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## Matching, entropy, holes and expansions

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# Stellingen

behorende bij het proefschrift

Matching, entropy, holes and expansions

van

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1. The natural extension of a system is a very powerful tool to find the invariant measure (Chapter 2 and 4).
2. Let  $\alpha \in [0, 1]$  and let  $T_\alpha(x) : (\alpha, 1] \rightarrow (\alpha, 1]$  be defined as  $T_\alpha(x) = \varepsilon_\alpha(x)(\frac{1}{x} - \lfloor \frac{1}{x} \rfloor) + \frac{1}{2}(1 - \varepsilon_\alpha(x))$  where  $\varepsilon_\alpha(x) = -1$  if  $x \in \cup_{n \geq 1}(\frac{1}{n+\alpha}, \frac{1}{n}]$  and  $\varepsilon_\alpha(x) = 1$  otherwise. For a large part of the parameter space the Krengel entropy and the wandering rate are proven to be independent of  $\alpha$ . Even though for different systems the same value is found and these observables give c-isomorphism invariants, the systems are not c-isomorphic (Chapter 2).
3. Let  $\alpha \in [0, 1]$  and let  $T_\alpha : [\alpha - 1, \alpha] \rightarrow [\alpha - 1, \alpha]$  be defined as  $T_\alpha(x) = \frac{1}{x} - \lfloor \frac{1}{x} + 1 - \alpha \rfloor$ . For this family, matching holds almost everywhere. Furthermore, the set for which matching does not hold is of full Hausdorff dimension (Chapter 3).
4. For (flipped or non-flipped)  $N$ -expansions the Gauss-Kuzmin-Levy based approximation method gives a good approximation of the invariant density for the corresponding dynamical system. This approximation scheme is fast, especially in the case of a low number of branches of the map (Chapter 4).
5. Let  $T_\beta$  be the greedy  $\beta$ -transformation. The set  $K_\beta(t) := \{x \in [0, 1) : T_\beta^n(x) \notin (0, t) \text{ for all } n \geq 0\}$  has both infinitely many accumulation points and infinitely many isolated points in any neighbourhood of zero for almost every  $\beta \in (1, 2]$ . Furthermore, the set of  $\beta \in (1, 2]$  for which  $K_\beta(t)$  has no isolated points has Hausdorff dimension zero (Chapter 5).

6. Matching is the next best thing after a Markov partition.
7. High entropy means fast convergence for the related expansions of the dynamical system. The Shannon-McMillan-Breiman-Chung Theorem illustrates exactly this.
8. Survivor sets often have interesting properties. Even more so, these sets are likely to be found in other contexts.
9. Dynamical systems are a great tool to gain information about number expansions.
10. Matching always has implications, although sometimes it is not clear what these implications are.
11. Statements concerning holes should not contain a h●le.