# Feature network models for proximity data : statistical inference, model selection, network representations and links with related models

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Feature Network Models for Proximity Data

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# Feature Network Models for Proximity Data

*Statistical inference, model selection, network representations and links with related models*

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in 1969

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*To my parents*

”On ne peut se flatter d’avoir le dernier mot d’une théorie, tant qu’on ne peut pas l’expliquer en peu de paroles à un passant dans la rue.”

[It is not possible to feel satisfied at having said the last word about some theory as long as it cannot be explained in a few words to any passer-by encountered in the street.]

*Joseph Diaz Gergonne, French mathematician* (Chasles, 1875, p. 115).

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# Notation and Symbols

## Notation conventions

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|---|---|
| matrices: | bold capital |
| vectors: | bold lowercase |
| scalars, integers: | lowercase |

## Symbols

| *Symbol* | *Description* |
|---|---|
| $O$ | an object or stimulus |
| $m$ | the number of objects, stimuli |
| $i$ | index $i = 1, \cdots, m$ |
| $j$ | index $j = 1, \cdots, m$ |
| $k$ | index $k = 1, \cdots, m$ |
| $n$ | the number of object pairs $= \frac{1}{2}m(m-1)$ |
| $l$ | index $l = 1, \cdots, n$ |
| $N$ | the number of replications of samples of size $n \times 1$ |
| $\ell$ | index $\ell = 1, \cdots, N$ |
| $f$ | a frequency value associated with an object pair |
| $\delta$ | a dissimilarity value associated with an object pair |
| $\hat{\delta}$ | an estimated dissimilarity value associated with an object pair |
| $\boldsymbol{\delta}$ | an $n \times 1$ vector with dissimilarities between all object pairs |
| $\hat{\boldsymbol{\delta}}$ | an $n \times 1$ vector with estimated dissimilarities between all object pairs |
| $\boldsymbol{\Delta}$ | an $m \times m$ matrix with dissimilarities |
| $\widetilde{\Delta}_{l\ell}$ | a random variable producing realisations $\tilde{\delta}_{l\ell}$ |
| $\tilde{\delta}_{l\ell}$ | a realisation of random variable $\widetilde{\Delta}_{l\ell}$ |
| $\widetilde{\boldsymbol{\Delta}}$ | an $n \times N$ matrix of random variables $\widetilde{\Delta}_{l\ell}$ |
| $\overline{\Delta}_l$ | mean of a row $l$ of $\widetilde{\boldsymbol{\Delta}}$ |
| $\varsigma$ | a similarity value associated with an object pair |
| $\boldsymbol{\varsigma}$ | an $n \times 1$ vector with similarities between all object pairs |
| $\boldsymbol{\Sigma}_\varsigma$ | an $m \times m$ matrix with similarities |
| $F$ | a feature, which is a binary $(0, 1)$ vector of size $m \times 1$ |
| $F_{\mathrm{C}}$ | a *cluster* feature, which is a binary $(0, 1)$ vector of size $m \times 1$ |
| $F_{\mathrm{U}}$ | a *unique* feature, which is a binary $(0, 1)$ vector of size $m \times 1$ |
| $T$ | the number of features |

| | |
|---|---|
| $T_{\mathrm{C}}$ | the number of *cluster* features |
| $T_{\mathrm{U}}$ | the number of *unique* features |
| $T_{\mathcal{D}}$ | the total number of distinctive features $= \frac{1}{2}(2^m) - 1$ |
| $t$ | index for the features: $t = 1, \cdots, T$ |
| $t_{\mathrm{C}}$ | index for the *cluster* features: $t_{\mathrm{C}} = 1, \cdots, T_{\mathrm{C}}$ |
| $t_{\mathrm{U}}$ | index for the *unique* features: $t_{\mathrm{U}} = 1, \cdots, T_{\mathrm{U}}$ |
| $\mathcal{S}_i$ | the set of features that represents object $O_i$ |
| $\mathbf{E}$ | an $m \times T$ matrix with columns representing features |
| $\mathbf{e}$ | a row vector from the matrix $\mathbf{E}$ |
| $e$ | an element of the matrix $\mathbf{E}$ |
| $\mathbf{E}_T$ | an $\mathbf{E}$ matrix with special feature structure that yields a tree representation |
| $\mathbf{E}_C$ | the part of $\mathbf{E}_T$ (size $m \times T_{\mathrm{C}}$) that represents the set of *cluster* features |
| $\mathbf{E}_U$ | the part of $\mathbf{E}_T$ (size $m \times T_{\mathrm{U}}$) that represents the set of *unique* features |
| $\mathbf{X}$ | an $n \times T$ matrix with featurewise distances obtained with $\mathbf{x}' = \lvert\mathbf{e}_{it} - \mathbf{e}_{jt}\rvert$ |
| $\mathbf{x}'$ | a row vector from the matrix $\mathbf{X}$ |
| $\mathbf{x}$ | a column vector from the matrix $\mathbf{X}$ |
| $\mathbf{X}_T$ | an $n \times T_{\mathrm{C}} + T_{\mathrm{U}}$ matrix with featurewise distances obtained with $\mathbf{E}_T$ |
| $\mathcal{D}$ | the complete set of featurewise distances |
| $d$ | a distance between an object pair |
| $\mathbf{d}$ | an $n \times 1$ vector of distances between all object pairs |
| $\hat{\mathbf{d}}$ | an $n \times 1$ vector of estimated distances between all object pairs |
| $\hat{\mathbf{d}}_{\mathrm{T}}$ | an $n \times 1$ vector of estimated distances between all object pairs for a tree structure |
| $\eta$ | feature discriminability parameter |
| $\eta_{\mathrm{OLS}}$ | true value of ordinary least squares feature discriminability parameter |
| $\eta_{\mathrm{ICLS}}$ | true value of inequality constrained least squares feature discriminability parameter |
| $\eta_{\mathrm{L}}$ | true value of Lasso feature discriminability parameter |
| $\eta_{\mathrm{PL}}$ | true value of Positive Lasso feature discriminability parameter |
| $\boldsymbol{\eta}$ | an $T \times 1$ vector of feature discriminability parameters |
| $\boldsymbol{\eta}_{\mathrm{OLS}}$ | an $T \times 1$ vector of true values $\eta_{\mathrm{OLS}}$ |
| $\boldsymbol{\eta}_{\mathrm{ICLS}}$ | an $T \times 1$ vector of true values $\eta_{\mathrm{ICLS}}$ |
| $\boldsymbol{\eta}_{\mathrm{L}}$ | an $T \times 1$ vector of true values $\eta_{\mathrm{L}}$ |
| $\boldsymbol{\eta}_{\mathrm{PL}}$ | an $T \times 1$ vector of true values $\eta_{\mathrm{PL}}$ |
| $\hat{\eta}$, $\hat{\eta}_{\mathrm{OLS}}$ | estimated values of $\eta$, $\eta_{\mathrm{OLS}}$, $\eta_{\mathrm{ICLS}}$, $\eta_{\mathrm{L}}$, $\eta_{\mathrm{PL}}$ |
| $C$ | the number of constraints necessary to obtain $\hat{\boldsymbol{\eta}}_{\mathrm{ICLS}}$ |
| $c$ | index $c = 1, \cdots, C$ |
| $\mathbf{r}$ | a $C \times 1$ vector with constraints |
| $\mathbf{A}$ | a $C \times T$ matrix of constraints of rank $c$ |
| $\boldsymbol{\lambda}_{\mathrm{KT}}$ | a $m \times 1$ vector with Kuhn-Tucker mutipliers |
| $\boldsymbol{\epsilon}$ | a $n \times 1$ vector with error values ($\boldsymbol{\epsilon} = \boldsymbol{\delta} - \mathbf{X}\boldsymbol{\eta}$) |
| $\hat{\boldsymbol{\epsilon}}$ | a $n \times 1$ vector with estimated error values ($\hat{\boldsymbol{\epsilon}} = \boldsymbol{\delta} - \mathbf{X}\hat{\boldsymbol{\eta}}$) |
| $\hat{\epsilon}$ | an element from the vector $\hat{\boldsymbol{\epsilon}}$ |
| $\sigma^2$, $\sigma$ | true variance and standard deviation of $\boldsymbol{\epsilon}$ |
| $\hat{\sigma}^2$, $\hat{\sigma}$ | estimated variance and standard deviation of $\hat{\boldsymbol{\epsilon}}$ |
| $\sigma^2_{\eta}$, $\sigma_{\eta}$ | true variance and standard error of $\eta$ |
| $\hat{\sigma}^2_{\eta}$, $\hat{\sigma}_{\eta}$ | estimated nominal variance and estimated nominal standard error of $\eta$ |
| $\hat{\sigma}^2_{\hat{\eta}}$, $\hat{\sigma}_{\hat{\eta}}$ | estimated nominal variance and nominal standard error of $\hat{\eta}$ |
| $\sigma^2_{\mathrm{OLS}}$, $\sigma_{\mathrm{OLS}}$ | true variance and standard error of $\hat{\eta}_{\mathrm{OLS}}$ |

| | |
|---|---|
| $\hat{\sigma}^2_{\mathrm{OLS}}, \hat{\sigma}_{\mathrm{OLS}}$ | estimated variance and standard error of $\hat{\eta}_{\mathrm{OLS}}$ |
| $\sigma^2_{\mathrm{ICLS}}, \sigma_{\mathrm{ICLS}}$ | true variance and standard error of $\hat{\eta}_{\mathrm{ICLS}}$ |
| $\hat{\sigma}^2_{\mathrm{ICLS}}, \hat{\sigma}_{\mathrm{ICLS}}$ | estimated variance and standard error of $\hat{\eta}_{\mathrm{ICLS}}$ |
| $B$ | number of bootstrap samples |
| $b$ | index $b = 1, \cdots, B$ |
| $\mathbf{b}_b$ | a bootstrap sample ($n \times 1$ vector) |
| $\mathbf{b}^*_b$ | a bootstrap sample, multivariate |
| $\tilde{\mathbf{b}}_b$ | a bootstrap sample, with sampled residuals |
| $sd_B$ | standard deviation of $B$ bootstrap samples |
| $S$ | number of simulation samples |
| $a$ | index $a = 1, \cdots, S$ |
| $\mathbf{s}^*$ | a simulation sample ($n \times 1$ vector) |
| $\kappa, p$ | parameters binomial distribution |
| $GCV$ | generalized cross-validation statistic |
| $GCV_{\mathrm{FNM}}$ | $GCV$ using inequality constrained least squares estimation |