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## Stochastic resetting and hierarchical synchronization

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# Stochastic resetting and hierarchical synchronization

Janusz Martin Meylahn

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# Stochastic resetting and hierarchical synchronization

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# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
§1.1	Stochastic Resetting . . . . .	2
§1.2	Example . . . . .	4
§1.3	Recent Results . . . . .	6
§1.4	Main results of Part I . . . . .	8
§1.5	The Stochastic Kuramoto model . . . . .	14
§1.6	Recent Results . . . . .	16
§1.7	Discrepancy . . . . .	19
§1.8	Main results of Part II . . . . .	22
<b>I</b>	<b>Large deviations of stochastic processes with resetting</b>	<b>27</b>
<hr/>		
<b>2</b>	<b>Large deviations for diffusions with resetting</b>	<b>29</b>
§2.1	Introduction . . . . .	30
§2.2	Problem . . . . .	31
§2.3	Results . . . . .	33
§2.4	Example . . . . .	35
§2.5	Conclusion . . . . .	39
<b>Appendix</b>		
§2.A	Spectral decomposition of the generating function . . . . .	40
<b>3</b>	<b>Properties of additive functionals of Brownian motion with resetting</b>	<b>43</b>
§3.1	Introduction . . . . .	44
§3.2	Two theorems . . . . .	45
§3.3	Two properties of the rate function . . . . .	48
§3.3.1	Zero rate function above the mean . . . . .	49
§3.3.2	Quadratic rate function below the mean . . . . .	53
§3.4	Positive occupation time . . . . .	54
§3.5	Area . . . . .	57
§3.6	Absolute area . . . . .	61
§3.7	Conclusion . . . . .	65
<b>Appendix</b>		

§3.A Large deviation principle . . . . .	65
§3.B Rate function of the absolute area for BM . . . . .	66
<b>II Synchronization on networks with community structure</b>	<b>67</b>
<hr/>	
<b>4 Synchronization of phase oscillators on the hierarchical lattice</b>	<b>69</b>
§4.1 Introduction . . . . .	70
§4.1.1 Mean-field Kuramoto model . . . . .	71
§4.1.2 McKean-Vlasov equation . . . . .	73
§4.1.3 Diffusive scaling of the average phase . . . . .	76
§4.1.4 Hierarchical lattice . . . . .	77
§4.1.5 Hierarchical Kuramoto model . . . . .	77
§4.2 Main results . . . . .	79
§4.2.1 Multi-scaling . . . . .	79
§4.2.2 Truncation approximation . . . . .	81
§4.2.3 Universality classes . . . . .	82
§4.3 Multi-scaling for the block average phases . . . . .	84
§4.3.1 Diffusive scaling of the average phase for mean-field Kuramoto	84
§4.3.2 Multi-scaling of the block average phases for hierarchical Kuramoto . . . . .	88
§4.4 Universality classes and synchronization levels . . . . .	94
§4.4.1 Properties of the renormalization map . . . . .	94
§4.4.2 Renormalization . . . . .	95
<b>Appendix</b>	
§4.A Numerical analysis . . . . .	99
<b>5 Two-community noisy Kuramoto model</b>	<b>103</b>
§5.1 Background and motivation . . . . .	104
§5.2 Basic properties . . . . .	105
§5.2.1 Model . . . . .	105
§5.2.2 McKean-Vlasov limit . . . . .	108
§5.2.3 Stationary solutions . . . . .	109
§5.3 Symmetric interaction with fixed phase difference . . . . .	111
§5.3.1 Without disorder . . . . .	112
§5.3.2 With disorder . . . . .	116
§5.4 Bifurcation of non-symmetric solutions . . . . .	118
§5.4.1 Existence and characterization of non-symmetric solutions . .	119
§5.4.2 Ordering of non-symmetric solutions . . . . .	124
§5.4.3 Properties of the bifurcation line . . . . .	125
§5.5 Simulation . . . . .	132
<b>Appendix</b>	
§5.A Concavity of ratio of modified Bessel functions of the first kind . . .	134

<b>6 The two-community noisy Kuramoto model as model for the Suprachiasmatic nucleus</b>	<b>139</b>
§6.1 Introduction . . . . .	140
§6.2 Model . . . . .	141
§6.3 Simulations . . . . .	144
§6.4 Discussion . . . . .	145
§6.5 Conclusion . . . . .	146
<b>Bibliography</b>	<b>151</b>
<b>Samenvatting</b>	<b>160</b>
<b>Acknowledgements</b>	<b>162</b>
<b>Curriculum Vitae</b>	<b>163</b>