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Division points in arithmetic

Javan Peykar, A.

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Author: Javan Peykar, A.

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Stellingen

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Division points in arithmetic

van Abtien Javan Peykar

1. Let K be a number field, let W be a finitely generated subgroup of K^* , and let $V \subset W$ be a subgroup such that W/V is finite. Let Ω be the set of maximal ideals of \mathcal{O}_K , and let $A(W, V)$ be the set of $\mathfrak{p} \in \Omega$ for which the reduction map from W to the multiplicative group of the residue class field at \mathfrak{p} is well-defined with a kernel that is contained in V . Then the set $A(W, V)$ has a natural density in Ω , and moreover, this density is a rational number.
2. Let K be a number field with algebraic closure \overline{K} , and let W be a finitely generated subgroup of K^* . Let $W^{1/\infty} = \{x \in \overline{K}^* : \text{there exists } m \in \mathbf{Z}_{\geq 1} \text{ such that } x^m \in W\}$. Then $\text{Gal}(K(W^{1/\infty})/K)$ embeds as an open subgroup in the profinite group of group automorphisms of $W^{1/\infty}$ that are the identity on W .
3. Let K be a number field with algebraic closure \overline{K} . Let E be an elliptic curve over K with $\mathcal{O} = \text{End}_K(E) \neq \mathbf{Z}$ a Dedekind domain. Let $W \subset E(K)$ be an \mathcal{O} -submodule, and let $W : \infty = \{P \in E(\overline{K}) : \text{there exists } a \in \mathcal{O} \setminus \{0\} \text{ such that } a \cdot P \in W\}$. Then $\text{Gal}(K(W : \infty)/K)$ embeds as an open subgroup in the profinite group of \mathcal{O} -module automorphisms of $W : \infty$ that are the identity on W .
4. Let K be a field of characteristic 0, let E be an elliptic curve over K with $\mathcal{O} = \text{End}_K(E) \neq \mathbf{Z}$ a Dedekind domain, let $W \subset E(K)$ be an \mathcal{O} -submodule, and let \mathfrak{a} be a nonzero ideal of \mathcal{O} . Let \overline{K} be an algebraic closure of K , and let $W : \mathfrak{a} = \{P \in E(\overline{K}) : \mathfrak{a} \cdot P \subset W\}$. Then $K(W : \mathfrak{a})$ is abelian over K if and only if

$\text{Ann}_{\mathcal{O}}(E(K)[\mathfrak{a}]) \cdot W \subset \mathfrak{a} \cdot E(K)$. This assertion is analogous to a theorem of Schinzel about the multiplicative group (see Theorem 1.1).

5. For every order A in a number field K and every prime number $l > \max\{2, [K : \mathbf{Q}]^2\}$ there is a prime number $p \leq \max\{2, 4(\log[K : \mathbf{Q}])^2\}$ and a prime ideal $\mathfrak{p} \subset A$ containing p such that the endomorphism $A_{\mathfrak{p}}^* \rightarrow A_{\mathfrak{p}}^*$ given by exponentiation with l is an automorphism of profinite groups, where $A_{\mathfrak{p}}$ denotes the completion of A at \mathfrak{p} .
6. Let G be a profinite group, let M be a profinite G -module, let $n \in \mathbf{Z}_{\geq 0}$, and let $H^n(G, M)$ be the n th continuous cochain cohomology group. Let, by functoriality, the $\widehat{\mathbf{Z}}$ -module structure on M induce a $\widehat{\mathbf{Z}}$ -module structure on $H^n(G, M)$. Then the annihilator of $H^n(G, M)$ in $\widehat{\mathbf{Z}}$ is a closed ideal of $\widehat{\mathbf{Z}}$.
7. Let K be a field of characteristic 0, let μ be the group of roots of unity in K , and let G be the Galois group of the maximal cyclotomic extension of K over K . Let M be a profinite abelian group on which G acts through its natural embedding in $\widehat{\mathbf{Z}}^*$. Then for all $n \in \mathbf{Z}_{\geq 0}$ the $\widehat{\mathbf{Z}}$ -module $H^n(G, M)$ from the previous *Stelling* is annihilated by the annihilator of μ in $\widehat{\mathbf{Z}}$.
8. Let G be a locally compact topological group, and let M be a topological abelian group. Let $C(G, M)$ be the group of continuous functions from G to M , endowed with the compact-open topology. Define a G -module structure on $C(G, M)$ by putting $g\varphi(h) = \varphi(hg)$ for $g, h \in G$ and $\varphi \in C(G, M)$. Then $C(G, M)$ is a topological G -module, and for all $n \in \mathbf{Z}_{\geq 1}$ the n th continuous cochain cohomology group $H^n(G, C(G, M))$ vanishes.