Bayesian inference for Gaussian models: Inverse problems and evolution equations

Yan, D.

Author: Yan, D.
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Part III

Evolution Equations

Heuristically speaking, a *stochastic evolution equation* in infinite dimensions describes a stochastic dynamical[1] system (a random process) whose trajectory (path) is in an infinite dimensional space. It naturally arises when the states of the dynamical system are infinite dimensional. Infinite dynamical systems pervasively exist in many quantitative fields. Examples of infinite dimensional evolution equations include population dynamics from biology, time dependent field equations emerged from physics, evolution of financial instruments such as the term structure of interest rates, whose details can be found in the introduction chapters in [23, 85]. Stochastic evolution equations are often used interchangeably[2] with *stochastic partial differential equations* (SPDEs), and we will also adopt this convention.

In this part, our goal is to use the Bayesian nonparametric approach to recover the parameters in the model, which is formally defined in Section 9.5, under the small noise asymptotic regime. That is, for $t \in [0, T]$, as n goes to infinity, we continuously observe the solution $X(t)$ of the SPDE

$$\begin{cases} dX^{(n)}(t) + \mathcal{L}X^{(n)}(t)\,dt = f(t)\,dt + \frac{1}{\sqrt{n}}B\;dW(t) \\ X^{(n)}(0) = u \in H \end{cases}, \tag{III.1}$$

where u is the initial condition. We will address that the parameter estimations of SPDEs are usually inverse problems, to which the general mechanism developed in Section 4.3 can be applied with some modifications. Consequently, the contraction rates are obtained for the recovery of the initial condition u and the drift f. For the purpose of streamlining the arguments, we only work with Gaussian priors, however it is noteworthy that the proof does not rely on any properties of the prior on conjugacy or Gaussianity, and hence, in principle, the method is applicable to other types of priors as well, such as random series priors.

The part is organized as follows. In Chapter 9, we introduce the necessary mathematical components to formalize the model (III.1). After that, we study the Bayesian statistical inference for SPDE in Chapter 10. The recovery of the initial condition u is examined Section 10.1, and the recovery of drift f is studied in Section 10.2.

[1]Differential dynamical system.

[2]Although arguably stochastic evolution equation is a broader term than SPDE.