ingoing and outgoing branches of the $n=2$ channel overlap, which we assume to give rise to partial reflection of the electrons moving in this channel. Assuming that this constitutes the only source of scattering, we write the two-terminal conductance of the current contact as

$$G_I = eI/(\mu_2 - \mu_1) = 2e^2(N_I + T_I)/h\,, \tag{2}$$

in which N_I denotes the number of fully transmitted channels and T_I $(0 < T_I < 1)$ denotes the partial transmission of the upper channel. The wide regions on either side of the point contacts are treated as reservoirs. Their electrochemical potentials μ_1, μ_2, and μ_3 are measured by the Ohmic contacts 1, 2, and 3. Equation (2) illustrates that quantization of the two-terminal conductance of a point contact[10,11] occurs in those intervals of B and gate voltage for which $T_I = 0$.

We now calculate $G_H = eI/(\mu_3 - \mu_1)$, employing the Landauer-Büttiker formalism for multiterminal measurements.[13] We put $\mu_1 = 0$ for convenience. Whenever $N_I < N_L$, the injected current is distributed unequally among the edge channels. The lowest N_I channels are fully occupied up to energy μ_2, carrying a current $(2e/h)N_I\mu_2$. Channel $N_I + 1$ is only partially occupied to μ_2, and carries a current $(2e/h)T_I\mu_2$. Channels $N_I + 2$ up to N_L are not populated and do not carry current. Interchannel scattering may occur in the region between the point contacts as a result of the presence of residual impurities or, more likely, at the exit of the current contact and the entrance of the voltage contact, where the confining potential changes rapidly. At this point we assume that this scattering is absent. For the calculation of μ_3, and G_H, three situations have to be considered. If $N_V > N_I$, the total injected current I enters the voltage point contact. The electrochemical potential μ_3, which will build up to compensate this current by an outgoing current of equal magnitude, is now determined by the two-terminal conductance of the voltage probe G_V, $\mu_3 = eI/G_V$, which yields

$$G_H = G_V = 2e^2(N_V + T_V)/h, \text{ for } N_V > N_I\,, \tag{3}$$

with N_V the number of fully transmitted channels in the voltage contact and T_V the partial transmission of the upper channel. In the case $N_I > N_V$ all channels entering the voltage contact are fully occupied to μ_2. Hence, $\mu_3 = \mu_2$, and G_H now equals G_I:

$$G_H = G_I = 2e^2(N_I + T_I)/h, \text{ for } N_I > N_V\,. \tag{4}$$

If $N_I = N_V = N$, the current entering the voltage contact as a result of the fully occupied channels is given by $(2e/h)N\mu_2$. Channel $N+1$ carried a current $(2e/h) \times T_I\mu_2$ of which only a fraction $(2e/h)T_IT_V\mu_2$ enters the voltage probe. The total current is given by $(2e/h) \times (N + T_IT_V)\mu_2$, which, after compensation with an equal outgoing current yields

$$G_H = \frac{2e^2}{h}\frac{(N+T_I)(N+T_V)}{N+T_IT_V}, \text{ for } N_I = N_V = N\,. \tag{5}$$

Equations (3)–(5) predict a quantization of G_H, whenever the point contact with the *largest* conductance is quantized. An exception[6] is formed by the case $N_I = N_V = 0$, to be discussed in a future paper, for which Eq. (5) predicts quantization independent of the values of T_I and T_V. It is striking that the number of occupied bulk Landau levels N_L does not appear in these expressions. This is due to the fact that edge channels, which are neither occupied by the current contact, nor detected by the voltage contact, are irrelevant for the electron transport.

In reverse magnetic fields the injected electrons move away from the voltage point contact and are detected by a bulk voltage contact (Ohmic contact 1), which has $N_V = N_L$. Therefore [Eq. (3)], we expect the regular quantization[14]: $G_H = (2e^2/h)N_L$.

The Hall conductivity has been measured as a function of B at 1.3 K for several fixed values of the gate voltage ($V_A = V_B$) in the setup of Fig. 1. Contacts 2 and 4 are current terminals. The voltage is measured between 1 and 3. For reverse fields the regular QHE is observed, independent of the applied gate voltage. In forward fields a gradual transition is observed from electron focusing at low fields to the formation of quantized plateaus at high fields.[12,15] In this case both the number of observed plateaus as well as their positions depend on the applied gate voltage.

For forward fields Eqs. (3)-(5) give relations between G_H, G_V, and G_I. We have investigated this experimentally by measuring these conductances as a function of gate voltage ($V_A = V_B$) at several *fixed* values of B. Figure 2 displays a comparison between G_I (two-terminal measurements between contacts 1 and 2), G_V (two-terminal measurement between contacts 1 and 3), and G_H. The presence of quantized plateaus in the same gate-voltage intervals illustrated that both point contacts exhibit almost equal behavior, showing different features in between the quantized plateaus only. At this magnetic field (3.3 T) two Landau levels are occupied in the wide 2D EG, which was determined from Hall measurements with wide contacts. (No gate voltage was applied in this case.) Despite the presence of two occupied bulk Landau levels, G_H follows G_I and G_V, and exhibits quantization at both $4e^2/h$ and $2e^2/h$, in a way which is consistent with Eq. (5). The rapid rise in G_H below -2.0 V is an artifact due to the complete pinch off of the point contacts.

For magnetic fields from 2.1 T up to our maximum field (7.3 T) the gate-voltage intervals in which G_H is quantized correspond to those in which G_I and G_V are quantized. The correlation shows that the quantization of G_H is governed by the point contacts and does not depend on the number of occupied bulk Landau levels. At fields below 2.1 T no plateaus are observed in G_H, although G_I and G_V are already quantized for $B > 1.4$ T. We attribute this to the onset of interchannel scattering in the region between the point contacts at low fields. This invalidates the relation between G_I, G_V, and G_H expressed in Eqs. (3)-(5).

The role of an individual point contact has been investigated by fixing both magnetic field and gate voltage V_B. In this way N_L is kept constant, as well as G_V. Figure 3(a) gives a comparison between G_H and G_I, both measured as functions of V_A. G_V has been fixed at $4e^2/h$. Whereas G_I drops from the $4e^2/h$ to the $2e^2/h$ plateau. G_H remains quantized at $4e^2/h$. This illustrates that G_H remains quantized whenever the largest conductance (G_V in this case) remains quantized [Eq. (3) with $N_V = 2$ and $T_V = 0$]. In Fig. 3(b) G_V has been fixed at $2e^2/h$. For $G_I > G_V$, the measured Hall conductance reproduces the features present in G_I. For $G_I < G_V$ the Hall conductance remains fixed at $2e^2/h$, until the current point contact is fully pinched off. These observations correspond to Eqs. (5) ($N = 1$, $T_V = 0$, $T_I \neq 0$) and (3) ($N_V = 1$, $T_V = 0$), respectively. These experiments

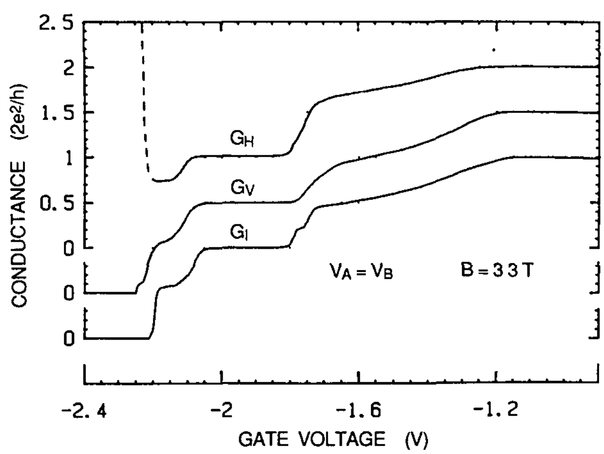

FIG. 2. Hall conductance G_H and two-terminal conductances G_I and G_V, measured as functions of gate voltage at a fixed magnetic field. The curves have been offset for clarity. Despite the presence of two occupied bulk Landau levels, G_H is quantized at $2e^2/h$ and $4e^2/h$, determined by G_V and G_I.

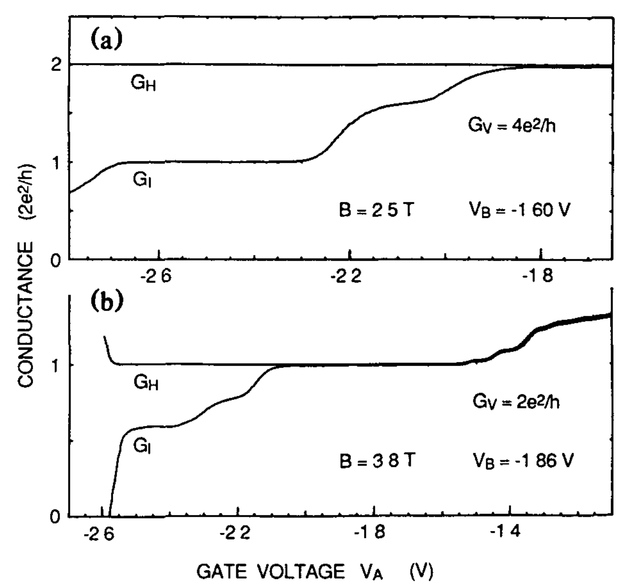

FIG. 3. Comparison between the Hall conductance G_H and the two-terminal conductance G_I, demonstrating the validity of Eqs. (3) and (5).

confirm that the anomalous quantum Hall effect is a result of the nonlocal transport. The Hall conductance changes, although the 2D EG in between the contacts is not affected (V_B as well as B are fixed).

In this Letter we have observed an anomalous quantum Hall effect, resulting from the selective population and detection of magnetic edge channels. It has been shown that the absence of scattering between different edge channels in high magnetic fields leads to a relation between the Hall conductance and the two-terminal conductances of current and voltage point contacts, the quantization of G_H being determined by the number of channels in the point contacts. Elastic scattering between occupied and unoccupied edge channels will destroy this anomalous quantum Hall effect. Electrons which are scattered into an initially unoccupied channel may flow past the voltage contact and fail to build up the quantized Hall voltage. This is in contrast with the regular QHE, which is insensitive to scattering between different edge channels located at the same boundary of the 2D EG,[5] since in this case all channels are equally populated.

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[1]K. v. Klitzing, G. Dorda, and M. Pepper, Phys. Rev. Lett. **45**, 494 (1980).

[2]*The Quantum Hall Effect*, edited by R. E. Prange and S. M. Girvin (Springer-Verlag, New York, 1987).

[3]G. Timp, A. M. Chang, P. Mankiewich, R. Behringer, J. E. Cunningham, T. Y. Chang, and R. E. Howard, Phys. Rev. Lett. **59**, 732 (1987); M. L. Roukes, A. Scherer, S. J. Allen, Jr., H. G. Craighead, R. M. Ruthen, E. D. Beebe, and J. P. Harbison, Phys. Rev. Lett. **59**, 3011 (1987); B. E. Kane, D. C. Tsui, and G. Weimann, Phys. Rev. Lett. **59**, 1353 (1987).

[4]P. Streda, J. Kucera, and A. H. MacDonald, Phys. Rev. Lett. **59**, 1973 (1987); J. K. Jain and S. A. Kivelson, Phys. Rev. Lett. **60**, 1542 (1988).

[5]M. Büttiker, Phys. Rev. B **38**, 9375 (1988).

[6]U. Sivan, Y. Imry, and C. Hartzstein (to be published).

[7]R. Landauer, Z. Phys. B **68**, 217 (1987).

[8]B. I. Halperin, Phys. Rev. B **25**, 2185 (1982).

[9]D. A. Syphers and P. J. Stiles, Phys. Rev. B **32**, 6620 (1985); S. Washburn, A. B. Fowler, H. Schmid, and D. Kern, Phys. Rev. Lett. **61**, 2801 (1988); R. J. Haug, A. H. MacDonald, P. Streda, and K. von Klitzing, Phys. Rev. Lett. **61**, 2797 (1988).

[10]B. J. van Wees, L. P. Kouwenhoven, H. van Houten, C. W. J. Beenakker, J. E. Mooij, C. T. Foxon, and J. J. Harris, Phys. Rev. B **38**, 3625 (1988); B. J. van Wees, H. van Houton, C. W. J. Beenakker, J. G. Williamson, L. P. Kouwenhoven, D. van der Marel, and C. T. Foxon, Phys. Rev. Lett. **60**, 848 (1988).

[11]D. A. Wharam, T. J. Thornton, R. Newbury, M. Pepper, H. Ahmed, J. E. F. Frost, D. G. Hasko, D. C. Peacock, D. A. Ritchie, and G. A. C. Jones, J. Phys. C **21**, L209 (1988).

[12]H. van Houten, B. J. van Wees, J. E. Mooij, C. W. J. Beenakker, J. G. Williamson, and C. T. Foxon, Europhys. Lett. **5**, 712 (1988); C. W. J. Beenakker, H. van Houten, and B. J. van Wees, Europhys. Lett. **7**, 359 (1988).

[13]M. Büttiker, Phys. Rev. Lett. **57**, 1761 (1986).

[14]As argued in Ref. 5, the presence of inelastic scattering in the region between the contacts may lead to a redistribution of the injected current over all N_L edge channels. This also gives the result $G_H = (2e^2/h)N_L$.

[15]H. van Houten, C. W. J. Beenakker, J. G. Williamson, M. E. I. Brockaart, P. H. M. van Loosdrecht, B. J. van Wees, J. E. Mooij, C. T. Foxon, and J. J. Harris (to be published).