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Brief communication

Disentangling rectangularization and life span extension with the moving rectangle method

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ABSTRACT

Purpose: The moving rectangle method is used to disentangle the contributions of rectangularization and life span extension to the increase in life expectancy. It requires the choice of an endpoint of the survival curve that approaches the maximum age at death. We examined the effect of choosing different end points on the outcomes of this method.
Methods: For five developed countries, survival curves from age 50 years were constructed per calendar year from 1922 onward. Survival values of 0.1, 0.01, and 0.001 were chosen as end points of the survival curve, and the contributions of rectangularization and life span extension to the increase in life expectancy were calculated using the moving rectangle method.
Results: The choice of different survival values as end points profoundly influenced the estimated contributions of rectangularization and life span extension to the increase in life expectancy. When choosing 0.001, rectangularization contributed most years, whereas when choosing 0.1, life span extension contributed most years.
Conclusions: When the moving rectangle method is used to estimate the contributions of rectangularization and life span extension to the increase in life expectancy, its outcomes depend on the choice of the endpoint of the survival curve.

Introduction

Over the past 170 years, life expectancy in developed countries has increased rapidly and steadily. Although in 1840, the highest recorded life expectancy in the world was 46 years for Swedish women [1], nowadays Japanese women can expect to live more than 86 years [2,3]. It is debated to what extent the increase in life expectancy is a result of rectangularization of the survival curve, which represents a shift of mortality from lower ages to the highest ages within the life span, or a result of life span extension, which denotes an increase in the maximum age at death [4–6]. For predicting trends in the increase in life expectancy and for assessing the impact of risk factors and diseases on these trends, it is important to accurately quantify the contributions of rectangularization and life span extension to the increase in life expectancy.

A method to disentangle the contributions of rectangularization and life span extension to the increase in life expectancy has been developed as an extension of the moving rectangle method [7]. The moving rectangle method is one of several comparable methods to study the process of rectangularization [4,8]. It requires the choice of an endpoint of the survival curve. Ideally, the maximum age at death is chosen, but this value is subject to high annual variation. Therefore, a less variable endpoint is chosen that corresponds with an age at which survival is very low. In a theoretical exploration of the moving rectangle method, a survival of 0.001 has been suggested as an appropriate endpoint [4]. In studies applying the method to disentangle rectangularization and life span extension, higher survival values of 0.01 and 0.1 have been chosen as end points [9–12]. Although the choice of the endpoint is regarded as

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arbitrary, it remains unknown whether this choice influences the estimations of the contributions of rectangularization and life span extension to the increase in life expectancy.

Therefore, we studied whether the choice of different survival values as end points of the survival curve influences the estimations of the contributions of rectangularization and life span extension to the increase in life expectancy using the moving rectangle method. We additionally measured the variability of the ages at the different survival values to compare their robustness as end points of the survival curve. Understanding these effects of choosing an endpoint is essential when using the moving rectangle method to disentangle rectangularization and life span extension.

Methods

We constructed survival curves per calendar year using period life tables obtained from the Human Mortality Database for five countries and for women and men separately [2]. We included data for France, the Netherlands, Sweden, and the United Kingdom from 1922 onward, the earliest year for which data were available for all countries. Japan, for which data were available from 1947 onward, was included because of the exceptional increase in its data/mortality rate. The most recent year for which data were available for all countries was 2009.

As depicted in Figure 1, we applied the moving rectangle method by which the smallest rectangle possible is fitted to each survival curve [4]. The left boundary of the rectangle is set at the age of 50 years to eliminate the influence of early mortality. Survival at the age of 50 years is set at 1 and determines the upper boundary of the rectangle. An endpoint of the survival curve is chosen at which survival is close to zero. This endpoint determines the right and lower boundaries of the rectangle. When the area of the rectangle that lies below the survival curve is divided by the area of the whole rectangle, the rectangularization index (R) is obtained. The age at the endpoint of the survival curve (E) is a direct measure of life span. The values of R and E were calculated from the life tables using a linear trapezoidal integration method in MATLAB 2012b (The MathWorks Inc, Natick, MA).

According to the extension of the moving rectangle method, which is explained in detail elsewhere [7], we calculated the life expectancy at the age of 50 years with exclusion of survival after E, hereafter simply referred to as life expectancy at the age of 50 years, from R and E by: 50 + R (E − 50). We decomposed the increase in the life expectancy at the age of 50 years between 1922 and 2009,

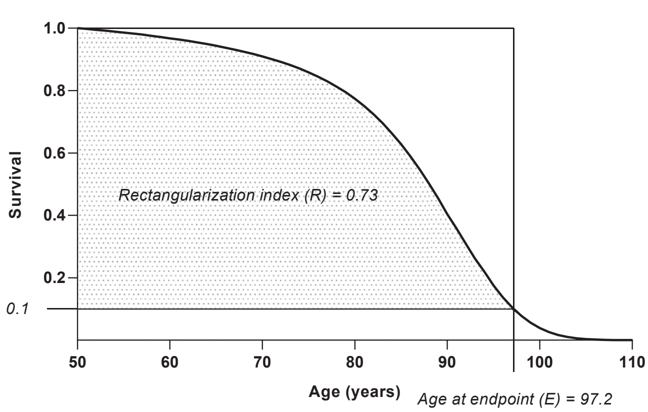

Fig. 1. A schematic overview of the moving rectangle method. Survival at age 50 years is taken as a reference and is set at one. A survival value is chosen close to zero, which provides the age at the endpoint of the survival curve (E). In this example, where a survival value of 0.1 is chosen, the age at the endpoint is 97.2. The rectangularization index (R) is calculated by dividing the shaded area of the rectangle under the curve by the entire rectangle, which is 0.73 in this example [4,7].

and between 1947 and 2009 for Japan, into the number of years attributable to either rectangularization or life span extension, separately for survival values of 0.1, 0.01, and 0.001. The number of years attributable to rectangularization was calculated by:

$$(R_{2009} - R_{1922})\left(\frac{(E_{1922} + E_{2009})}{2} - 50\right).$$

The number of years attributable to life span extension was calculated by:

$$(E_{2009} - E_{1922})\ \frac{(R_{1922} + R_{2009})}{2}.$$

In addition, to assess the variability of the age at the endpoint E of the survival curves between 1922 and 2009, separately for survival values of 0.1, 0.01, and 0.001, we calculated its moving standard deviation by:

$$\sqrt{\frac{\sum_{T=1923}^{2008}\left(E_T - \frac{E_{T-1}+E_T+E_{T+1}}{3}\right)^2}{n}}$$

with E_T being the value of E in year T, and n being the number of years in the period of interest minus the first and last year. We excluded the years from 1939 to 1946 because of the substantial influence of the Second World War on survival. We repeated the calculations using cohort data from 1850 until 1900, which were available for France, the Netherlands, and Sweden. We applied the same procedure to the highest age at which a death was recorded in each year. Confidence intervals were calculated based on the inverse χ^2 distribution.

Results

Figure 2 shows the increase in life expectancy at age 50 years for five developed countries between 1922 and 2009, and between 1947 and 2009 for Japan, decomposed into the number of years attributable to rectangularization and the number of years attributable to life span extension, separately for men and women. These data are described in more detail in Appendix A. The total increase in life expectancy differed between countries and sexes, but similar patterns were observed regarding the decomposition. With different end points of the survival curve chosen, the contributions of rectangularization and life span extension for all countries and sexes crossed the identity line, which indicates equal contributions. With a survival value of 0.1, we observed a greater contribution of life span extension. With a survival value of 0.01, the contributions of rectangularization and life span extension were comparable. With a survival value of 0.001, we observed a greater contribution of rectangularization.

The variabilities of the ages at the end points of the survival curves chosen at different survival values are given in Appendix B. Mean standard deviations were 0.30, 0.39, and 0.44 years for women and 0.28, 0.38, and 0.44 years for men for survival values of 0.1, 0.01, and 0.001. The maximum age at death was more variable with a mean standard deviation of 1.43 years for women and 1.49 years for men. For individual countries, a similar pattern was observed in both sexes. As period data in the Human Mortality Database are smoothened, we also compared the variabilities of these ages using cohort data, which have not been smoothened, and found similar patterns (Appendix C).

Discussion

In this study, we investigated the moving rectangle method as a method to disentangle the contributions of rectangularization and

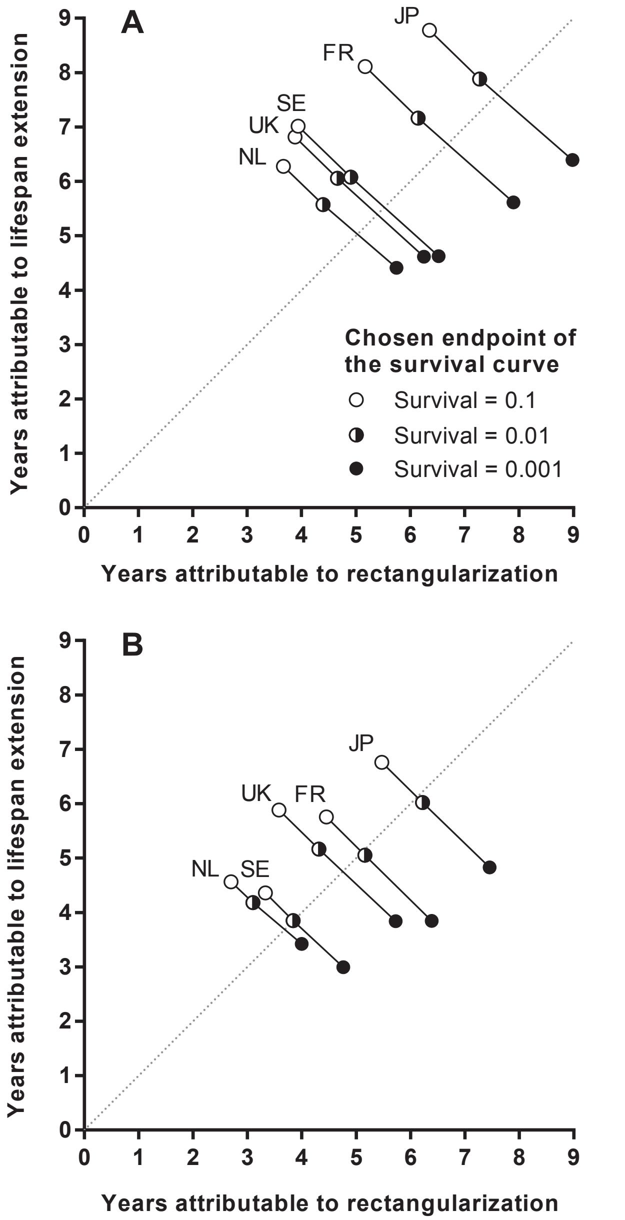

Fig. 2. The contributions of rectangularization and life span extension to the increase in life expectancy. The decomposition of the increase in life expectancy at age 50 years between 1922 and 2009 is dependent on the chosen endpoint of the survival curve. Data are shown for five developed countries: France (FR), Japan (JP), the Netherlands (NL), Sweden (SE), and the United Kingdom (UK), separately for women (A) and men (B). The different survival values that were chosen in the analyses are represented by the different symbols and are connected to illustrate the influence of this choice on the calculated contributions of rectangularization and life span extension. The identity line indicates equal contributions of rectangularization and life span extension to the increase in life expectancy.

life span extension to the increase in life expectancy. The method requires the choice of a survival value as an endpoint of the survival curve. We showed that the choice of this endpoint profoundly influences the outcomes of the method.

Our results can firstly be explained empirically. When the endpoint of the survival curve was chosen at a lower survival value, the number of years by which the endpoint increased was smaller. As a consequence, life span extension contributed less to the increase in life expectancy than rectangularization (Appendix A). Secondly, our results can be explained methodologically. As a consequence of the sharp-tailed shape of a survival curve, the choice of an endpoint at a lower survival value yields a lower rectangularization index as calculated by the moving rectangle method [7]. It follows that an increase in life expectancy by the same number of years is then attributed to a greater contribution of rectangularization or, conversely, a smaller contribution of life span extension. The method assumes that, if both rectangularization and life span extension contribute to the increase in life expectancy, the survival curve shifts to higher ages while retaining its shape [7]. Alternative methods have been proposed that distinguish between rectangularization and a shift of the survival curve [13].

According to the theoretical foundation of the moving rectangle method, the maximum age at death is ideally used as the endpoint of the survival curve [4,7]. However, this age varies highly from year to year as it is defined by a single case. Moreover, the maximum age at death can often not be deduced from survival data, as life tables are generally ended at younger ages. Therefore, as an approximation of this age, an endpoint of the survival curve is chosen at which survival is very low. Although it has theoretically been proposed to use a survival of 0.001 as an endpoint [4], studies have chosen survival values of 0.01 and 0.1 [9–12]. A low endpoint as close as possible to the maximum age at death is preferable, but the most appropriate choice of an endpoint depends on the survival data that are used. Absence, unreliability, or high variability of low survival values due to censored, incomplete, or scarce survival data at high ages may necessitate the choice of an endpoint at a higher survival value.

A previous study applied the moving rectangle method to assess how many years of the increase in life expectancy in European countries could be attributed to either rectangularization or life span extension [9]. A survival of 0.1 was chosen as an endpoint of the survival curve without a clear rationale. Life span extension was found to have had a greater contribution than rectangularization to the increase in life expectancy. Our results indicate that the opposite would have been found if 0.001 was chosen as the endpoint of the survival curve. Meanwhile, our results indicate that, although end points at lower survival values were more variable than end points at higher survival values, their variabilities were comparably small when compared with the greater variability of the maximum age at death, justifying the choice of an endpoint at a lower survival value.

Recently in this journal, a study applied the moving rectangle method to assess whether smoking affected rectangularization and life span extension in European countries [12]. A survival of 0.1 was chosen as an endpoint of the survival curve. Smoking was found to counteract rectangularization more than life span extension. Rectangularization was found to have had a smaller contribution to the increase in life expectancy for mortality not related to smoking than to the increase in life expectancy for all-cause mortality. When the contributions of rectangularization and life span extension are compared between populations—such as smokers and nonsmokers—they might not always be dependent on the choice of the endpoint. As becomes apparent from our results (Appendix A), the absolute contributions of rectangularization and life span extension differed with different choices of the endpoint, but the relations of these absolute contributions between countries remained similar. However, the relative contributions of rectangularization and life span extension as percentages of the increases in life expectancy differed and the relations of these relative contributions between countries also differed with different choices of the endpoint.

Rectangularization and life span extension both contribute to the increase in life expectancy. It remains a matter of scrutiny to what extent each has contributed to this increase and to what extent each is affected by risk factors and diseases. This study shows that the estimation of the contributions of rectangularization and life span extension to the increase in life expectancy is profoundly influenced by the chosen endpoint of the survival curve when using the moving rectangle method. Choosing a higher survival value as an endpoint yields a smaller contribution of rectangularization and a greater contribution of life span extension to the increase in life expectancy. Because studies often choose relatively high survival values, they may have underestimated the contribution of rectangularization and overestimated the contribution of life span extension to the increase in life expectancy.

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Appendix

Appendix A
Contributions of rectangularization and life span extension to the increase in life expectancy between 1850 and 1900 as dependent on the chosen endpoint of the survival curve

Endpoint of the survival curve (E)	Survival = 0.1			Survival = 0.01			Survival = 0.001		
	ΔLE50	Rectangul.	Life span ext.	ΔLE50	Rectangul.	Life span ext.	ΔLE50	Rectangul.	Life span ext.
Women									
France	13.5	5.6 (41.6)	7.9 (58.4)	13.3	7.2 (53.8)	6.1 (46.2)	13.3	8.1 (61.1)	5.2 (38.9)
Japan	15.4	6.4 (41.6)	9.0 (58.4)	15.2	7.9 (52.0)	7.3 (48.0)	15.1	8.8 (58.0)	6.4 (42.0)
The Netherlands	11.1	4.6 (41.5)	6.5 (58.5)	11.0	6.1 (55.3)	4.9 (44.7)	11.0	7.0 (64.1)	3.9 (35.9)
Sweden	10.2	4.4 (43.4)	5.7 (56.6)	10.0	5.6 (55.9)	4.4 (44.1)	9.9	6.3 (63.1)	3.7 (36.9)
United Kingdom	10.9	4.6 (42.5)	6.3 (57.5)	10.7	6.1 (56.5)	4.7 (43.5)	10.7	6.8 (63.7)	3.9 (36.3)
Men									
France	10.2	3.8 (37.6)	6.4 (62.4)	10.2	5.1 (49.5)	5.2 (50.5)	10.2	5.8 (56.4)	4.5 (43.6)
Japan	12.3	4.8 (39.3)	7.5 (60.7)	12.2	6.0 (49.2)	6.2 (50.8)	12.2	6.8 (55.3)	5.5 (44.7)
The Netherlands	7.8	3.0 (38.6)	4.8 (61.4)	7.7	3.9 (50.1)	3.8 (49.9)	7.7	4.4 (56.7)	3.3 (43.3)
Sweden	7.4	3.4 (46.1)	4.0 (53.9)	7.3	4.2 (57.4)	3.1 (42.6)	7.3	4.6 (62.9)	2.7 (37.1)
United Kingdom	9.6	3.8 (40.1)	5.7 (59.9)	9.5	5.2 (54.5)	4.3 (45.5)	9.5	5.9 (62.2)	3.6 (37.8)

The increase in life expectancy at the age of 50 years (ΔLE50), and the contributions to this increase of rectangularization (Rectangul.) and life span extension (Life span ext.) are given in years with percentages.

Appendix B
Variability of the age at the endpoint of the survival curve between 1922 and 2009 for different values of survival

Endpoint of the survival curve (E)	Women				Men			
	Survival = 0.1	Survival = 0.01	Survival = 0.001	Maximum age at death	Survival = 0.1	Survival = 0.01	Survival = 0.001	Maximum age at death
France	0.34 (0.30–0.41)	0.43 (0.37–0.51)	0.50 (0.42–0.58)	1.11 (0.96–1.32)	0.29 (0.25–0.35)	0.39 (0.34–0.46)	0.44 (0.38–0.53)	1.59 (1.38–1.89)
Japan	0.24 (0.20–0.29)	0.30 (0.25–0.36)	0.33 (0.28–0.40)	2.31 (1.95–2.83)	0.26 (0.22–0.32)	0.35 (0.29–0.42)	0.42 (0.36–0.52)	2.10 (1.78–2.57)
The Netherlands	0.34 (0.30–0.41)	0.46 (0.39–0.54)	0.53 (0.46–0.63)	1.05 (0.91–1.25)	0.34 (0.29–0.40)	0.44 (0.38–0.53)	0.52 (0.45–0.62)	1.23 (1.06–1.46)
Sweden	0.24 (0.21–0.28)	0.31 (0.27–0.31)	0.36 (0.31–0.43)	1.24 (1.08–1.48)	0.21 (0.19–0.25)	0.32 (0.27–0.38)	0.40 (0.34–0.47)	1.02 (0.88–1.21)
United Kingdom	0.34 (0.30–0.41)	0.43 (0.37–0.51)	0.47 (0.41–0.56)	NA	0.29 (0.25–0.35)	0.40 (0.34–0.47)	0.43 (0.37–0.51)	NA

NA = not available.
Standard deviations are given in years of age with 95% confidence intervals.

Appendix C

Variability of the age at the endpoint of the survival curve between 1850 and 1900 for different values of survival based on cohort survival data

Endpoint of the survival curve (E)	Women			Men		
	Survival = 0.1	Survival = 0.01	Survival = 0.001	Survival = 0.1	Survival = 0.01	Survival = 0.001
France	0.06 (0.05–0.07)	0.07 (0.06–0.09)	0.12 (0.10–0.15)	0.05 (0.05–0.07)	0.06 (0.05–0.08)	0.13 (0.11–0.16)
The Netherlands	0.07 (0.06–0.09)	0.11 (0.09–0.14)	0.29 (0.24–0.36)	0.07 (0.06–0.09)	0.12 (0.10–0.15)	0.31 (0.26–0.39)
Sweden	0.05 (0.04–0.07)	0.11 (0.09–0.14)	0.26 (0.22–0.33)	0.07 (0.06–0.09)	0.11 (0.09–0.14)	0.26 (0.22–0.32)

Standard deviations are given in years of age with 95% confidence intervals. Data were not available for Japan and the United Kingdom.