Division points in arithmetic

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Stellingen

behorende bij het proefschrift

Division points in arithmetic

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1. Let K be a number field, let W be a finitely generated subgroup of K^*, and let $V \subset W$ be a subgroup such that W/V is finite. Let Ω be the set of maximal ideals of $\mathcal{O}_K$, and let $A(W, V)$ be the set of $\mathfrak{p} \in \Omega$ for which the reduction map from W to the multiplicative group of the residue class field at $\mathfrak{p}$ is well-defined with a kernel that is contained in V. Then the set $A(W, V)$ has a natural density in Ω, and moreover, this density is a rational number.

2. Let K be a number field with algebraic closure $\overline{K}$, and let W be a finitely generated subgroup of K^*. Let $W^{1/\infty} = \{x \in \overline{K}^* : \text{there exists } m \in \mathbf{Z}_{\geq 1} \text{ such that } x^m \in W\}$. Then $\mathrm{Gal}(K(W^{1/\infty})/K)$ embeds as an open subgroup in the profinite group of group automorphisms of $W^{1/\infty}$ that are the identity on W.

3. Let K be a number field with algebraic closure $\overline{K}$. Let E be an elliptic curve over K with $\mathcal{O} = \mathrm{End}_K(E) \neq \mathbf{Z}$ a Dedekind domain. Let $W \subset E(K)$ be an $\mathcal{O}$-submodule, and let $W{:}\infty = \{P \in E(\overline{K}) : \text{there exists } a \in \mathcal{O} \setminus \{0\} \text{ such that } a \cdot P \in W\}$. Then $\mathrm{Gal}(K(W{:}\infty)/K)$ embeds as an open subgroup in the profinite group of $\mathcal{O}$-module automorphisms of $W{:}\infty$ that are the identity on W.

4. Let K be a field of characteristic 0, let E be an elliptic curve over K with $\mathcal{O} = \mathrm{End}_K(E) \neq \mathbf{Z}$ a Dedekind domain, let $W \subset E(K)$ be an $\mathcal{O}$-submodule, and let $\mathfrak{a}$ be a nonzero ideal of $\mathcal{O}$. Let $\overline{K}$ be an algebraic closure of K, and let $W{:}\mathfrak{a} = \{P \in E(\overline{K}) : \mathfrak{a} \cdot P \subset W\}$. Then $K(W{:}\mathfrak{a})$ is abelian over K if and only if

$\mathrm{Ann}_{\mathcal{O}}(E(K)[\mathfrak{a}]) \cdot W \subset \mathfrak{a} \cdot E(K)$. This assertion is analogous to a theorem of Schinzel about the multiplicative group (see Theorem 1.1).

5. For every order A in a number field K and every prime number $l > \max\{2, [K : \mathbf{Q}]^2\}$ there is a prime number $p \leq \max\{2, 4(\log[K : \mathbf{Q}])^2\}$ and a prime ideal $\mathfrak{p} \subset A$ containing p such that the endomorphism $A_{\mathfrak{p}}^* \longrightarrow A_{\mathfrak{p}}^*$ given by exponentiation with l is an automorphism of profinite groups, where $A_{\mathfrak{p}}$ denotes the completion of A at $\mathfrak{p}$.

6. Let G be a profinite group, let M be a profinite G-module, let $n \in \mathbf{Z}_{\geq 0}$, and let $\mathrm{H}^n(G, M)$ be the nth continuous cochain cohomology group. Let, by functoriality, the $\widehat{\mathbf{Z}}$-module structure on M induce a $\widehat{\mathbf{Z}}$-module structure on $\mathrm{H}^n(G, M)$. Then the annihilator of $\mathrm{H}^n(G, M)$ in $\widehat{\mathbf{Z}}$ is a closed ideal of $\widehat{\mathbf{Z}}$.

7. Let K be a field of characteristic 0, let μ be the group of roots of unity in K, and let G be the Galois group of the maximal cyclotomic extension of K over K. Let M be a profinite abelian group on which G acts through its natural embedding in $\widehat{\mathbf{Z}}^*$. Then for all $n \in \mathbf{Z}_{\geq 0}$ the $\widehat{\mathbf{Z}}$-module $\mathrm{H}^n(G, M)$ from the previous *Stelling* is annihilated by the annihilator of μ in $\widehat{\mathbf{Z}}$.

8. Let G be a locally compact topological group, and let M be a topological abelian group. Let $\mathrm{C}(G, M)$ be the group of continuous functions from G to M, endowed with the compact-open topology. Define a G-module structure on $\mathrm{C}(G, M)$ by putting $g\varphi(h) = \varphi(hg)$ for $g, h \in G$ and $\varphi \in \mathrm{C}(G, M)$. Then $\mathrm{C}(G, M)$ is a topological G-module, and for all $n \in \mathbf{Z}_{\geq 1}$ the nth continuous cochain cohomology group $\mathrm{H}^n(G, \mathrm{C}(G, M))$ vanishes.