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A Statistical Investigation of the Properties of Quasars

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Summary. A discussion is given of most of the observational material concerning QSO's and QSS's. From the $N(m)$-relation and the V/V_{max}-test it is found that the density of *both* classes of objects increases with increasing distance; on the basis of redshifts (through the V/V_{max}-test) it is found that the co-moving density is proportional to $(1+z)^{6\pm1}$.
An analysis of the bi-variate luminosity function of QSS's shows that this function can be conveniently described by postulating a correlation between the radio luminosity function and the optical luminosity such that the radio luminosity function depends on the ratio of radio and optical luminosity. Such a correlation makes it possible to have identical optical luminosity functions for QSO's and QSS's. This formulation gives a good account of the 3 CR, 4 C and B 2 data over a range of 1.6 decades in optical luminosity. The data show that QSO's with lower optical luminosities are very rarely radio-emitters. From the QSS-observations it is found that there exists a lower limit to the ratio between radio and optical luminosity of $10^{2.3}$. This means that radio observations of sufficiently high sensitivity should result in the detection of about one third of all QSO's in an optically complete sample. The actual number of detections falls short of this prediction by a factor of about three. A possible explanation for this effect is that at earlier epochs the radio emission of an average quasar was relatively less prominent (as compared to its optical emission) than it is now.

Key words: quasars – density of quasars – luminosity function

I. Introduction

Since the discovery of QSO's (i.e. objects similar to quasi-stellar sources, but detected by optical techniques alone) there has been a fair amount of discussion about the relationship between the two classes of objects. It soon became evident that the surface density of QSO's brighter than a certain limiting magnitude is much larger than that of QSS's in the current radio catalogues (3 CR, 4 C and Parkes) above the same optical limit.

An important question raised by this observation is whether the surface density of QSS's will rapidly approach that of QSO's when the detection limits of the radio surveys become lower, or whether even an increase in sensitivity by a factor of $\simeq 100$ will fail to lead to an appreciable increase in the surface density of QSS's.

In order words, are QSO's and QSS's essentially the same objects having slightly different ratios of radio to optical power or is the majority of QSO's in reality emitting negligible amounts of radio radiation.

In the present article a discussion is given of essentially all relevant observational material concerning QSO's and QSS's. The material can be divided into two groups:
A) QSS-*Data*, consisting of optical identifications of radio sources from the 3 CR, 4 C and B 2 catalogues (Schmidt, 1968; Lynds and Wills, 1972; Bergamini *et al.*, 1972). Redshifts are available for well defined samples of QSS's in the 3 CR and 4 C catalogues.
B) QSO-*Data*, consisting of

1) optically complete samples of QSO's (Sandage and Luyten, 1969; Braccesi and Formiggini, 1969; Braccesi *et al.*, 1970). Redshifts have been measured for only a limited number of these objects.

2) high-sensitivity radio observations (Katgert *et al.*, 1972) of the optically selected sample of QSO's studied by Braccesi *et al.*

* The authors want to state explicitly that, although they are to be held responsible for the final presentation of the discussion, this article would not have appeared if it were not for the work done by many other people in Bologna and Leiden (cf. Bergamini *et al.*, 1972 and Katgert *et al.*, 1972).

II. Observed $N(m)$-Relations for QSO's and QSS's

An important feature of the QSO-data is that the slope of the cumulative number magnitude relation is significantly steeper than 0.6. Sandage and Luyten find

$$\log N(B) = 0.75\,B - 9.4$$

while Braccesi and Formiggini find

$$\log N(b) = 0.72(\pm 0.05)\,b - 8.7(\pm 0.1)$$

where we refer the reader to the original articles for the definition of the parameters. Taking into account the difference between B and b, it can be seen that the two relations agree well for $17.0 < B < 19.5$. Moreover, Sandage and Luyten find some additional evidence that the above relation may still hold for $B = 22$.

It is interesting to compare this result with the $N(m)$-relation that may be derived from the QSS-data. To this end we grouped the sources in the three radio catalogues according to the ratio between the observed radio and optical flux densities, in order to account properly for both the optical and radio selection. In fact we used a parameter r which is defined by the relation:

$$r = m_b + 2.5 \log S_{408}$$

where m_b-values are photographic blue magnitudes and S_{408} is the 408 MHz flux density in units of $10^{-26}\,\mathrm{W\,m^{-2}\,Hz^{-1}}$. For the different classes of r present in the samples used we solved for K, which for samples with well-defined limits is defined by the following relations:

$$\log N(m) = Km + \text{constant}$$

$$\log N(S) = -2.5\,K \log S + \text{constant}$$

(cf. Rees and Schmidt, 1972; also Bergamini *et al.*, 1972).

We find:

r	K
17.25 – 18.25	0.71 ± 0.06
18.25 – 19.25	0.69 ± 0.04
19.25 – 20.25	0.68 ± 0.05
> 20.25	0.64 ± 0.09

The agreement between the values of K for the different categories of sources is considered satisfactory. There is some indication of a slight correlation between K and r, a fact that is not too surprising since the sources with high r-values on the average are optically fainter than those with low values of r and consequently show the flattening of the $N(m)$-relation at fainter magnitudes. We therefore conclude that the spatial distribution of sources having $r > 17.25$ is possibly similar for different classes of r.

The weighted mean value of K for $r > 17.25$ is found to be 0.68 ± 0.04. This is in good agreement with what we obtained for the slope of the $N(m)$-relations of the QSO-samples. The fact that this value and that for QSO's are both well above 0.6 can be most simply interpreted to mean that the co-moving density of both QSO's and QSS's increases with increasing distance.

III. Observed $\langle V/V_{max} \rangle$

In the preceding paragraph we ignored the redshift information that is available for a large fraction of QSS's as well as for some of the QSO's. We will now assume these redshifts to be completely cosmological. Using conventional Friedmann models, we can directly relate redshift to distance and thus statistically investigate the distribution of quasars with respect to distance. We did V/V_{max}-tests (cf. e.g. Schmidt, 1968) for the following three samples: 3 CR, 4 C and the complete sample of QSO's for which redshifts have been obtained by Lynds (Braccesi, 1972).

We used the Friedmann model having $\sigma_0 = q_0 = 1$ and $H_0 = 100\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. In this test V simply represents the volume corresponding to the redshift z of the object, while V_{max} is the volume corresponding to the redshift z_{max} beyond which the object would no longer be included in the complete sample (in those cases where z_{max} was larger than 2.5 we nevertheless used the volume corresponding to a redshift of 2.5; however the results are insensitive to the exact choice of this cut-off redshift).

The results for the three samples are:

Sample	Number of sources	Completeness limits Radio	Completeness limits Optical	$\langle V/V_{max} \rangle$
3 CR	33	$S_{178} > 9$	$m_v < 18.4$	0.70 ± 0.05
4 C	30	$S_{178} > 2$	$m_v < 19.5$	0.67 ± 0.05
QSO	23	–	$m_i < 17.65$	0.63 ± 0.06

The quoted errors in $\langle V/V_{max} \rangle$ are equal to $(12\,n)^{-\frac{1}{2}}$ (cf. Rowan-Robinson, 1969). Regarding the statistical errors the agreement between the values for the three samples is good. Moreover, since all three values are well above 0.5 we conclude that, *assuming the redshifts to be completely cosmological*, we get the same indication from $\langle V/V_{max} \rangle$ as we got from the slope of the $N(m)$-relations namely that the (co-moving) quasar density increases with increasing distance. We should mention that it is not surprising to find for the QSO-sample a value that is slightly lower than that for the 3 CR and 4 C samples. Such differences are to be expected because the redshift distributions of the three samples are not the same. In particular the QSO-sample contains relatively many objects with *small* redshifts, which consequently have rather low optical luminosities. If we exclude from the QSO-sample those nine objects having $\log F$ (2500) < 22.4 (i.e. objects having optical luminosities that are *not* present in the 3 CR-sample) we obtain $\langle V/V_{\max}\rangle_{\rm QSO} = 0.71 \pm 0.08$ instead of 0.63 ± 0.06

IV. The Dependence of Density on Distance

In order to investigate in more detail the way in which the density changes with distance, we will assume that the dependence of density on distance is the same for *all* optical luminosities present in the various samples. At this stage such an assumption is permitted because we have no clear evidence to the contrary. We will therefore separate the general tri-variate distribution function $\varrho(z, P_o, P_r)\, dz\, dP_o\, dP_r$, which in principle gives the most complete description of the present data, as follows:

$$\varrho(z, P_o, P_r) = \varrho(z)\, \Phi(P_o, P_r)$$

in which $\varrho(z, P_o, P_r)$ is the number of sources in the interval ΔP_o, ΔP_r around P_o, P_r per co-moving unit of volume at a distance corresponding to a redshift of z; P_o and P_r denote $F(2500)$ and $P(500)$ which were introduced by Schmidt. This kind of representation, in which the z-dependence of $\varrho(z, P_o, P_r)$ is the same for all parts of the bi-variate luminosity function $\Phi(P_o, P_r)$ is generally known as pure density evolution.

The functional dependence of density on redshift may now be obtained, using all objects in the three samples for which redshifts are available. Fig. 1 shows

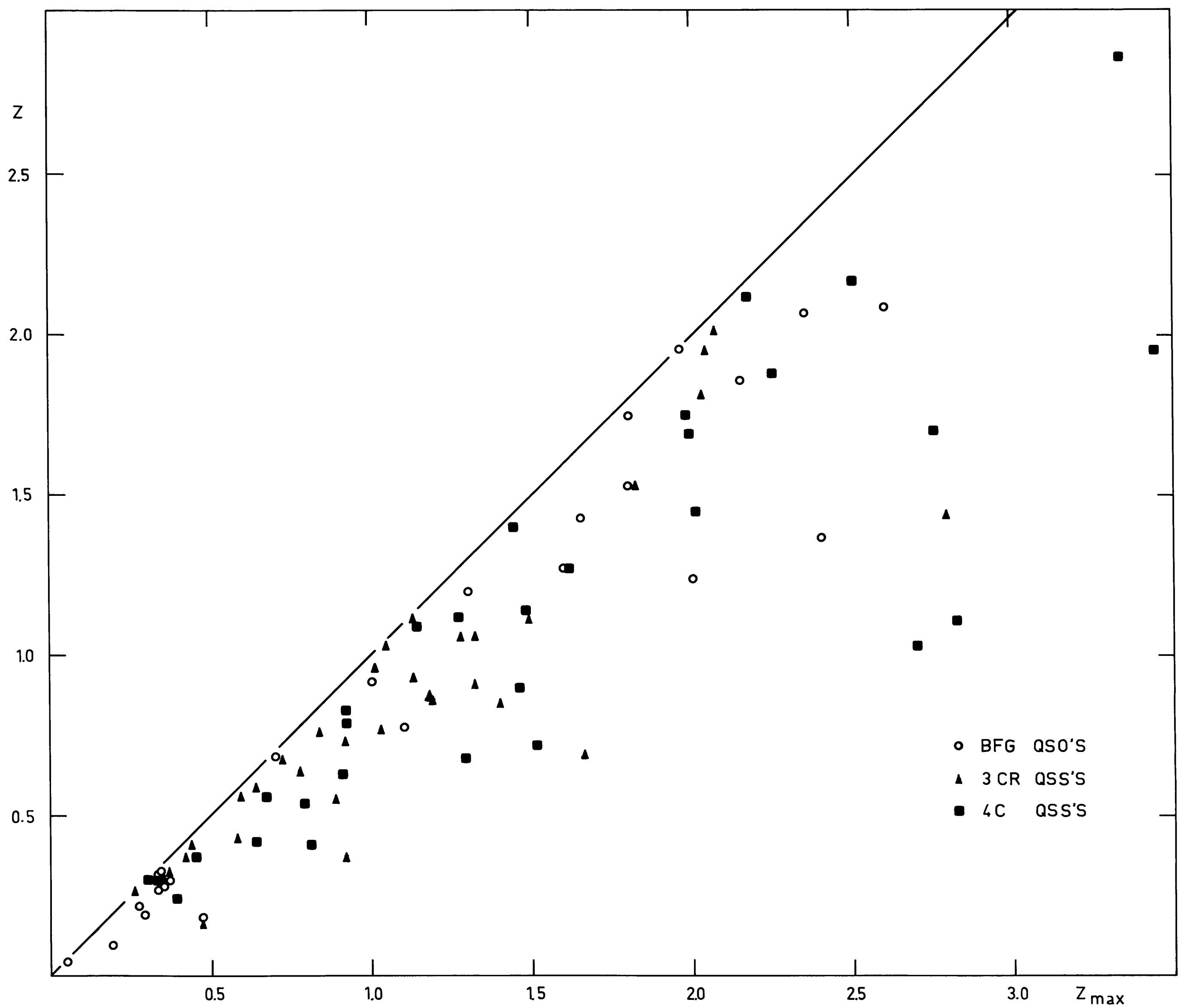

Fig. 1. The distribution in the $(z, z_{\max})$-plane of all 86 objects, from the three well-defined samples, for which redshifts are available

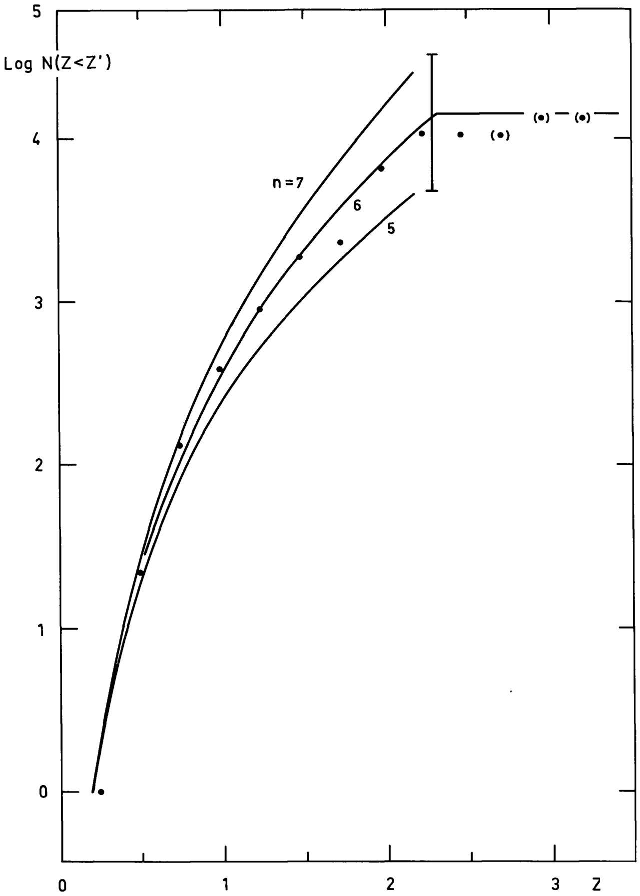

Fig. 2. The true cumulative distribution of quasars with respect to redshift (dots); three model distributions, based on a density that is proportional to $(1+z)^n$, are drawn in

the distribution of these objects in the (z, z_{max})-plane. By applying a close approximation to the so-called C-method (Lynden-Bell, 1971) we derive $N(z<z')$ i.e. the true cumulative distribution of the objects with respect to z'. $N(z<z')$ has been normalized with respect to the number of objects out to $z'=0.19$. (cf. Braccesi, 1972, for details of the procedure). The result, which is displayed in Fig. 2 has been corrected for redshift-dependent effects of incompleteness that are present if one selects upon colours. In certain intervals of red-shift $u-b$ may become less negative than -0.35 (the observational completeness limit), due to the presence of strong emission lines in the b-band.

Figure 2 also contains predictions of $N(z<z')$ based on a density which depends on redshift in the following way: $\varrho(z) \propto (1+z)^n$. It is seen that $n=6$ gives the best fit to the observations, up to about $z=2.3$. Monte Carlo trials show that the uncertainty in n is about 1 at the 1σ confidence level.

It should be noted that we purposely did not consider a density law of the form $N(V) \propto V^{2\frac{1}{4}}$. Although Lynden-Bell (1971) found that such a density law gives a better representation of the 3 CR data than does a $(1+z)^n$-law, it implies zero density at $z=0$, which seems to be in conflict with the QSO data.

In the following we shall therefore adopt:

$$\varrho(z) \propto (1+z)^6 \quad z \leqq 2.3\,,$$
$$\varrho(z) = 0 \quad z > 2.3\,.$$

V. Optical Luminosity Function of QSO's and the $N(m)$-Relation

We may now ask whether the density relation derived above is consistent with all observational evidence. It is important to predict the $N(m)$-relation for QSO's, on the basis of the assumed density function, and see whether it compares well with the observations. In order to be able to do this, we must first determine the optical luminosity function of QSO's. Because it was assumed that the luminosity function is, at least to a first approximation, independent of redshift one may simply compute the local space density of QSO's of various optical luminosities in the following way:

$$\phi(P_o) = \sum_{i=1}^{n} \delta(P_o - P_{o_i}) \left\{ \int_0^{z_{\max,}} dz\,(1+z)^6\, dV/dz \right\}^{-1} .$$

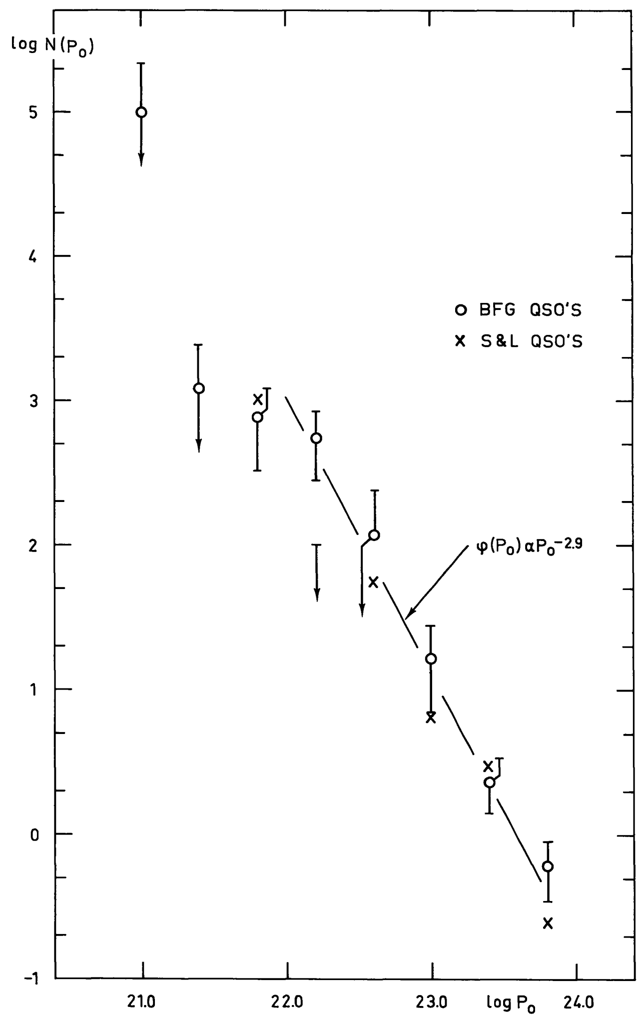

Fig. 3. The optical luminosity function of QSO's, given in the form $N(P_o) = \int_{P_o \cdot 10^{-0.2}}^{P_o \cdot 10^{+0.2}} dP\, \phi(P)$. Open circles refer to the sample studied by Braccesi *et al.*, while crosses derive from the sample studied by Sandage and Luyten (cf. Schmidt, 1970)

The result is given in Fig. 3 which also contains the unsmoothed $\phi(P_o)$ as derived by Schmidt (1970) from the Sandage and Luyten data. The two functions are seen to agree quite well. We approximated the observations by the following power law:

$$\phi(P_o)\,dP_o = \text{constant}\; P_0^{-2.9}\,dP_o$$

which represents the data satisfactorily in the range $22.0 < \log P_o < 24.0$.

In general, the slope of the $N(m)$-relation is not directly related to the slope of the luminosity function. However, in the special case where the slope of the luminosity function is the same over a very large range in luminosity, the two are related such that a -2.9 slope of the luminosity function would correspond to a -0.76 slope of the $N(m)$-relation. In practice, this means that the slope of the luminosity function implies an upper limit to the slope of the $N(m)$-relation.

The agreement between the observed and predicted slopes of the $N(m)$-relation is taken to mean that the density function used in the determination of $\phi(P_o)$ is consistent with the optical QSO-counts.

VI. $\langle V/V_{max}\rangle$ Corrected for the Density Increase

Another way to test the validity of the derived density function is to compute $\langle V/V_{max}\rangle$ using pseudo-volumes V' and V'_{max} which are defined as follows:

$$V'(z) = \int_0^z dz(1+z)^6\, dV/dz\,.$$

We obtain the following results:

Sample	$\langle V'/V'_{max}\rangle$
3 CR	0.50 ± 0.05
4 C	0.43 ± 0.05
QSO	0.52 ± 0.06

Evidently the correction for the density increase gives quite satisfactory results. Nevertheless, we consider $\langle V'/V'_{max}\rangle$ to be slightly low for the 4 C-sample. The reason could of course be that the density increase of the 4 C QSS's is somewhat less pronounced than that of quasars in the 3 CR and QSO samples. However the effect can be explained by statistical fluctuations alone. It appears that the number of 3 CR QSS's contained in the 4 C-sample is about three times larger than what one expects on the basis of the average 3 CR QSS surface density (we expect 2.5 objects instead of the 7 that have been found). These 3 CR QSS's have, of course, in a 4 C-sample low V'/V'_{max}-values. Fluctuations in their surface density may therefore have a marked effect on $\langle V'/V'_{max}\rangle$. If, in calculating $\langle V'/V'_{max}\rangle$ for the 4 C-sample, one uses the 2.5 3 CR QSS's *expected* together with the average V'/V'_{max} of the seven objects actually *found*, the value of $\langle V'/V'_{max}\rangle$ for the 4 C-sample becomes 0.47 ± 0.06.

Summarizing the preceding discussion one may conclude that the proposed separation of $\varrho(z, P_o, P_r)$ into $\varrho(z=0)\;(1+z)^6\;\Phi(P_o, P_r)$ in no way contradicts the observations. We will therefore adopt this model in the analysis of the bi-variate luminosity function of QSO's and QSS's.

VII. The Bi-Variate Luminosity Function of QSS's

Having derived $\varrho(z)$, we can now proceed to determine the bi-variate luminosity function $\Phi(P_o, P_r)$ for both the 3 CR and 4 C samples. The computational procedure is analogous to the one used in deriving the optical luminosity function of QSO's. The result is given in Table 1. (Lynds and Wills, 1972, determined $\Phi(P_o, P_r)$ at $z=1$, using the same data but a different density law.) In Table 1 each $\log P_o$, $\log P_r$-box contains two values of $\Phi(P_o, P_r)$, the first one derives from 3 CR QSS's while the second one derives from 4 C QSS's. The unit in which $\Phi(P_o, P_r)$ is given is: number of objects per cubic Gpc. It should be noted that the 3 CR QSS's in the 4 C-sample have been entered with a weight of 0.36 (2.5/7.0), for reasons which were given in the discussion of $\langle V'/V'_{max}\rangle$. In this way we secure consistency between the 3 CR and 4 C samples at the highest flux density levels. If no sources have been observed in a certain $\log P_o$, $\log P_r$ interval an upper limit is given that is based on the maximum volume over which objects in that interval would have been included in the sample.

Since the agreement between the 3 CR and 4 C luminosity functions is satisfactory, we computed the weighted mean of the two luminosity functions. The applied weights are proportional to the maximum volume over which a certain $\log P_o$, $\log P_r$ – interval is represented in the respective samples. The average bi-variate luminosity function is given in Table 2 where in the bottom row the optical luminosity function $\varphi(P_o)$ of OSO's has been entered for comparison.

If one compares the optical luminosity function of QSS's (i.e. $\varphi(P_o, P_r)$ integrated over P_r) directly with the QSO optical luminosity function, given at the bottom of Table 2 one immediately sees that for $\log P_o > 23.2$ the two are almost equal. This means that almost all QSO's with $\log P_o > 23.2$ are QSS's above the limits of the 3 CR and 4 C catalogues. Below $\log P_o = 23.2$ the percentage of radio detected QSO's decreases strongly with decreasing P_o. From Table 2 it is easy to see that all the QSO's which are radio quiet at the 4 C detection limit must populate the lower left-hand part of the (P_o, P_r)-plane. This means that they have low radio luminosities as well as low optical luminosities, because at the higher optical

Table 1. The bi-variate luminosity functions from the 3 CR and 4 C samples

$\log P_r$ \ $\log P_o$	21.6–22.0	22.0–22.4	22.4–22.8	22.8–23.2	23.2–23.6	23.6–24.0
28.8–28.4	$<1.9\cdot10^{0}$ $<4.8\cdot10^{0}$	$<4.0\cdot10^{-1}$ $<8.3\cdot10^{-1}$	$<7.0\cdot10^{-2}$ $<1.4\cdot10^{-1}$	$<1.2\cdot10^{-2}$ $<2.1\cdot10^{-2}$	$<1.6\cdot10^{-3}$ $<2.8\cdot10^{-3}$	**$3.9\cdot10^{-4}$** $<1.9\cdot10^{-3}$
28.4–28.0	$<1.9\cdot10^{0}$ $<4.8\cdot10^{0}$	$<4.0\cdot10^{-1}$ $<8.3\cdot10^{-1}$	$<7.0\cdot10^{-2}$ $<1.4\cdot10^{-1}$	**$1.3\cdot10^{-2}$** $<2.1\cdot10^{-2}$	**$2.3\cdot10^{-3}$** **$3.0\cdot10^{-3}$**	**$1.1\cdot10^{-3}$** **$8.3\cdot10^{-4}$**
28.0–27.6	$<1.9\cdot10^{0}$ $<4.8\cdot10^{0}$	$<4.0\cdot10^{-1}$ $<8.3\cdot10^{-1}$	$<7.0\cdot10^{-2}$ $<1.4\cdot10^{-1}$	**$3.9\cdot10^{-2}$** $<2.1\cdot10^{-2}$	**$1.6\cdot10^{-2}$** **$5.4\cdot10^{-3}$**	**$8.6\cdot10^{-3}$** $<2.6\cdot10^{-3}$
27.6–27.2	$<1.9\cdot10^{0}$ $<4.8\cdot10^{0}$	$<4.0\cdot10^{-1}$ **$1.6\cdot10^{-1}$**	**$7.6\cdot10^{-2}$** $<1.4\cdot10^{-1}$	$<2.9\cdot10^{-2}$ **$9.0\cdot10^{-2}$**	**$1.0\cdot10^{-1}$** **$5.3\cdot10^{-3}$**	$<2.9\cdot10^{-2}$ **$3.9\cdot10^{-2}$**
27.2–26.8	$<1.9\cdot10^{0}$ $<4.8\cdot10^{0}$	$<4.0\cdot10^{-1}$ $<8.3\cdot10^{-1}$	**$1.5\cdot10^{-1}$** **$8.0\cdot10^{-2}$**	$<1.7\cdot10^{-1}$ **$3.2\cdot10^{-1}$**	**$1.8\cdot10^{-1}$** **$5.3\cdot10^{-2}$**	**$1.1\cdot10^{-1}$** **$1.4\cdot10^{-1}$**
26.8–26.4	$<1.9\cdot10^{0}$ $<4.8\cdot10^{0}$	$<8.7\cdot10^{-1}$ $<7.2\cdot10^{-1}$	**$2.6\cdot10^{-1}$** $<6.3\cdot10^{-1}$	**$7.7\cdot10^{-1}$** **$8.2\cdot10^{-1}$**	**$2.9\cdot10^{-1}$** $<6.3\cdot10^{-1}$	$<8.7\cdot10^{-1}$ $<6.3\cdot10^{-1}$
26.4–26.0	$<4.2\cdot10^{0}$ **$2.0\cdot10^{0}$**	$<4.2\cdot10^{0}$ $<4.0\cdot10^{0}$	$<4.2\cdot10^{0}$ $<4.0\cdot10^{0}$	$<4.2\cdot10^{0}$ $<4.0\cdot10^{0}$	$<4.2\cdot10^{0}$ **$3.3.10^{0}$**	$<4.2\cdot10^{0}$ $<4.0\cdot10^{0}$
26.0–25.6	$<1.9\cdot10^{1}$ $<1.9\cdot10^{1}$	$<1.9\cdot10^{1}$ $<1.9\cdot10^{1}$	$<1.9\cdot10^{1}$ **$1.4\cdot10^{1}$**	$<1.9\cdot10^{1}$ $<1.9\cdot10^{1}$	$<1.9\cdot10^{1}$ $<1.9\cdot10^{1}$	$<1.9\cdot10^{1}$ $<1.9\cdot10^{1}$

luminosities essentially all QSO's have been detected above the limit of the 4 C catalogue.

From the QSS's in the $\log P_o$-interval 23.2 to 23.6 we may derive an almost unbiased estimate of the dependence of $\Phi(P_o, P_r)$ on P_r. Separating $\Phi(P_o, P_r)\, dP_o\, dP_r$ into $\Theta(P_o)\, dP_o\, \chi(P_r)\, dP_r$, thus implying that there is *no* correlation between P_o and P_r, we find

$$\chi(P_r)\, dP_r = \text{constant}\ P_r^{-2.3}\, dP_r.$$

Using all values of $\Phi(P_o, P_r)$ with $\log P_r > 26.4$ we may then derive

$$\Theta(P_o)\, dP_o = \text{constant}\ P_0^{-1.3}\, dP_o$$

so that one finally has

$$\Phi(P_o, P_r)\, dP_o dP_r = \text{constant}\ P_0^{-1.3} P_r^{-2.3}\, dP_o dP_r.$$

In this representation where there is *no* correlation between P_o and P_r the domain of definition is

$$P_{o_{\min}} < P_o < P_{o_{\max}}; \quad P_{r_{\min}} < P_r < P_{r_{\max}}.$$

As a result we find that in this representation the optical luminosity function of QSS's is intrinsically different from that of QSO's.

The only way to have the two optical luminosity functions identical is by assuming a correlation between P_o and the radio luminosity function. We assume a correlation such that the optical luminosity function scales with the ratio between P_o and P_r, which is consistent with the fact that the radio quiet QSS's should populate the lower left-hand corner of the (P_o, P_r)-plane. This means that we will separate $\Phi(P_o, P_r)$ as follows:

$$\Phi(P_o, P_r)\, dP_o dP_r = \Theta(P_o)\, \Psi(R)\, dP_o dP_r$$

in which $R = P_r/P_o$ and $dR = 1/P_o \cdot dP_r$. In this case the domain of definition becomes

$$P_{o_{\min}} < P_o < P_{o_{\max}}; \quad R_{\min} < R < R_{\max}.$$

If we now postulate

$$\Theta(P_o) = \phi(P_o)$$

Table 2. The average bi-variate luminosity function from 3 CR and 4 C

$\log P_r$ \ $\log P_o$	21.6–22.0	22.0–22.4	22.4–22.8	22.8–23.2	23.2–23.6	23.6–24.0
28.8–28.4						$2.9 \cdot 10^{-4}$
28.4–28.0				$7.6 \cdot 10^{-3}$	$2.6 \cdot 10^{-3}$	$1.0 \cdot 10^{-3}$
28.0–27.6				$2.3 \cdot 10^{-2}$	$1.0 \cdot 10^{-2}$	$3.7 \cdot 10^{-3}$
27.6–27.2		$7.0 \cdot 10^{-2}$	$4.4 \cdot 10^{-2}$	$4.7 \cdot 10^{-2}$	$4.0 \cdot 10^{-2}$	$2.5 \cdot 10^{-2}$
27.2–26.8			$1.2 \cdot 10^{-1}$	$1.9 \cdot 10^{-1}$	$1.0 \cdot 10^{-1}$	$1.3 \cdot 10^{-1}$
26.8–26.4			$1.2 \cdot 10^{-1}$	$8.0 \cdot 10^{-1}$	$1.3 \cdot 10^{-1}$	
26.4–26.0	$1.0 \cdot 10^{0}$				$1.8 \cdot 10^{0}$	
26.0–25.6			$7.0 \cdot 10^{0}$			
	$(2.6 \cdot 10^{3})$	$4.5 \cdot 10^{2}$	$7.5 \cdot 10^{1}$	$1.2 \cdot 10^{1}$	$2.3 \cdot 10^{0}$	$4.0 \cdot 10^{-1}$

thereby forcing the optical luminosity function of QSS's to be identical to that of QSO's, we get

$$\Phi(P_o, P_r)\, dP_o dP_r \propto P_0^{-2.9} \frac{P_r^{-2.3}}{P_0^{-2.3}} P_0^{-0.7} dP_o dP_r$$

which is, apart from a factor $P_o^{0.3}$, equal to

$$\Phi(P_o, P_r)\, dP_o dP_r \propto P_o^{-2.9} R^{-2.3} dP_o dR\,.$$

Admittedly this determination of $\Psi(R)$ is rather crude, but below $\Psi(R)$ will be derived directly from the observations.

It should be noted here that already in 1970, Schmidt introduced the concept of a correlation between the optical luminosity and the radio luminosity function, i.e. of $\Psi(R)$, in order to account for the fact that the redshift distributions of 18$^{\text{th}}$ magnitude QSO's and QSS's are indistinguishable.

VIII. The $\Psi(R)$-Function as Derived from the 3 CR and 4 C Samples

The determination of $\Psi(R)$ in the preceding section is rather incomplete in the sense that we only derived the slope of $\Psi(R)$ using the two partial derivatives of $\Phi(P_o, P_r)$, neither of which is very accurately known. A better way to determine $\Psi(R)$ is to compute it directly as the fraction $\Phi(P_o, P_r)/\Theta(P_o)$ in various $\Delta \log P_o = \Delta \log R = 0.4$ – intervals. These fractions have been entered in Table 3 together with $\Phi(P_o, P_r)$ (in parentheses the number of 3 CR and 4 C QSS's is given). It is seen that there is a good agreement between the values of $\Psi(R)$ at different optical luminosities. The two rightmost columns of Table 3 contain $\Psi(R)$ averaged over the range in optical luminosities $\log P_o = 22.4 - 24.0$, and $G(R)$ which is defined as follows: $G(R) = \int_R^{R_{max}} dR\ \Psi(R)$, where $\log R_{max} = 5.2$. Fitting a power law to $\Psi(R)$ in the range $2.8 < \log R < 5.2$, one finds: $\Psi(R)\, dR =$ constant $R^{-2.3} dR$ which is in good agreement with what we found in the previous section.

Having derived $\Psi(R)$ for $\log P_o > 22.4$ one may ask whether the same $\Psi(R)$-function also applies for $\log P_o < 22.4$. In order to answer this important question, we computed the number of sources, in each $\log P_o$, $\log R$-interval, that we should have observed. We assumed $\Theta(P_o) = \phi(P_o)$ and postulated the derived $\Psi(R)$-function to be valid for $\log P_o < 22.4$. For $\log P_o = 22.4 - 24.0$, $\log R = 2.8 - 5.2$ we predict 50.2 sources in the 3 CR and 4 C samples, while the number of objects effectively observed is 56.2 (the reason for this number being non-integer is explained in Section VI.) Of course the agreement is not surprising since $\Psi(R)$ was determined from the observations in this part of the ($\log P_o$, $\log R$)-plane. On the contrary, we predict 28.2 objects for $\log P_o = 21.6 - 22.4$,

Table 3. The bi-variate luminosity function and the $\Psi(R)$-function from 3 CR and 4 C

log R \ log P_o	21.6–22.0	22.0–22.4	22.4–22.8	22.8–23.2	23.2–23.6	23.6–24.0	$\Psi(R)$	$G(R)$
5.2								
		$7.0 \cdot 10^{-2}$ (0+1) $1.5 \cdot 10^{-4}$		$1.5 \cdot 10^{-2}$ (3+0) $1.3 \cdot 10^{-3}$			$3.3 \cdot 10^{-4}$	
4.8								$3.3 \cdot 10^{-4}$
			$1.7 \cdot 10^{-1}$ (3+1) $2.3 \cdot 10^{-3}$	$2.1 \cdot 10^{-2}$ (4+1) $1.8 \cdot 10^{-3}$	$8.4 \cdot 10^{-3}$ (5+4) $3.7 \cdot 10^{-3}$	$1.3 \cdot 10^{-3}$ (5+1) $3.2 \cdot 10^{-3}$	$2.8 \cdot 10^{-3}$	
4.4								$3.1 \cdot 10^{-3}$
	$1.0 \cdot 10^{0}$ (0+1) $3.8 \cdot 10^{-4}$		$1.1 \cdot 10^{-1}$ (1+0) $1.5 \cdot 10^{-3}$	$6.2 \cdot 10^{-2}$ (0+5) $5.2 \cdot 10^{-3}$	$6.2 \cdot 10^{-3}$ (3+1) $2.7 \cdot 10^{-3}$	$1.5 \cdot 10^{-3}$ (1+0) $3.8 \cdot 10^{-3}$	$3.3 \cdot 10^{-3}$	
4.0								$6.4 \cdot 10^{-3}$
			$3.9 \cdot 10^{-1}$ (1+0) $5.2 \cdot 10^{-3}$	$6.2 \cdot 10^{-1}$ (0+4) $5.2 \cdot 10^{-2}$	$7.6 \cdot 10^{-2}$ (3+2) $3.3 \cdot 10^{-2}$	$7.4 \cdot 10^{-3}$ (1+2) $1.8 \cdot 10^{-2}$	$2.7 \cdot 10^{-2}$	
3.6								$3.3 \cdot 10^{-2}$
	$\Phi(P_o, P_r)$ $\Psi(R)$		$7.0 \cdot 10^{0}$ (0+2) $9.3 \cdot 10^{-2}$		$2.0 \cdot 10^{-1}$ (2+0) $8.7 \cdot 10^{-2}$	$6.1 \cdot 10^{-2}$ (1+2) $1.5 \cdot 10^{-1}$	$8.3 \cdot 10^{-2}$	
3.2								$1.2 \cdot 10^{-1}$
					$1.6 \cdot 10^{0}$ (0+1) $7.0 \cdot 10^{-1}$	$8.0 \cdot 10^{-2}$ (0+1) $2.0 \cdot 10^{-1}$	$2.2 \cdot 10^{-1}$	
2.8								$3.4 \cdot 10^{-1}$
	$(2.6 \cdot 10^{3})$	$4.5 \cdot 10^{2}$	$7.5 \cdot 10^{1}$	$1.2 \cdot 10^{1}$	$2.3 \cdot 10^{0}$	$4.0 \cdot 10^{-1}$		

$\log R = 2.8 - 5.2$ of which only 1.4 have been actually observed. We interpret this deficiency of sources to mean that one cannot extrapolate $\Psi(R)$ to optical powers below $\log P_o = 22.4$. In other words: QSO's with $\log P_o < 22.4$ seem to be very rarely radio-emitters. From the preceding we conclude that:

a) $\Psi(R)$-representation is very useful for describing the bi-variate luminosity function of 3 CR and 4 C QSS's.

b) QSO's with low optical luminosities very seldom are radio-emitters.

c) From 3 CR and 4 C QSS's we find that, of a sample of QSO's complete to a given magnitude, about one third is radio emitting with $\log R > 2.8$.

IX. QSS's in the B 2 Identification Sample

It obviously is important to investigate whether the $\Psi(R)$-representation can still be used for QSS-samples with considerably lower radio detection limits. Recently an extensive identification program of sources in the B 2 catalogue was carried out by the Bologna group (Bergamini *et al.*, 1972). Identifications were obtained for 70 QSS's. The sample is thought to be complete for $m_b < 21$ and $S_{408} > 0.2$ flux units. The distribution of the objects in the (m_b, S_{408})-plane is given in Fig. 4 where lines of constant r (see above) are also drawn in.

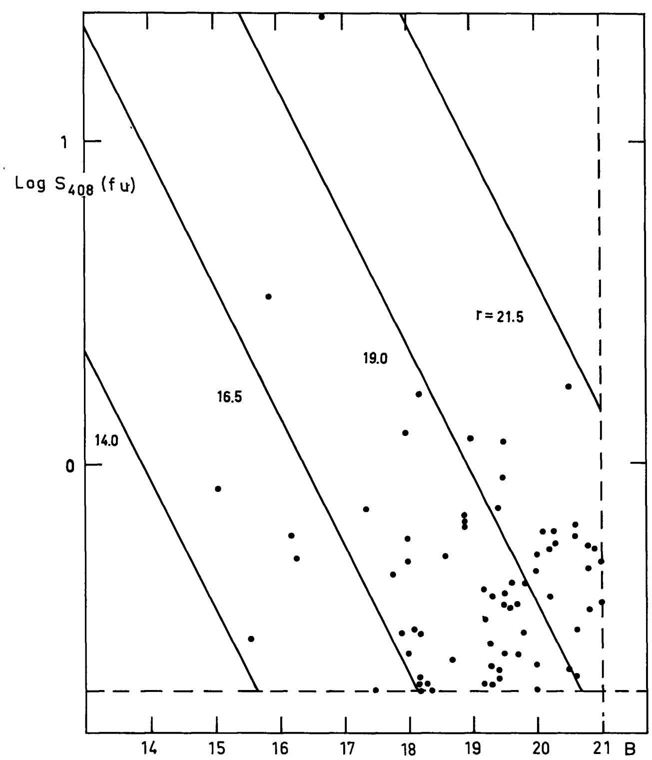

Fig. 4. The distribution of the seventy QSS's in the B2-sample according to S_{408} and B. Lines of constant $r = 2.5 \log S_{408} + B$ have been drawn

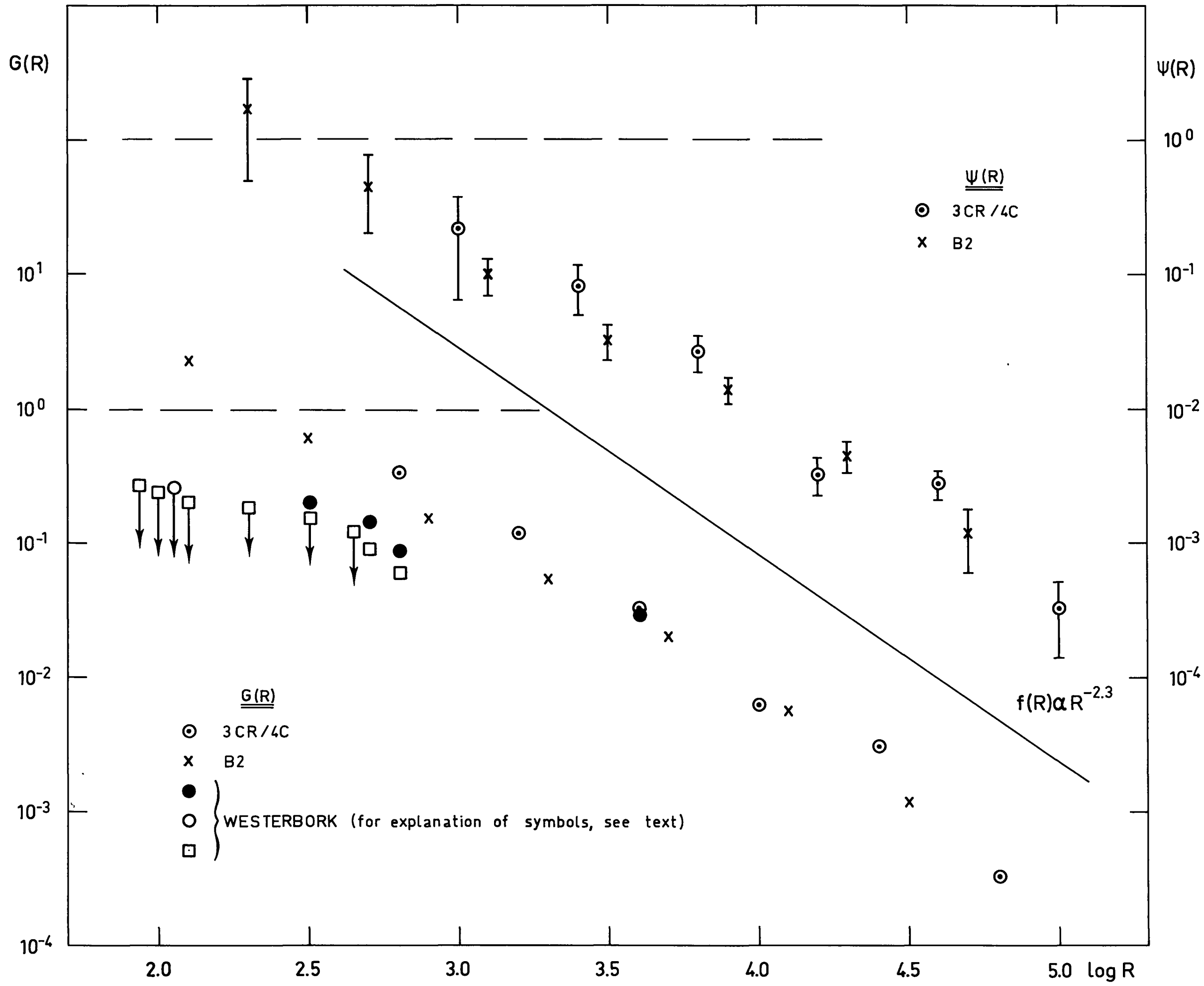

Fig. 5. The $\Psi(R)$- and $G(R)$-functions as derived from the different QSO and QSS samples

Although no redshifts are available for these QSS's (i.e. we do not know P_o and P_r) we may statistically derive R from r. More specifically, because it happens that the slope of the average QSS-spectrum is almost identical in the optical and radio domains, the relation between r and R for an average QSS is: $r=2.5\log R+9.0$.

Assuming $\Psi(R)$ to be z-independent, we can now determine it from the B 2 sample by computing $-\frac{d}{dr}\left\{\frac{N(m,r)}{N_{\mathrm{QSO}}(m)}\right\}$ where $N_{\mathrm{QSO}}(m)$ is the number of QSO's with a magnitude brighter than m, and $N(m,r)$ is the number of QSS's brighter than m with a ratio between radio and optical flux larger than $10^{0.4r}$. It must be noted that $N_{\mathrm{QSO}}(m)$ has been computed from $\varrho(z)$ and from the optical luminosity function $\phi(P_o)$ which has been truncated below $\log P_o=22.4$ for reasons given in the preceding section (the $\Psi(R)$-function given by Bergamini *et al.* differs from the one given here, because they used the observed number-magnitude relation given by Braccesi and Formiggini). Fig. 5 contains both $\Psi(R)$ and $G(R)$ as derived from the 3 C and 4 C samples as well as from the B 2 sample. The good agreement between the two $\Psi(R)$-functions indicates that the $\Psi(R)$-representation is still valid for samples of QSS's with a limiting flux density that is about ten times smaller than that of the 4 C-sample.

X. High-Sensitivity Radio Observations of an Optically Selected Sample of QSO's

Extrapolating $G(R)$ as determined from the 3 CR, 4 C and B 2 samples, one sees that essentially all QSO's with $\log P_o>22.4$ are expected to have $\log R$-values larger than about 2.3. This means that if one can lower the radio detection limits so far that the $\log R$ corresponding to $(S_{\mathrm{lim}}, m_{\mathrm{lim}})$ becomes less than 2.3, radio observations of QSO's brighter than m_{lim} should reveal all of them (with $\log P_o>22.4$) to be radio emitting above S_{lim}. More specifically, for $m_{\mathrm{lim}}\simeq 19.5$, roughly one third of all QSO's brighter than m_{lim} have $\log P_o>22.4$ and therefore should be detected radio wise.

In order to check this prediction high-sensitivity radio observations were made with the Westerbork Synthesis

Radio Telescope of the optically selected sample of QSO's studied by Braccesi *et al.* The observations are described elsewhere (Katgert *et al.*, 1972). The effective $S_{\lim}$ of 0.006 fu at 1415 MHz implies that about one third of all QSO's with $m_b < 19.3$ should have been detected.

The actual $G(R)$ derived from the Westerbork observations has also been drawn in Fig. 5. Filled circles derive from positive detections within the -3 dB contours, while the open circle represents one possible detection within the -3 dB contours. Rectangles derive from both positive and possible detections within the -10 dB contours (for an explanation of the terminology we refer to Katgert *et al.*). The inclusion of possible detections results in upper limits for $G(R)$. Above $\log R = 2.3$ the Westerbork observations are essentially complete, even though the sensitivity is not uniform in those measurements.

It is seen that below $\log R \simeq 3.0$ $G(R)$ for QSO's is systematically lower than the previously determined $G(R)$. Actually, for $\log R = 2.5$ $G(R)$ is only of the order of 0.2 as compared with 0.6 for the B2 sample at the same value of R. This means that a high percentage of the QSO's in the sample studied by Braccesi *et al.* have values of $\log R$ considerably lower than 2.5. This conclusion is corroborated by the upper limits on $\log R$ for the undetected QSO's, which for some objects are as low as 1.4.

XI. Discussion

Taking the observational data at face value, we are inclined to consider the discrepancy between $G(R)$ from the 3 CR, 4 C and B 2 samples on the one hand and that from the QSO-sample on the other hand, to be significant. We think that the range of R for which $\Psi(R)$ is significantly different from zero is definitely larger for the QSO-sample than it is for the QSS-samples, i.e. it seems to us that the $\Psi(R)$-function can no longer be regarded as a universal function, but that it depends on the radio detection limit of a given sample of quasars.

One could of course argue that $\Psi(R)$ should, by definition, be independent of the observational limits of a sample of quasars. In that case one has to explain the observed discrepancy below $\log R \simeq 3.0$ by effects which cause an overestimation of $\Psi(R)$ from the QSS-samples. Although the possibility of an overestimation of $\Psi(R)$ can not be excluded completely, we think that the good agreement between the QSS-samples for $\log R = 3.0 - 4.0$ indicates that systematic effects probably can not account for the discrepancy. We find for example that in order to make both $\Psi(R)$ agree below $\log R = 2.9$, at least four out of five B 2 QSS-identifications brighter than $m_b \simeq 17.5$ should have to be incorrect.

A possible explanation for the dependence of $\Psi(R)$ on radio detection limit could be that $\Psi(R)$, as opposed to what we have assumed throughout, is *not* independent of redshift. Because about two thirds of the objects in the QSS-samples are radio limited against essentially none in the QSO-sample, the pseudo-volume contributing to the determination of $\Psi(R)$ is on the average much larger for QSO's than it is for QSS's (viz. by factors of up to ten). As a result the observed dependence of $\Psi(R)$ on limiting flux density can be attributed to a dependence of $\Psi(R)$ on redshift, in the sense that the range of R for which $\Psi(R)$ is significantly different from zero becomes larger at larger redshifts. This would mean that at earlier epochs the radio emission of an average quasar was relatively less prominent (as compared to its optical emission) than it is now.

Clearly, this explanation puts limits on the domain of validity of the scheme that we have used in interpreting the data: i.e. we can no longer assume $\Phi(P_o, P_r)$ to be independent of redshift. However, an increase of the overall density of quasars with distance remains necessary, although it is not sufficient to explain the discrepancy by a factor of three between the two $\Psi(R)$-functions.

Unfortunately, the present data are statistically insufficient to even attempt to derive a possible dependence of $\Phi(P_o, P_r)$ on redshift. It seems to us that radio observations of well-defined QSO-samples with a sensitivity better than that of the Westerbork measurements are required for a further study of the bi-variate luminosity function of quasars.

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