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Stochastic resetting and hierarchical synchronization

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Stochastic resetting and hierarchical synchronization

Janusz Martin Meylahn

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Stochastic resetting and hierarchical synchronization

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