Bayesian inference for Gaussian models: Inverse problems and evolution equations

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Part II

Inverse Problems

In Part II, under the Bayesian framework, we study the statistical linear inverse problem, i.e. the Gaussian linear model of the following form,

$$Y^{(n)} = \mathcal{A}^{(n)} f + \xi^{(n)},$$

where the operator $\mathcal{A}^{(n)}$ has no bounded inverse. The investigation is conducted focusing on the following aspects. First, we study the problem in a general framework, which in particular covers the singular value decomposition framework[1]. Second, we are interested in a unified evaluation procedure for the posterior contraction of priors, both conjugate and non-conjugate. Lastly, we consider the inference of inverse problems with discrete observations.

This part is organised as follows. In Chapter 5, we formulate the inverse problem in the smoothness scales from Chapter 2. In particular, we introduce the *Galerkin* projection method, which serves as an important tool in demonstrating posterior contractions. After the problem has been properly stated, in Chapter 6 we study the continuous model contaminated by the white noise, in which the posterior contractions are obtained using a variant of the general testing approach (Theorem 4.10), without invoking any conjugacy. The rest of the chapters in this part tackle the inverse problem with discrete observations from two angles. Chapter 7 utilises the Gaussian conjugacy in linear models to study the posterior performance, and Chapter 8 extends the results obtained in Chapter 6 to the regression model.

[1] See the discussion in Section 5.4 and the definition in Section 7.1.