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Bayesian inference for Gaussian models: Inverse problems and evolution equations

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Dong Yan

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Bayesian Inference for Gaussian Models

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TER VERKRIJGING VAN

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*To my grandparents, who enlightened me
To my mother, for everything*

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