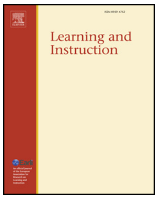
Learning and Instruction 67 (2020) 101311

Contents lists available at ScienceDirect

Learning and Instruction

journal homepage: www.elsevier.com/locate/learninstruc



Fourth graders' adaptive strategy use in solving multidigit subtraction problems

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ARTICLE INFO

Keywords:
Mathematics education
Multidigit subtraction
Strategy flexibility
Strategy adaptivity
Choice/no-choice method

ABSTRACT

Using the choice/no-choice methodology we investigated Dutch fourth graders' ($N = 124$) adaptive use of the indirect addition strategy to solve subtraction problems. Children solved multidigit subtraction problems in one choice condition, in which they were free to choose between direct subtraction and indirect addition, and in two no-choice conditions, in which they had to use either direct subtraction or indirect addition. Furthermore, children were randomly assigned to mental computation, written computation, or free choice between mental and written computation. One third of the children adaptively switched their strategy according to the number characteristics of the problems, whereas the remaining children consistently used the same strategy. The likelihood to adaptively switch strategies decreased when written computation was allowed or required, compared to mandatory mental computation. On average, children were adaptive to their own speed differences but not to the accuracy differences between the strategies.

1. Introduction

Children and adults have been known to use a variety of strategies for cognitive tasks, including arithmetic problems (Siegler, 2007). For instance in the domain of multidigit subtraction they use various ways to solve problems such as 812–784 = _, including *indirect addition* which involves adding-on from 784, or *direct subtraction* by sequentially subtracting 700, 80, and 4 (Blöte, van der Burg, & Klein, 2001; Threlfall, 2009; Torbeyns, Ghesquière, & Verschaffel, 2009). This variability is related to adaptive expertise: the ability to solve mathematics problems flexibly with a variety of meaningful strategies and adaptively by selecting the optimal strategy (Baroody, 2003; Hatano, 2003; McMullen et al., 2016; Verschaffel, Luwel, Torbeyns, & Van Dooren, 2009). This contrasts with routine expertise where children apply standard procedures in an inflexible way (Kilpatrick, Swafford, & Findell, 2001). Since adaptive expertise is an indicator of deeper mathematical understanding and a key aspect in the development of later mathematical competence (Kieran, 1992; Star et al., 2015; Xu et al., 2017), it has become an important aim of mathematics education worldwide and has received ample attention from researchers (Sievert, van den Ham, Niedermeyer, & Heinze, 2019).

In the domain of multidigit subtraction, researchers have focused on the use of the indirect addition strategy, also called subtraction-by-addition, which is assumed to be particularly efficient for small difference problems where the difference is (much) smaller than the subtrahend, such as in 812–784 = _. Although adults have been shown to use indirect addition efficiently and adaptively (Torbeyns, de Smedt, Peters, Ghesquière, & Verschaffel, 2011; Torbeyns, De Smedt, Stassens, et al., 2009), children seem much less inclined to switch to indirect addition on small difference problems (Heinze, Marschick, & Lipowsky, 2009; Hickendorff, 2018; Selter, 2001; Torbeyns, Hickendorff, & Verschaffel, 2017). However, one study using a different methodology showed that children, Flemish sixth graders, can use indirect addition efficiently and adaptively (Torbeyns, Peters, De Smedt, Ghesquière, & Verschaffel, 2018). Hence, the current study aims to replicate the latter study with a different sample (Dutch fourth graders) and in different settings, to give more insights into how adaptively children solve mathematics problems and under what circumstances.

1.1. Solution strategies for multidigit subtraction problems

Children can apply diverse strategies to solve multidigit subtraction problems (De Smedt, Torbeyns, Stassens, Ghesquière, & Verschaffel, 2010; Peltenburg, van den Heuvel-Panhuizen, & Robitzsch, 2012; Selter, Prediger, Nührenbörger, & Hußmann, 2012; Torbeyns, Ghesquière, et al., 2009). In the current study we focus on the so-called number-based strategies (also called informal or mental strategies), thus excluding digit-based written algorithms. A first classification

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https://doi.org/10.1016/j.learninstruc.2020.101311
Received 8 October 2019; Received in revised form 11 February 2020; Accepted 17 February 2020

dimension is by the operation that underlies the solution process: taking the subtrahend away from the minuend in *direct subtraction* or adding on from the subtrahend to the minuend in *indirect addition*. A second, complementary dimension is how the numbers are dealt with. In *sequential* strategies the minuend is unsplit: e.g., 812–684 via 812–600 = 212; 212–80 = 132; 132–4 = 128. In *decomposition* strategies both minuend and subtrahend are decimally decomposed, and hundreds, tens, and units are subtracted separately: 800–600 = 200; 10–80 = −70; 2–4 = −2; so the outcome is 200–70–2 = 128. Finally, in *varying* strategies the minuend and/or the subtrahend are changed to get an easier problem. An example is the compensation strategy where the subtrahend is rounded up to a nearby round number: e.g. subtract 700 instead of 684 and then add 16 since that is the amount that has been subtracted too much: 812–700 = 112; 112 + 16 = 128.

Both direct subtraction and indirect addition can be executed by means of sequential or decomposition strategies, although not all combinations are common or suitable (Hickendorff, Torbeyns, & Verschaffel, 2019; Peltenburg et al., 2012; Torbeyns et al., 2018; Van Zanten & van den Heuvel-Panhuizen, 2014). In mathematics instruction in the Netherlands, as well as in Flanders, the standard strategy for multidigit subtraction is sequential direct subtraction (Van Zanten & van den Heuvel-Panhuizen, 2014). Indirect addition using a sequential approach does not receive much instructional attention, but can be a highly efficient strategy (Torbeyns et al., 2017). In the current study we therefore focus on sequential strategies and investigate children's adaptive use of the indirect addition strategy versus direct subtraction. The results could lead to implications for including indirect addition in mathematics instruction.

1.2. *Strategy competence*

Contemporary mathematics education aspires children to acquire adaptive expertise, characterized by flexible and adaptive problem solving with a variety of meaningful strategies (Baroody, 2003; Hatano, 2003; McMullen et al., 2016; Verschaffel et al., 2009). An individual's strategy competence is defined as his or her repertoire, frequency, efficiency, and adaptivity of strategy use (Lemaire & Siegler, 1995). *Strategy repertoire* involves which strategies an individual knows and uses, *strategy distribution* involves the frequency with which the strategies are used, *strategy efficiency* involves the speed and accuracy of executing the strategies, and *strategy adaptivity* involves adapting the choice for a strategy to task and/or individual strategy efficiency characteristics. Importantly, strategy flexibility and adaptivity are not only procedural in nature but also have a basis in and effect on conceptual knowledge and deeper understanding of the relations between numbers and procedures (Baroody, 2003; Schneider, Rittle-Johnson, & Star, 2011), which explains that it is a key aspect in the development of later mathematical competence.

To investigate all strategy competence parameters simultaneously Siegler and Lemaire (1997) proposed the choice/no-choice design (see aso Luwel, Onghena, Torbeyns, Schillemans, & Verschaffel, 2009). In the first condition, the choice condition, individuals are free to select their strategy on each problem. Next, there are two or more no-choice conditions in which they have to use one strategy to solve all problems. The function of the no-choice conditions is that selection effects cannot bias strategy efficiency estimates, such as when a particular strategy is used more often by highly competent children and/or on relatively easy problems. Furthermore, it is possible to identify which strategy is optimal *for each individual on each problem*, and relate that to the individuals’ strategies used in the choice condition. As such, the choice/no-choice design is the only way in which adaptivity to individual strategy efficiency can be charted and it has been applied successfully in various mathematical domains (e.g., Fagginger Auer, Hickendorff, & van Putten, 2016; Luwel, Verschaffel, Onghena, & De Corte, 2003; Torbeyns & Verschaffel, 2016; Torbeyns, Verschaffel, & Ghesquière, 2004).

1.3. *Empirical findings of adaptive use of the indirect addition strategy*

Several studies with a choice condition only have shown that the frequency of using indirect addition is low (usually below ten percent), even when children have received implicit or explicit instruction in indirect addition (Torbeyns, De Smedt, Ghesquière, & Verschaffel, 2009a, 2009b; Blöte et al., 2001; Csíkos, 2016; De Smedt et al., 2010; Heinze, Arend, Gruessing, & Lipowsky, 2018; Heinze et al., 2009; Hickendorff, 2018; Peltenburg et al., 2012; Selter, 2001; Torbeyns et al., 2017). These results contrast with findings from Flemish choice/no-choice studies showing that adults (Torbeyns et al., 2011; Torbeyns, De Smedt, Stassens, et al., 2009) as well as children (sixth graders; Torbeyns et al., 2018) have indirect addition in their repertoire, use it frequently (40–70 percent) and as least as efficiently as direct subtraction, and are adaptive to task characteristics and to their own strategy efficiency data. These contradictory findings call for replication of the choice/no-studies in other, younger samples who have less experience in solving multidigit subtraction problems (Torbeyns et al., 2018). Furthermore, it is also relevant to widen the scope of educational settings. In the Netherlands, mathematics education is largely influenced by the principles of realistic mathematics education with flexible and adaptive strategy use as central elements (Freudenthal, 1973; Treffers, 1993; van den Heuvel-Panhuizen & Drijvers, 2014). So the Dutch instructional context is particularly favorable for developing strategy adaptivity.

So far, all choice/no-choice studies into direct subtraction and indirect addition, and the majority of the studies using only a choice design, required mental computation: i.e., all computations had to be done in the head without using paper and pencil (exceptions are studies by Blöte et al., 2001; Heinze et al., 2018, 2009; Hickendorff, 2018). Writing down a strategy or intermediate answers reduces cognitive load and supports schematizing the solution process (Ruthven, 1998). Therefore, strategy competence might be affected by mental versus written computation, as supported by findings from choice/no-choice studies in the domain of multidigit division (Fagginger Auer et al., 2016; Hickendorff, van Putten, Verhelst, & Heiser, 2010). These latter studies showed that non-algorithmic and varying strategies were more common in mental computation than in written computation (strategy repertoire and distribution), that mental computation was faster but less accurate than written computation (strategy efficiency), and that written strategies were chosen more for their accuracy advantage but mental strategies more for their speed advantage (strategy adaptivity).

The fact that most of the findings on multidigit subtraction are restricted to mandatory mental computation might give a one-sided view on all parameters of strategy competence, with a large emphasis on strategy speed. Furthermore, it does not reflect educational practices in Dutch mathematics instruction and assessment, where children are allowed to write down their solution steps (Scheltens, Hemker, & Vermeulen, 2013). Therefore, it is important to include and compare a wider and more representative range of settings regarding mental versus written computation.

1.4. *Current study*

The current study aims to give further insights into the extent to which children make adaptive strategy choices in the domain of multidigit subtraction. The first aim was to replicate Torbeyns et al. (2018)'s study in a younger, less experienced, sample (fourth compared to sixth grade) from a different educational background (Dutch compared to Flemish). The second aim was to include and compare more varied and representative settings regarding mental and written computation. As such, the current study provides a broader perspective on how adaptively children solve mathematics problems and under what circumstances, which could lead to suggestions for mathematics instruction fostering adaptive expertise.

A sample of 124 fourth graders solved small difference and large difference multidigit subtraction problems in a choice/no-choice design focusing on indirect addition versus direct subtraction. Children were assigned to one of three conditions: mandatory written computation, free choice between mental and written computation, and mandatory mental computation. There were four research questions (RQs) organized around the four parameters of strategy competence:

RQ1: What is children's strategy repertoire and distribution and how is that affected by mental or written computation? We hypothesized that most children would use indirect addition at least once but that direct subtraction would be the most frequent strategy (Hickendorff, 2018; Selter, 2001; Torbeyns, De Smedt, Ghesquière, et al., 2009a; Torbeyns et al., 2018). Furthermore, we expected indirect addiction to be less frequent when written computation was allowed or required, compared required mental computation, because informal strategies are more common in mental strategies than in written ones (Blöte et al., 2001; Fagginger Auer et al., 2016; Hickendorff et al., 2010).

RQ2: What is the strategy efficiency difference between the two strategies, and how is that affected by mental or written computation? We hypothesized indirect addition to be at least equally accurate but faster than direct subtraction, particularly on small difference problems (Torbeyns et al., 2018), and that this speed advantage would be lower when allowing or requiring written strategies, compared to mandatory mental computation (Fagginger Auer et al., 2016; Hickendorff et al., 2010).

RQ3: What is the strategy adaptivity to task characteristics, and how is that affected by mental or written computation? We hypothesized that on average indirect addition would be used relatively more often on small difference problems compared to large difference problems (Torbeyns et al., 2011; Torbeyns, De Smedt, Stassens, et al., 2009) but that there would be individual differences in strategy adaptivity between children (Hickendorff, Torbeyns, & Verschaffel, 2018; Torbeyns et al., 2017). Furthermore, we hypothesized that allowing or requiring written strategies would lead to relatively lower levels of adaptivity because the advantage of using an efficient shortcut strategy is lower when it is possible to write down solution steps to reduce cognitive load (Ruthven, 1998).

RQ4: What is the strategy adaptivity to individual strategy efficiency characteristics, and how is that affected by mental or written computation? We hypothesized that children selected their strategy according to both their individual speed and accuracy differences (Torbeyns et al., 2018), and that speed would be relatively less important when allowing or requiring written strategies, compared to required mental computation (Fagginger Auer et al., 2016).

2. Materials and methods

2.1. Participants

Participants were 124 fourth graders (68 girls; mean age 9y6m) from 12 schools in the Netherlands. The research protocol was approved by the Institute's IRB [ECPW2016-128] and only children with written parental consent participated. We collected children's most recent score on the standardized mathematics subtest of CITO's student monitoring system (Hop, Janssen, & Engelen, 2016): 14 children (11.3%) scored in percentile 0–20, 15 children (12.1%) scored in percentile 20–40, 24 children (19.4%) scored in percentile 40–60, 29 children (23.4%) scored in the percentile 60–80, and 34 children (27.4%) scored in the percentile 80–100; there were missing values for 8 children.

2.2. Materials

All participants solved three parallel series of 13 multidigit subtractions in the number domain up to 1000. Like in Torbeyns et al. (2018), there were five small difference (SD) problems with format three-digit minuend, three-digit subtrahend, and two-digit difference ranging from 12 to 28 (e.g., 903–886 = _), and five large difference (LD) problems with format three-digit minuend, two-digit subtrahend ranging from 12 to 28, and three-digit difference (e.g., 502–18 = _), see the Appendix for the full problem set. The remaining three problems were buffer items with format three-digit minuend, three-digit subtrahend, and three-digit difference where the subtrahend and the minuend both had values close to half of the minuend (e.g., 554–268 = _). All problems required crossing both tens and units. We did not include problems with unit values 0, 5 or 9 in the minuend, subtrahend or difference to prevent the use of strategies other than direct subtraction or indirect addition. To control the difficulty level of the three series, we selected problems with approximately equal mean values for the minuend, subtrahend and difference across the three series. For each problem series two orders were created.

2.3. Design

The design had one within-subject factor with three levels: choice condition, no-choice indirect addition and no-choice direct subtraction. In the choice condition, on each problem children were free to choose between direct subtraction or indirect addition. Both strategies were verbally explained by the experimenter, supported by a visual representation. In the two respective no-choice conditions, children had to solve all 13 problems with a mandatory strategy: either direct subtraction or indirect addition. Verbal instruction including practice trials and visual support were used to force children to use the required strategy.

There was one between-subject factor with three levels to which children were randomly assigned: mandatory written computation (WR condition; $n = 41$), free choice between written and mental computation (W/M condition; $n = 43$), and mandatory mental computation (ME condition; $n = 39$). In the WR condition instructions were "*In solving these problems you have to write down how you have computed the answer. Write down your steps in such a way that a classmate can understand what you have done.*" In the W/M condition instructions were "*You are free to decide whether you compute the answer in your head or that you use the test booklet to write down your steps*." In the ME condition children instructions were "*In solving these problems you cannot write your steps down on paper, only the answer. So you have to compute in your head.*"

2.4. Procedure

In all conditions, children were instructed to solve the problems as fast and accurately as possible. The three within-subject conditions took place in three different sessions on different days minimally one day and maximally one week apart. All children started with the choice condition; the order of the two no-choice conditions was balanced across children. The three problem series in two orders were balanced across the three choice/no-choice conditions. In each session children were presented an A5-sized test booklet with two practice problems and the 13 test problems. Each problem was printed on a separate page. As in an earlier choice/no-choice study in the domain of multidigit division (Fagginger Auer et al., 2016), trained experimenters recorded the solution time (time between presenting the problem and finishing writing down the answer on the designated answer line) with a stopwatch. After each problem, the experimenter checked whether the solution strategy was clear from the written notes (WR and W/M condition only) and if not, asked the child to report verbally how (s)he solved the problem. These verbal reports were audio-recorded. All strategies were coded as direct subtraction, indirect addition or other strategies (e.g., decomposition strategies) based on the solution steps written down and/or verbally reported.

2.5. Analyses

The buffer problems were excluded from the analyses, and all data were grouped across problem series and across order of no-choice condition. For the first research question (strategy repertoire and distribution), we report the frequency of direct subtraction and indirect addition and compared that between the three setting conditions. For the second research question (strategy efficiency), we ran multilevel logistic (accuracy) or linear (speed) regression analyses on accuracy and solution time data (trimmed at 150 s) of the twenty problems across the no-choice conditions with random intercepts for schools, children, and items (Fagginger Auer et al., 2016). To account for skewness of solution times a Gamma distribution was specified (Lo & Andrews, 2015). Predictor variables were no-choice condition (DS, IA), problem type (LD, SD), and setting (WR, W/M, ME). Effects were tested using Likelihood Ratio (LR) statistics.

For the third research question (adaptivity to task characteristics), we investigated whether *on average*, children used indirect addition more on small difference than on large difference problems. To analyze *individual differences* in strategy choice patterns we conducted latent class analysis (LCA), a model-based clustering technique for categorical data (Collins & Lanza, 2010; Hickendorff, Edelsbrunner, McMullen, Schneider, & Trezise, 2018). Input variables were the observed strategy choices (direct subtraction, indirect addition, other) on the ten problems in the choice condition. A bias-adjusted three-step approach (e.g., Bray, Lanza, & Tan, 2015) was used to analyze differences between the settings (WR, W/M, ME) in the prevalence of the identified strategy use patterns. All LCAs were conducted in Latent Gold 5.0 (Vermunt & Magidson, 2013).

For the fourth research question (strategy adaptivity to individual strategy efficiency) we used two approaches. First we computed the correlation between on the one hand the frequency of using indirect addition in the choice condition, and on the other hand (a) the accuracy difference and (b) the speed difference between indirect addition and direct subtraction (data from the no-choice conditions) and compared these correlations between conditions (WR, W/M, ME). In the second approach we combined accuracy and speed data by identifying for each child and for each problem what the optimal strategy for that specific problem was, by comparing the efficiency on the two parallel problems in the no-choice conditions. This information was used to evaluate whether children used that optimal strategy in the choice condition (adaptive) or not (counter-adaptive). Finally, we compared the frequency of adaptive strategy choices between settings (WR, W/M, ME).

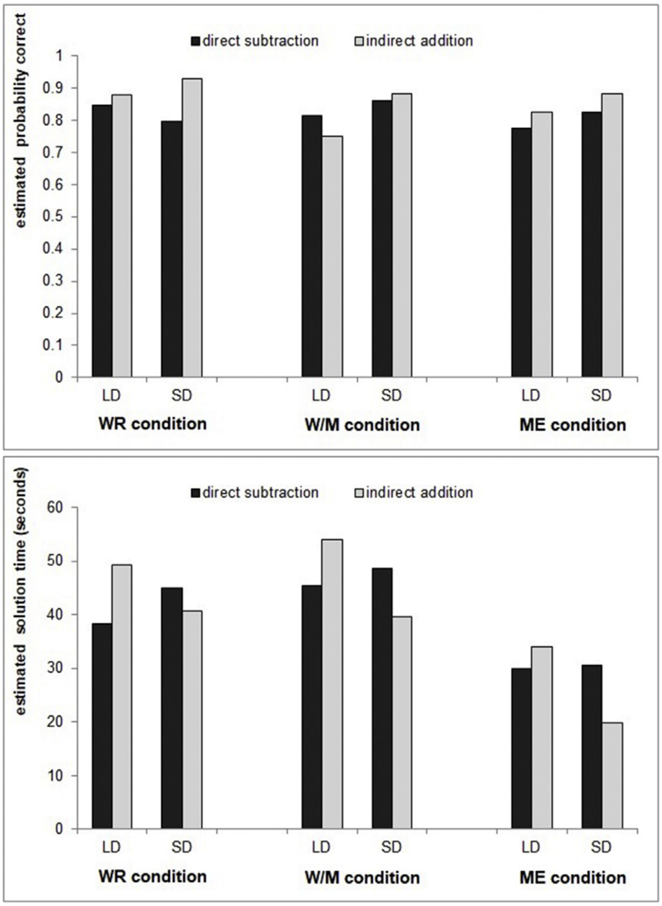

Fig. 1. Estimated accuracy (top panel) and solution times (bottom panel) in the two no-choice conditions by problem type (LD = large difference problems, SD = small difference problems) and setting (WR = written computation, W/M = free choice between written or mental computation, ME = mental computation).

3. Results

3.1. RQ1: Strategy repertoire and distribution

In the choice condition, 52 children (42%) never used indirect addition whereas 72 children (58%) used it at least once, of whom 16 children used it on all ten problems. On average, significantly more problems were solved by direct subtraction ($M = 6.10$, $SD = 3.71$) than by indirect addition ($M = 3.35$, $SD = 3.67$), t (123) = 4.236, $p < .001$. The frequency of indirect addition did not differ significantly between settings (WR: $M = 2.56$, $SD = 3.61$; W/M: $M = 3.65$, $SD = 3.85$; ME: $M = 3.85$, $SD = 3.48$), F (2,121) = 1.479, $p = .232$.

3.2. RQ2: strategy efficiency

Two children did not complete the no-choice conditions and were excluded from the analyses. Additionally, four children were excluded because they used the required strategy on fewer than eight of the ten problems. We excluded an additional 14 trials where children did not use the required strategy. This yielded an effective sample size of 2346 trials from $N = 118$ children.

3.2.1. Accuracy

First, an empty multilevel logistic regression model (i.e., without predictor variables) was estimated on accuracy data of the twenty problems across the no-choice conditions with random intercepts for schools, children and items. The random school intercept, accounting for children being nested in schools, did not improve the fit and was therefore excluded from further analyses. The random item intercept was small and including it led to convergence problems in the subsequent more complex models, so it was also excluded from further analyses. Next, the predictor variables were added in a forward model building procedure, starting with adding no-choice condition (DS, IA) to the empty model, then problem type (SD, LD), and finally setting (WR, W/M, ME), keeping significant main effects and interactions in the model. The final model included main effects of all three predictors and two-way interactions between no-choice condition and problem type, and between no-choice condition and setting. The other two-way interaction between problem type and setting and the three-way interaction were not significant. Fig. 1 (top panel) presents the estimated accuracies of direct subtraction and indirect addition for each of the combinations of problem type and setting. Post-hoc tests showed that the two strategies did not differ significantly in accuracy except for the small difference problems in the WR condition where indirect addition was more accurate than direct subtraction ($z = 4.248$, $p < .001$).

3.2.2. Speed

First, an empty multilevel regression model was estimated on trimmed speed data of the twenty problems across the no-choice conditions with random intercepts for schools, children and items. Again, the random school intercept did not improve the fit and was therefore excluded from the remaining analyses, and the random item intercept was excluded because it led to convergence problems. Next, the predictor variables were added in the same forward model building procedure as in the accuracy analyses. The final model included main effects of all three predictors and all two-way interactions; the three-way interaction was not significant. Fig. 1 (bottom panel) presents the estimated solution times of direct subtraction and indirect addition for each of the combinations of problem type and setting. Post-hoc tests showed that in each setting indirect addition was the fastest strategy on the small difference problems (WR: $z = -3.286$, $p = .001$; W/M: $z = -8.908$, $p < .001$; ME: $z = -13.802$ $p < .001$); this speed advantage increased from required written to required mental computation. By contrast, direct subtraction was the fastest strategy on the large difference problems (WR: $z = 8.262$, $p < .001$; W/M: $z = 6.202$, $p < .001$; ME: $z = 3.770$, $p < .001$) but this speed advantage decreased from required written to required mental computation.

3.3. *RQ3: strategy adaptivity to task characteristics*

Indirect addition was used significantly more often on small difference problems ($M = 2.22$, $SD = 2.18$) than on large difference problems ($M = 1.14$, $SD = 1.97$), $t\,(123) = 6.174$, $p < .001$. Thus, on average children adapted their strategy choice to number characteristics of the problems. However, investigating means may obscure individual differences in this adaptivity pattern. To identify these individual differences latent class analysis (LCA) was performed on the variables coding the strategy choice on each of the ten problems in the choice conditions. On the basis of fit statistics BIC, AIC3 and CAIC and conceptual appeal of the solution a model with three classes was selected. This model had a very good fit with mean classification error of 0.006 and R-square entropy of .983.

Fig. 2 presents the estimated probability of using each strategy on each problem, per class. Children in class 1 (45% of the sample) had a very high likelihood of using direct subtraction on all ten problems and this class was therefore labeled *Consistent use of direct subtraction.* Children in class 2 (23%) had a very high likelihood of using indirect addition on all ten problems and this class was therefore labeled *Consistent use of indirect addition.* Finally, children in class 3 (33%) were likely to use direct subtraction on the problems with a large difference, but indirect addition on the problems with a small difference. Hence, these children switched adaptively between the two strategies according to the number characteristics of the problems and this class was therefore labeled *Adaptive switchers.* The LCA thus gave an important insight that is obscured by analyzing means: the differences in strategy use between large difference and small difference problems should be attributed to one-third of the children only; the majority was very consistent in their strategy use across the problems.

Next, the relation between strategy profile class and condition (WR, W/M, ME) was significant (Wald = 12.012, $p = .017$). As Fig. 3 shows, the prevalence of the *Consistent use of direct subtraction* class was highest in the WR condition and lowest in the ME condition. The prevalence of the *Adaptive switchers* class showed the opposite pattern: it was highest in the ME condition and lowest in the WR condition. The prevalence of the *Consistent use of indirect addition* class was rather unaffected by condition. Thus, the likelihood to switch to indirect addition on the small difference problems decreased when written computation was allowed or required, compared to mental computation.

3.4. *RQ4: strategy adaptivity to individual strategy efficiency*

We first analyzed accuracy and speed separately. The correlation between frequency of indirect addition and the accuracy difference (positive values when indirect addition was more accurate than direct subtraction) was not significant: $r\,(116) = 0.109$, $p = .241$, whereas the correlation between frequency of indirect addition and the speed difference (positive values when indirect addition was faster than direct subtraction) was significant: $r\,(116) = 0.584$, $p < .001$. These correlations imply adaptivity with respect to strategy speed but not to accuracy.

Table 1 shows the correlations split by setting condition. To test whether the correlations differed significantly between conditions, we conducted ANCOVAs with frequency of indirect addition as dependent variable, accuracy or speed difference as covariate, and condition as factor. The interaction effects between condition and accuracy difference and between condition and speed difference were both not significant (F (2, 112 = 0.78, $p = .461$; $F\,(2, 112) = 0.40$, $p = .670$, respectively), indicating that the correlations did not differ significantly between conditions for either accuracy or speed.

In the second approach we combined accuracy and speed data by identifying for each child and for each problem what the optimal strategy for that specific problem was, by comparing the efficiency on the two parallel problems in the no-choice conditions. In total 1125 solutions with valid responses (i.e., either direct subtraction or indirect addition used in choice condition and the required strategy used in the no-choice conditions) were analyzed. Of the 308 instances in which only one of the two strategies resulted in a correct answer, this strategy was selected in 160 instances (adaptive to accuracy). Of the 744 instances in which both strategies yielded the correct answer the fastest strategy was selected in 509 instances (adaptive to speed). In the remaining 81 trials both strategies yielded an incorrect answer and it was not possible to determine the optimal strategy. In total, 669 (59.4%) of all strategy choices were adaptive whereas 383 (34.0%) were counter-adaptive. The difference between settings in the average number of adaptive strategy choices was not significant, $F\,(2,115) = 2.21$, $p = .114$ (ME condition: $M = 5.05$, $SD = 2.52$; W/M condition: $M = 5.71$, $SD = 2.46$; WR condition: $M = 6.20$, $SD = 2.20$).

4. Discussion

To get further insight into children's adaptive use of the indirect addition strategy in different circumstances, the current study used the choice/no-choice design to investigate the four parameters of strategy competence and compared them between three conditions: mandatory written computation, mandatory mental computation, and free choice between mental and written computation. Across conditions, one third of the children were adaptive to the problem's number characteristics whereas the remaining children were not. Adaptive switching was more frequent in mental than in written computation. On average, children were adaptive to their own speed differences between the strategies but did not take accuracy differences into account. Considering accuracy and speed simultaneously resulted in 60 percent adaptive strategy choices and 34 percent counter-adaptive choices. Adaptivity to individual efficiency data was unaffected by mental versus written computation. All in all, these results are largely in line with our hypotheses but also diverge on some aspects.

4.1. *Strategy repertoire, distribution, efficiency, and adaptivity*

The findings support our expectations regarding strategy repertoire and distribution. The majority of the children used indirect addition at least once but direct subtraction was the most frequent strategy. One-third of all problems were solved with indirect addition, which is much higher than in previous studies having a ‘choice only’ design (Blöte et al., 2001; Csíkos, 2016; De Smedt et al., 2010; Heinze et al., 2009, Heinze, Arend, Gruessing, & Lipowsky, 2018; Hickendorff, 2018; Peltenburg et al., 2012; Selter, 2001; Torbeyns, De Smedt, Ghesquière, et al., 2009b, 2009a; Torbeyns et al., 2017), but is comparable with the results of the choice/no-choice study in Flemish sixth graders (Torbeyns et al., 2018). Possibly, the fact that indirect addition was explicitly offered as a viable strategy changed the ‘didactical contract’ of what strategies can be used (Verschaffel et al., 2009; Verschaffel, Greer, & De Corte, 2007). This could make children less hesitant to use a non-standard strategy such as indirect addition than in a choice only study

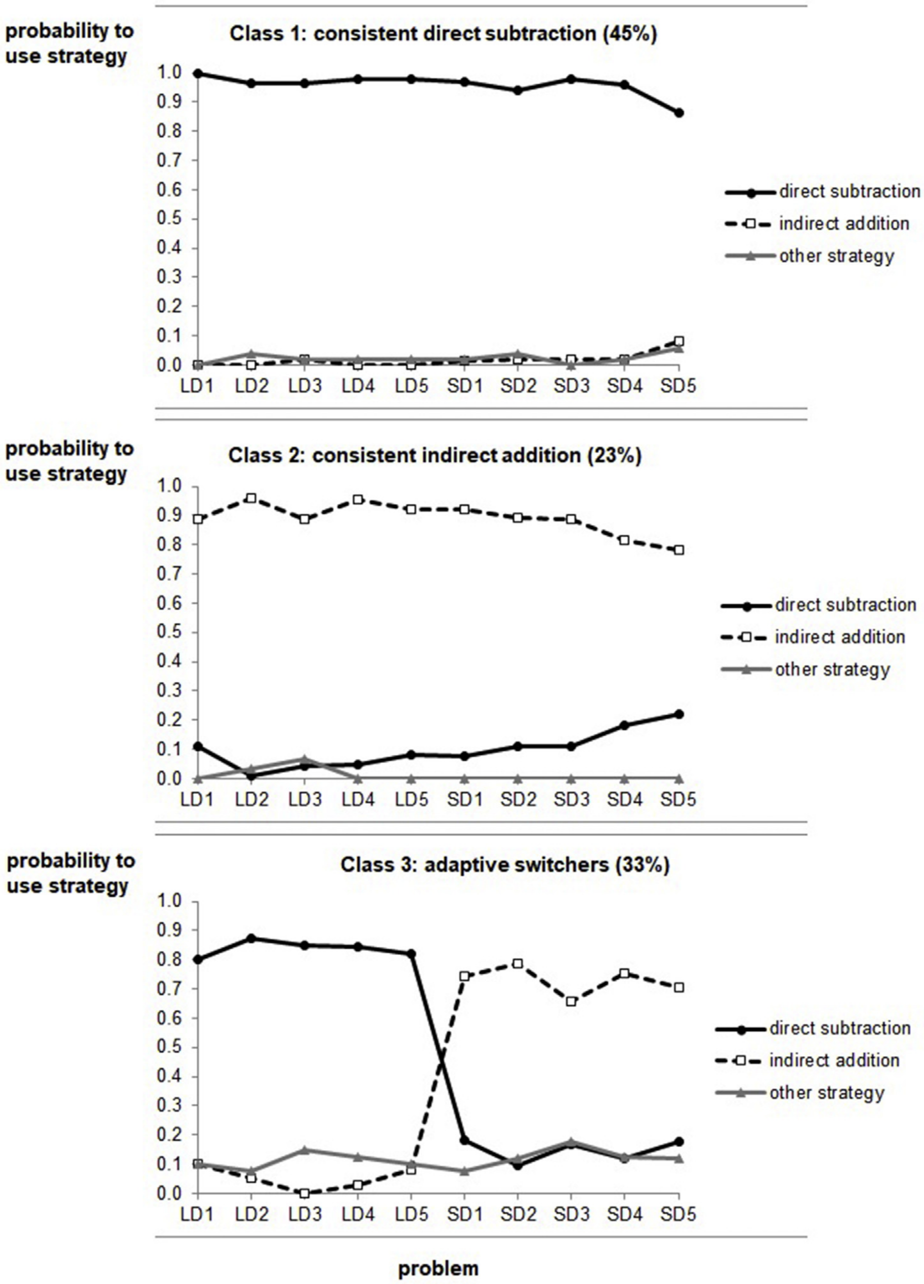

Fig. 2. Strategy use probabilities in the three latent classes. LD = large difference problems, SD = small difference problems.

where this strategy is not offered explicitly. Another potential explanation of the higher occurrence of indirect addition in the current study compared to previous choice only studies is that because indirect addition was verbalized by the experimenter, children's difficulties of verbalizing an untaught strategy might be reduced (Peters, De Smedt, Torbeyns, Ghesquière, & Verschaffel, 2012, 2013). Irrespective of the explanation it implies that there is hidden potential in the use of indirect addition, which may be uncovered by explicit instruction that indirect addition can be used.

We expected that indirect addition would be used most frequently in mandatory mental computation because mental strategies have been found to be more informal than written ones (Blöte et al., 2001; Fagginger Auer et al., 2016; Hickendorff et al., 2010). Although the means followed the hypothesized pattern, the differences did not reach statistical significance. A possible explanation is that *requiring* mental computation may result in different strategy use than *choosing* mental computation, because children may choose to compute in their heads *because* they selected a shortcut strategy such as indirect addition (Hickendorff et al., 2010).

Second, in line with our hypotheses regarding strategy efficiency, indirect addition was at least as accurate as direct subtraction (and even more accurate on the small difference problems when written computation was required) and it was the fastest strategy on the small difference problems. As expected, this speed advantage was lower when allowing or requiring written strategies. However, contrary to our expectations, direct subtraction was the fastest strategy on the large difference problems, in particular when children had to use a written strategy. This diverges from previous findings that indirect addition was faster on all problems (e.g., Torbeyns et al., 2011, Torbeyns et al., 2018), which can partly be explained by the fact that these earlier studies included mental computation only. A rational task analysis shows that on large difference problems such as 502–18 = _ indirect

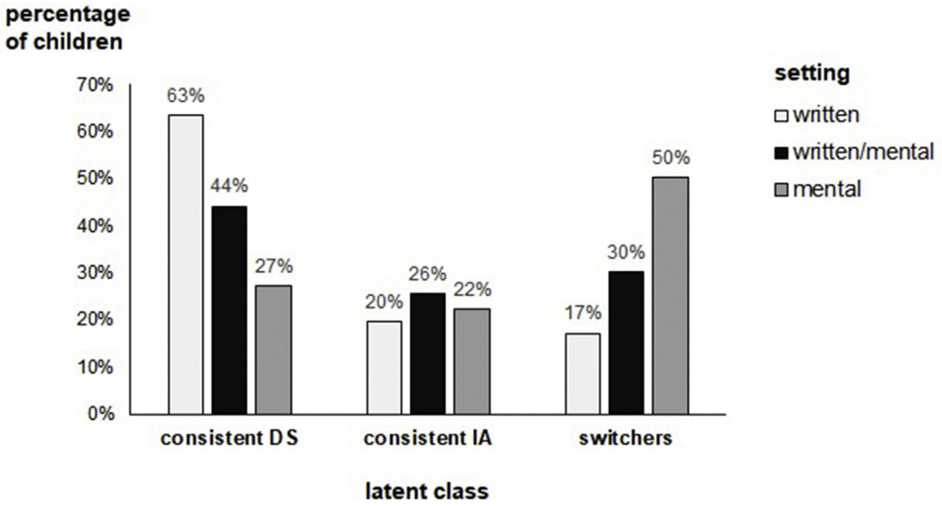

Fig. 3. Class prevalences by setting condition

Table 1

Correlations between frequency of indirect addition in the choice condition, and accuracy and speed differences from the no-choice conditions, split by setting (WR = written computation, W/M = free choice between written or mental computation, ME = mental computation).

setting	accuracy difference	speed difference
WR condition	*r* (40) = .20 (*ns*)	*r* (38) = .57 (*p* < .001)
W/M condition	*r* (38) = .20 (*ns*)	*r* (38) = .55 (*p* < .001)
ME condition	*r* (35) = .08 (*ns*)	*r* (35) = .64 (*p* < .001)

addition requires more solution steps (18 + 82 = 100; 100 + 402 = 502; so 82 + 402 = 484) than direct subtraction (502–10 = 492; 492–8 = 484), which may explain why it costs more time to execute indirect addition than direct subtraction when one has to write down the steps. A more general explanation may be that since the fourth graders in the current study had ample instruction in direct subtraction in the two preceding years (Van Zanten & van den Heuvel-Panhuizen, 2014), this may have resulted in relatively higher efficiency in direct subtraction than sixth graders who have been taught the digit-based strategy, which tends to replace number-based strategies such as direct subtraction (Selter, 2001; Torbeyns et al., 2017). This could explain that in fourth graders direct subtraction may be more efficient than indirect addition on some problems, whereas in sixth graders it is not (anymore).

Third, children were on average adaptive to task characteristics since they used indirect addition more often on the small difference than on the large difference problems. However, a latent class analysis showed that there were large individual differences between children: almost half quite consistently used direct subtraction and almost a quarter quite consistently used indirect addition across all problems. Only the remaining one third of the children adaptively switched from direct subtraction on the large difference problems to indirect addition on the small difference problems. As hypothesized, adaptive strategy switchers were most common in mandatory mental computation, possibly because the higher cognitive load of mandatory mental computation induces children to adaptively switch to a shortcut strategy such as indirect addition. This signifies the importance of taking contextual variables into account when studying strategy competence (Ellis, 1997; Verschaffel et al., 2009).

Fourth, we investigated strategy adaptivity to individual strategy efficiency. Overall, children were, as expected, adaptive to speed differences. However, contrary to our expectations based on results in Flemish sixth graders (Torbeyns et al., 2018), they did not take accuracy differences into account. These findings are similar to those in the domain of multidigit division where Dutch sixth graders were adaptive to speed but not accuracy (Fagginger Auer et al., 2016). A tentative explanation is that the educational context in the Netherlands which emphasizes strategy variability and shortcut strategies (Torbeyns et al., 2017) may induce Dutch children valuing speed over accuracy more than their Flemish peers.

Combining accuracy and speed showed that 60 per cent of all strategy choices were adaptive: either the most accurate strategy was selected, or the fastest in case both strategies yielded a correct answer. Conversely, one-third of all strategy choices were counter-adaptive. These findings illustrate that the correlations reported for speed and accuracy separately might present a too optimistic picture. Regarding differences between mental/written conditions in adaptivity to individual efficiency data, we found that although the correlations followed the expected pattern, they did not reach statistical significance. So we did not find support for the hypothesis that speed would be less important when allowing or requiring written computation, compared to mental computation (Fagginger Auer et al., 2016).

4.2. Methodological considerations

Three methodological aspects warrant further attention. First, the choice/no-choice methodology has drawbacks (for an extensive discussion, see Luwel et al., 2009). An important disadvantage is that the researcher has to decide *a priori* which strategies are focal and that the choice condition entails a forced choice rather than a free choice between strategies. As a consequence we might underestimate children's strategy adaptivity: children might be able to efficiently and adaptively use other efficient strategies such as compensation, which has been found to be easier to self-invent than indirect addition (Heinze et al., 2018). Further research may address the possibilities of individualizing the no-choice conditions, so that each child follows no-choice conditions only for the strategies that (s)he used in a free choice condition. This would enable charting the extent to which children spontaneously choose the strategy that is most efficient for them, out of their own individual strategy repertoire.

Second, verbal strategy reports can be inaccurate and can influence children's spontaneous strategy choices (Kirk & Ashcraft, 2001; Robinson, 2001). An important drawback in the context of the current study is that children may have difficulty verbalizing a (largely) untaught strategy such as indirect addition (Peters, De Smedt, Torbeyns, Verschaffel, & Ghesquière, 2014). However, since the indirect addition strategy was explicitly offered as a viable strategy and a verbal explanation of this strategy was modeled by the experimenter, we aimed to minimize these difficulties. Furthermore, the chances of forgetting or fabricating strategies have been minimalized by immediate retrospective strategy reports, which have been shown to yield veridical and non-reactive reports in children's subtraction problem solving (Robinson, 2001).

The third methodological point concerns the use of person-centered statistical approaches. Since learning and development are characterized by qualitatively different patterns or profiles of behavior it is necessary to use statistical tools that can capture this heterogeneity (Hickendorff et al., 2018; Torbeyns et al., 2017). The current study illustrates that traditional statistical analyses focusing on means obscure relevant patterns, which latent class analysis was able to uncover.

4.3. Educational implications

One educationally-relevant conclusion is that children might prefer speed over accuracy. Since sociomathematical norms, including the value attached to thoughtful versus fast performance, may impact children's strategy use (Ellis, 1997), an instructional implication is that teachers have to be cautious when stimulating fast computation, because it may go at the cost of accuracy. A second point relates to mental versus written computation. The current findings indicate that requiring mental computation may be more favorable for strategy adaptivity to number characteristics than written computation, and that it did not go at the cost of performance. However, previous studies in the domain of multidigit division showed that mental computation may lead to lower performance (Fagginger Auer et al., 2016; Hickendorff et al., 2010). A recommendation to teachers is to occasionally require mental computation to stimulate strategy adaptivity to number characteristics, but at the same time also advocate the advantages of writing down solution steps or intermediate answers, particularly in more demanding arithmetic domains.

A third educational implication concerns the position and value of indirect addition in mathematics education. Like earlier studies in Flemish adults and children, Dutch fourth graders use indirect addition frequently, efficiently, and to some extent adaptively. This raises questions to the dominance of the sequential direct subtraction strategy in mathematics instruction specifically (Selter et al., 2012; Torbeyns et al., 2018), and the (procedural) focus on only one strategy more generally. Importantly, both conceptual and procedural knowledge contribute to strategy flexibility and adaptivity (Schneider et al., 2011). Instruction and practice in alternative strategies, such as indirect addition, may stimulate the development of deeper mathematical understanding of numbers and procedures, and particularly comparing and contrasting worked examples of alternative strategies is a promising approach (Rittle-Johnson & Star, 2007; Rittle-Johnson, Star, & Durkin, 2009). A very influential source of learning opportunities is the mathematics textbook (Mullis, Martin, Foy, & Hooper, 2016; Sievert et al., 2019), and a recent study showed that mathematics textbooks differ in their quality of learning opportunities for strategy adaptivity and that this affects children's adaptive use of task-appropriate strategies (Sievert et al., 2019). An educational recommendation is therefore that textbooks include explicit instruction and practice in indirect addition. Furthermore, presenting problems in an adding-on context might elicit the indirect addition strategy in a more implicit way (Peltenburg et al., 2012). Finally, teaching practices that foster children's flexible and adaptive strategy use are stimulating children to invent, reflect, and discuss strategies, and teachers asking open questions (Blöte et al., 2001; Heinze et al., 2009; Star et al., 2015). These techniques stimulate thinking about relations between numbers and procedures and as such may contribute to the deeper understanding that strategy flexibility and adaptivity should ultimately contribute to.

5. Conclusion

The current study showed that Dutch fourth graders can use the indirect addition strategy frequently, efficiently, and to some extent adaptively. Most children were consistent in their strategy choices but one-third switched strategies according to the number characteristics, which was more common in mandatory mental than in mandatory written computation. Children may fit their strategy choice more to speed advantages than to accuracy advantages. In conclusion, indirect addition is a computationally efficient strategy and it is educationally promising to stimulate its adaptive use in mathematics textbooks and instruction, taking into account that different children have different strategy competence profiles calling for differentiated instruction.

CRediT authorship contribution statement

Marian Hickendorff: Conceptualization, Methodology, Formal analysis, Investigation, Resources, Data curation, Writing - original draft, Writing - review & editing, Visualization, Project administration.

Declaration of competing interest

None.

Acknowledgments

We are indebted to all undergraduate students who helped in data collection and to all schools and children participating in this study.

Appendix. The three sets of large difference (LD) and small difference (SD) problems used

	item set A	item set B	item set C
LD1	502–18 =	512–24 =	504–26 =
LD2	616–28 =	603–26 =	603–16 =
LD3	704–17 =	713–27 =	702–16 =
LD4	803–26 =	806–28 =	816–28 =
LD5	913–27 =	902–18 =	903–27 =
SD1	504–476 =	514–486 =	514–488 =
SD2	606–588 =	604–577 =	602–578 =
SD3	714–687 =	703–686 =	704–677 =
SD4	802–778 =	802–788 =	804–787 =
SD5	903–886 =	913–887 =	912–884 =

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