Matching, entropy, holes and expansions

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Stellingen

behorende bij het proefschrift

Matching, entropy, holes and expansions

van

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1. The natural extension of a system is a very powerful tool to find the invariant measure (Chapter 2 and 4).

2. Let $\alpha \in [0,1]$ and let $T_\alpha(x) : (\alpha,1] \to (\alpha,1]$ be defined as $T_\alpha(x) = \varepsilon_\alpha(x)(\frac{1}{x} - \lfloor \frac{1}{x} \rfloor) + \frac{1}{2}(1-\varepsilon_\alpha(x))$ where $\varepsilon_\alpha(x) = -1$ if $x \in \cup_{n\geq 1}(\frac{1}{n+\alpha}, \frac{1}{n}]$ and $\varepsilon_\alpha(x) = 1$ otherwise. For a large part of the parameter space the Krengel entropy and the wandering rate are proven to be independent of α. Even though for different systems the same value is found and these observables give c-isomorphism invariants, the systems are not c-isomorphic (Chapter 2).

3. Let $\alpha \in [0,1]$ and let $T_\alpha : [\alpha-1,\alpha] \to [\alpha-1,\alpha]$ be defined as $T_\alpha(x) = \frac{1}{x} - \lfloor \frac{1}{x} + 1 - \alpha \rfloor$. For this family, matching holds almost everywhere. Furthermore, the set for which matching does not hold is of full Hausdorff dimension (Chapter 3).

4. For (flipped or non-flipped) N-expansions the Gauss-Kuzmin-Leŷy based approximation method gives a good approximation of the invariant density for the corresponding dynamical system. This approximation scheme is fast, especially in the case of a low number of branches of the map (Chapter 4).

5. Let T_β be the greedy β-transformation. The set $K_\beta(t) := \{x \in [0,1) : T_\beta^n(x) \notin (0,t) \text{ for all } n \geq 0\}$ has both infinitely many accumulation points and infinitely many isolated points in any neighbourhood of zero for almost every $\beta \in (1,2]$. Furthermore, the set of $\beta \in (1,2]$ for which $K_\beta(t)$ has no isolated points has Hausdorff dimension zero (Chapter 5).

6. Matching is the next best thing after a Markov partition.
7. High entropy means fast convergence for the related expansions of the dynamical system. The Shannon-McMillan-Breiman-Chung Theorem illustrates exactly this.
8. Survivor sets often have interesting properties. Even more so, these sets are likely to be found in other contexts.
9. Dynamical systems are a great tool to gain information about number expansions.
10. Matching always has implications, although sometimes it is not clear what these implications are.
11. Statements concerning holes should not contain a h•le.