Aspects of cosmic acceleration

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INTRODUCTION

Cosmological research is about the global, large-scale properties of the universe. It is one of the most actively developing fields of modern physics. This rapid flourishing of the field is partly motivated by the Nobel Prize winning discovery of cosmic acceleration in 1998 [1, 2], and partly by the fact that cosmology can serve as a uniquely fascinating laboratory for testing various aspects of fundamental theories of physics. Indeed, it is already widely acknowledged that the cosmological observations suggest tests at regimes which are by far not accessible at the laboratory setups.

All the wealth of cosmological observations are consistently explained by a phenomenological model referred to as the *cosmological standard model*. This model assumes, first of all, that the universe is homogeneous and isotropic at the largest scales. Additionally, it is now well measured that the biggest share in the energy budget of the universe, about 68%, belongs to the *cosmological constant*, Λ - a constant energy density component with a negative pressure. Such a component causes the universe to expand with increasing rate, a phenomenon known as *cosmic acceleration*. In addition to this, about 27% of the universe is composed of a non-relativistic, pressureless gas called *cold dark matter*, which interacts gravitationally, but does not interact electromagnetically, and hence can be observed only through its gravitational effects. The conventional baryonic matter and radiation together make only about 5% of the universe's energy budget. This matter content, together with a hypothesized short period of very rapid expansion of spacetime in the very early universe, known as *cosmic inflation*, provides a beautifully simple interpretation of practically all the currently available cosmological observations in the context of *General Theory of Relativity (GR)*. This cosmological model is often referred to as the Λ-Cold Dark Matter (ΛCDM) model.

The rough timeline of the universe is that it experienced a rapid (inflationary) expansion during its earliest stages. This expansion caused most of the inhomogeneity and anisotropies in spacetime to reduce, and the spatial curvature to flatten out (see later in this chapter). After the inflationary stage the universe *reheats*, i.e it becomes dominated by a relativistic plasma. As universe expands, the energy density of this relativistic plasma decreases and the universe enters the epoch dominated by non-relativistic particles - baryons and dark matter. At some point the energy of collisions in cosmic plasma decreases so much that neutral atoms are formed, and the residual photons, unable to Compton-scatter on free electrons anymore, freestream through the entire universe. Later on, as the universe becomes dominated by dark matter, the small fluctuations in density start to grow, eventually leading to formation of galaxies and galaxy clusters. The matter dominated epoch then is followed by an accelerated expansion caused by yet unknown mechanism. Phenomenologically the simplest candidate for this unknown mechanism is the cosmological constant mentioned above.

Even though phenomenologically extremely successful, the cosmological standard model is in fact very difficult to incorporate into fundamental physics. The 95% of the universe's energy budget, namely the cosmological constant and the dark matter sectors are still waiting for their theoretical explanations. A completely satisfactory model for cosmic inflation is also still a subject of active research. In this thesis our primary interest will be the phenomenon of comic acceleration, and the nature of dark matter, while also very important and interesting, is beyond the scope of this thesis.

It is a very curious fact that the standard framework of quantum field theory already leads to accelerated expansion of the universe. Indeed, quantum mechanically we expect a non-zero vacuum energy, which behaves exactly like a cosmological constant. If the theoretically estimated value of the vacuum energy density would agree with the cosmological observations, this would have been one of the most elegant predictions in theoretical physics. Unfortunately the reality is by far not as simple as that. The trouble is that the theoretical expectation for the value of this vacuum energy is at least tens of orders of magnitude larger than the value inferred from cosmological observations (see [3] for a pedagogical treatment of the topic). Besides the quantum mechanical contribution, there is also a classical contribution to the vacuum energy density originating, e.g., from the minima of scalar field potentials. The huge value of the quantum mechanical vacuum energy can in principle be cancelled against the classical contributions. This cancellation between two huge values, however, is highly unsatisfactory as we would need a very precise, finely-tuned cancellation.

In the last decades this problem has motivated a substantial effort in exploring possible modifications to the standard model of cosmology. This effort can be overall split into two main categories. One category is dubbed as *dynamical dark energy scenario*. In this scenario the cosmological constant sector is replaced by a field which evolves during cosmic history and is responsible for late-time cosmic acceleration. Another category goes under the name of *modified gravity*, where one constructs gravitational theories which posses so-called self-accelerating solutions, i.e. they can explain the accelerated expansion without the need of cosmological constant. Both of these possibilities, of course, do not provide an explanation for the abovementioned difficulty with the quantum-field theoretical vacuum energy. The typical attitude is to assume that there is a yet unknown symmetry or mechanism which makes the vacuum energy exactly zero, and instead achieve the cosmic acceleration via either the dynamical dark energy or the appropriate modifications of General Relativity.

The line of research of exploring the alternatives to the cosmological standard model, while originating from the need of explaining the accelerated expansion, has now to some extent diverged from its origins. Indeed, now a big part of research in this direction is devoted to using cosmological observations for testing various theoretical models, without necessarily requiring these models to give cosmic acceleration in absence of cosmological constant.

The theme of this dissertation is largely motivated by the phenomenon of cosmic acceleration and is devoted to understanding various properties of the fundamental laws of nature by exploiting the cosmological phenomena. Before moving to the main chapters of this thesis, let us quickly review the main ideas in modern cosmology.

1.1 THE COSMOLOGICAL STANDARD MODEL IN A NUTSHELL

Homogeneous and isotropic universe

In order to understand the basic dynamical properties of the universe, we should note that the most relevant interaction at such large scales is the gravity. Our current picture of the latter is dominated by the fact that spacetime is a dynamical object described by the metric tensor $g_{\mu\nu}$ (we use Greek indices for denoting the 4-dimensional spacetime coordinates). In this thesis we will employ the $(-,+,+,+)$ sign convention for the metric.

Cosmological observations suggest that on very large scales (larger than $\mathcal{O}(100)$ megaparsecs) the universe is described by a spatially homogeneous and isotropic manifold, first presented by Friedmann [4, 5]. The most general metric compatible with spatial homogeneity and isotropy is known as Friedmann-Lemaître-Robertson-Walker (FLRW) metric and can be written as

$$\mathrm{d}s^2 = -N(t)^2\mathrm{d}t^2 + a(t)^2\left(\frac{\mathrm{d}r^2}{1-\kappa r^2} + r^2\mathrm{d}^2\Omega\right), \tag{1.1}$$

where t is the time coordinate, r is a radial coordinate on the spatial hypersurfaces, $\mathrm{d}^2\Omega$ is the metric of a two-sphere and κ is introduced for accounting for the spatial curvature of the metric. As we see, we need to introduce two functions of time, $N(t)$ and $a(t)$ known as the *lapse function* and the *scale factor* of the universe. The former is related to the time-reparametrization invariance of the metric, and can be safely fixed to any functional form. This reparametrization invariance originates from the symmetries of General Theory of Relativity to be discussed below. Two important choices for $N(t)$ are the so-called cosmic time, corresponding to $N(t) = t$ and the conformal time, corresponding to $N(t) = a(t)$. The scale factor keeps track of how length intervals on spatial slices of spacetime shrink or expand over cosmic time t. For example, the ratio of physical distances between two galaxies at times t_1 and t_2 is simply given by $a(t_1)/a(t_2)$. This change between the distances is an inherent feature of an FLRW metric and should not be confused with the change caused by the peculiar motion of galaxies, which can be, for example, due to the gravitational force exerted on the considered galaxies by their neighbouring mass. An additional comment on terminology is appropriate here. The radial coordinate r in FLRW metric is typically referred to as a *comoving coordinate.* This reflects the fact that the distance r between two point does not change during the cosmic evolution. The physical distance between two points, $r_{\mathrm{phys}} = a(t)r$, however, of course changes as the universe expands or contracts.

It is worth noting that the metric given in Eq. (1.1) is left invariant under the following rescalings

$$a(t) \to \sigma a(t), \quad r \to r/\sigma, \quad k \to \sigma^2\kappa, \tag{1.2}$$

where σ is a constant. This property, rather conveniently, allows us to rescale the radial coordinate in such a way that the scale factor at present time is equal to unity, i.e. $a_0 = 1$.

Observationally it is well-known that $a(t)$ is in fact an increasing function of time, i.e. the observable universe is expanding. This fact is established by noticing that the spectra of distant galaxies are redshifted, i.e. a spectral line with a restframe wavelength λ_{rest} is observed to have $\lambda_{\text{observed}} > \lambda_{\text{rest}}$. This is expected in an expanding universe, as the electromagnetic waves are stretched alongside with cosmic evolution. An important relation between the redshift factor z and the cosmic scale factor a is given by

$$z \equiv \frac{\lambda_{\text{observed}}}{\lambda_{\text{rest}}} - 1 = \frac{a_0}{a(t_\star)} - 1, \tag{1.3}$$

where a_0 is the present-time scale factor and $a(t_\star)$ is the value of the scale factor when the wave has been emitted.

The redshift of galaxy spectra can be interpreted as a result of *Doppler effect*. When the considered galaxy moves much slower than the speed of light, then the corresponding Doppler redshift of spectral lines would be given by $z \approx v/c$, where c is the speed of light in vacuum and v is the speed of the galaxy with respect to the observer.

As discussed above, in an expanding FLRW universe the physical distances between two points at fixed comoving distance r is given by $r_{\text{phys}} = a(t)r$. This then leads to the recession speed of a galaxy at the distance r_{phys} from the observer to be

$$v = Hr_{\text{phys}} \approx H_0 r_{\text{phys}}, \tag{1.4}$$

where $H \equiv \dot{a}/a$, with a dot denoting a derivative with respect to cosmic time t, is known as the *Hubble function*. In the last part of this equation we have assumed the galaxy to have a small redshift, so that the Hubble function can be assumed to be approximately constant and equal to its present-day value of H_0. This result is the celebrated Hubble's law of cosmic expansion discovered in 1920's. We recommend Ref. [6] for an interesting summary of the story behind the discovery of this law.

Dynamics of the FLRW universe

In the context of Einstein's theory of General Relativity [7], the dynamics of the metric tensor field can be derived from the Einstein-Hilbert action, given by

$$S = \frac{M_{\mathrm{Pl}}^2}{2} \int \mathrm{d}^4x \sqrt{-g} R + S_{\mathrm{m}}(g_{\mu\nu}, \Psi_i), \tag{1.5}$$

where g is the determinant of the metric tensor, $R \equiv g^{\mu\nu} R_{\mu\nu}$ is the Ricci scalar constructed from the metric tensor $g_{\mu\nu}$ and the corresponding Ricci tensor $R_{\mu\nu}$. S_{m} is the action describing the dynamics of matter fields, collectively denoted by Ψ_i. Additionally, we have introduced the reduced Planck mass, defined by $M_{\mathrm{Pl}} \equiv \sqrt{\frac{\hbar c}{8\pi G_{\mathrm{N}}}}$, with $\hbar$ being the reduced Planck constant and G_{N} - the Newton's constant. It should be noted that the central property of GR is that all the matter species Ψ_i are universally coupled to the metric. This coupling is proportional to the Newtonian gravitational constant G_{N}.

An additional observation at this point is that the symmetries of the action (1.5), namely, the invariance under general coordinate transformations, or, the diffeomorphism invariance, allow us to add a constant term in the Einstein-Hilbert action. This term, known as the cosmological constant discussed earlier, is an essential piece for constructing the phenomenologically simplest cosmological model which is compatible with all the currently known experimental and observational evidence, namely the ΛCDM model.

We are going to assume that the matter content of the universe is described by a perfect fluid with an energy density $\rho(a)$ and pressure $p(a)$. Our next step is to derive the equations of motion which govern the dynamics of this metric. For that purpose we can plug our FLRW metric ansatz Eq. (1.1) into the Einstein-Hilbert action (including the cosmological constant term -2Λ and the matter energy density $\rho(a)$) and obtain the so-called *minisuperspace* action. Varying the action with respect to the lapse function $N(t)$ yields the energy constraint equation, which is the celebrated *first Friedmann equation*

$$3M_{\rm Pl}^2 H^2 = M_{\rm Pl}^2 \Lambda + \rho(a) - 3M_{\rm Pl}^2 \frac{\kappa}{a^2}. \tag{1.6}$$

Additionally, the variation of the action with respect to the scale factor $a(t)$ gives

$$\dot{H} + H^2 = -\frac{1}{6M_{\rm Pl}^2}\left(\rho(a) + 3p(a)\right) + \frac{\Lambda}{3}. \tag{1.7}$$

where we have set $N(t) = 1$ and defined the pressure as

$$p(a) = -\rho(a) - \frac{1}{3} a \frac{\delta\rho(a)}{\delta a}. \tag{1.8}$$

An important consequence of the diffeomorphism invariance is the automatic conservation of the energy-momentum tensor, given the Einstein field equations are satisfied. This conservation is given by $\nabla^\mu T_{\mu\nu} = 0$, where ∇^μ is the covariant derivative compatible with the metric $g_{\mu\nu}$. For the perfect fluids considered here this equation takes the form $\dot{\rho} + 3H\rho(1+w) = 0$, where $w \equiv p/\rho$ is the equation of state of the considered fluid. From this simple relation it follows that the energy densities of dark matter with $w = 0$, radiation with $w = 1/3$ and cosmological constant with $w = -1$ (which are assumed to be non-interacting, hence are conserved separately) are evolving as

$$\rho_{\rm r} = \rho_{\rm r}(a_0) a^{-4}, \tag{1.9}$$

$$\rho_{\rm m} = \rho_{\rm m}(a_0) a^{-3}, \tag{1.10}$$

$$\rho_\Lambda = \rho_\Lambda(a_0), \tag{1.11}$$

where a_0 is the present-day value of the scale factor.

Let us note that radiation dilutes away faster than non-relativistic matter, which means that no matter how subdominant the latter is initially, it will dominate over radiation at some later stage. Additionally, both radiation and non-relativistic matter will eventually become subdominant compared to cosmological constant. This shows that in the ΛCDM model the universe asymptotically approaches an epoch described by a constant Hubble function. This spacetime metric at this epoch is known as the *de Sitter metric*.

It is also useful to introduce the dimensionless density parameters as

$$\Omega_{\mathrm{r}}(a) \equiv \rho_{\mathrm{r}}(a)/3H^2 M_{\mathrm{Pl}}^2, \tag{1.12}$$

$$\Omega_{\mathrm{m}}(a) \equiv \rho_{\mathrm{m}}(a)/3H^2 M_{\mathrm{Pl}}^2, \tag{1.13}$$

$$\Omega_{\Lambda}(a) \equiv \Lambda/3H^2, \tag{1.14}$$

$$\Omega_{\kappa}(a) \equiv -\kappa/H^2 a^2. \tag{1.15}$$

In terms of these dimensionless parameters the first Friedmann equation can be rewritten as

$$\Omega_{\mathrm{m}}(a) + \Omega_{\mathrm{r}}(a) + \Omega_{\Lambda}(a) + \Omega_{\kappa}(a) = 1. \tag{1.16}$$

Let us mention that the cosmological observations tightly constrain the spatial curvature κ to be tiny [8]. In this thesis we will mainly assume it being exactly zero.

Perturbing the FLRW universe

As we mentioned above, the FLRW metric provides a valid description of the universe on scales larger than $\mathcal{O}(100)$ megaparsecs. On smaller scales, however, the universe is no longer homogeneous and isotropic. Various observational surveys have particularly seen a web of clustered matter, known as the *cosmic web* or cosmic large scale structure (LSS) of the universe. This means that after specifying *the cosmological background*, the next important step is to consider perturbations around it. Of course, in complete generality one would aim at solving the full Einstein's equations, which are, in general, are highly non-linear partial differential equations.

However, it is a fortunate property of the universe that at large enough scales the perturbations of the relevant fields are small enough, so we can make use of the perturbation theory. The starting point for this perturbative approach is to specify the form of the perturbed metric. Naively, one would start perturbing all the components of the metric tensor, which would lead to extremely complicated calculations. However, as we mentioned earlier, one of the central properties of GR is its invariance under general coordinate transformations. For a given calculation in the framework of GR we can choose a particularly suitable coordinate system, where the given problem is solved the easiest. This coordinate freedom is known as the *gauge freedom* of GR, and the particular coordinate choice is often called a gauge choice for the metric.

In the previous subsection, when deriving the Firedmann equations, we did not make direct use of the Einstein's field equations. For deriving the equations of motion for the perturbed quantities we can proceed similarly and first derive the action which would then directly lead to the equations of motion for the desired perturbation variables. For example, if we are interested in the linear order perturbations, then we would need to expand the Einstein-Hilbert action to second order in these perturbations. Such a second order action then would lead to linear equations of motion. Alternatively, we could derive the full equations of motion and perturb them to the desired order. In the bulk of this thesis we have used both of these approaches. Here, in order to demonstrate the main features of the standard cosmological model at perturbative level, let us make use of the latter approach.

The starting point are the Einstein's field equations, derived from Eq. (1.5) by varying with respect to the metric tensor. They read as

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{1}{M_{\rm Pl}^2} T_{\mu\nu}, \tag{1.17}$$

where $T_{\mu\nu}$ is the energy-momentum tensor of the matter fields defined as

$$T_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}} \frac{\delta S_{\mathrm{m}}}{\delta g^{\mu\nu}}. \tag{1.18}$$

The first step of our perturbative treatment is to write the metric as

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}, \tag{1.19}$$

where $\bar{g}_{\mu\nu}$ is the FLRW background metric and $\delta g_{\mu\nu}$ is a perturbation around it. We will then plug it in the left hand side of Eq. (1.17) and keep only the terms up to first order in $\delta g_{\mu\nu}$. Such a background-perturbation splitting is an arbitrary choice, but is perhaps the most intuitive one from the point of view of a generic observer in a Hubble flow. The most general form of the metric is

$$\mathrm{d}s^2 = -(1+2\phi)\mathrm{d}t^2 + 2aB_i \mathrm{d}t\mathrm{d}x^i + a^2\left(\delta_{ij} - h_{ij}\right)\mathrm{d}x^i\mathrm{d}x^j, \tag{1.20}$$

where one can show that ϕ, B_i and h_{ij} are, respectively, 3−scalar, vector and tensor. It turns out that the perturbative calculations simplify significantly if we decompose these perturbations into scalar, vector and tensor degrees of freedom. For the vectors this decompositions is well known from general physics. Namely, any 3−vector can be written as

$$B_i = \partial_i B + S_i, \tag{1.21}$$

where B transforms as a 3−scalar, while S_i is a divergence-free 3−vector. Similar decomposition is possible for higher-rank objects, namely for h_{ij}:

$$h_{ij} = 2\psi\delta_{ij} + 2E_{,ij} + F_{i,j} + F_{j,i} + \tilde{h}_{ij}, \tag{1.22}$$

where ψ and E are two additional 3−scalars, F_i is a devergence-free 3−vector and the 3−tensor $\tilde{h}_{ij}$ is such that

$$\tilde{h}^i_i = 0 = \partial_i \tilde{h}^i_j. \tag{1.23}$$

As such, we have decomposed the 10 independent components of the symmetric 4×4 metric $\delta g_{\mu\nu}$ into 4 scalar functions (namely, ϕ, ψ, B, E), 4 vector modes (encoded in the 6 components of B_i and F_i and the corresponding divergence-free conditions), and 2 tensor degrees of freedom (encoded in the 6 components of $\tilde{h}_{ij}$ and the corresponding conditions given in Eq. (1.23)).

The significant advantage of such a decomposition is that it turns out that the linearized Einstein's equations lead to decoupled dynamics of these scalar, vector and tensor sectors. The formation of the large scale structure of the universe is largely given by the scalar sector of the metric, and now we will be considering only this sector. Let us mention, however, that the dynamics of the tensor sector characterizes the propagation of gravitational waves, and hence, even though not relevant for the large scale structure formation, contains valuable information by its own.

The most general way to write the scalar-perturbed metric is as follows

$$ds^2 = -(1+2\phi)dt^2 + 2a\partial_i B dt dx^i + a^2 \left[(1-2\psi)\delta_{ij} + 2\partial_i\partial_j E\right] dx^i dx^j. \quad (1.24)$$

A widely used gauge choice is the Newtonian gauge, specified by $E = 0 = B$.

Our next step is to include the perturbed energy-momentum tensor $T_{\mu\nu}$. The latter for a generic perfect fluid can be written as

$$T_{\mu\nu} = (\rho + P)u_\mu u_\nu + p g_{\mu\nu}, \quad (1.25)$$

where, u_μ is the four-velocity of the fluid element as seen by a comoving observer, ρ is its energy density, p - its pressure. Here we will assume any deviations from the perfect fluid approximation to be exactly zero. The perturbed sector of the energy-momentum tensor is given by

$$\delta T^0{}_0 = -\delta\rho, \quad (1.26)$$

$$\delta T^0{}_i = -\delta T^i{}_0 = (1+w)\bar{\rho} v_i, \quad (1.27)$$

$$\delta T^1{}_1 = \delta T^2{}_2 = \delta T^3{}_3 = c_s^2 \delta\rho. \quad (1.28)$$

Here we have denoted the spatially averaged energy density as $\bar{\rho}$, and the perturbations around this background are denoted by $\delta\rho \equiv \rho(x) - \bar{\rho}(t)$. Additionally, v^i are the components of the three-velocity and $c_s^2 \equiv \delta p/\delta\rho$ denotes the square of the sound speed of the considered fluid.

The linearly perturbed Einstein equations have the following form (see e.g. Ref. [9])

$$6H^2\phi - \frac{2}{a^2}\partial^i\partial_i\psi + 6H\dot{\psi} = \frac{1}{M_{\rm Pl}^2}\delta T^0_0, \tag{1.29}$$

$$-2\partial_i\left(\dot{\psi} + H\phi\right) = \frac{1}{M_{\rm Pl}^2}\delta T^0_i, \tag{1.30}$$

$$\ddot{\psi} + 3H\dot{\psi} + H\dot{\phi} + (3H^2 + 2\dot{H})\phi + \frac{1}{3a^2}\partial^i\partial_i\left(\phi - \psi\right) = \frac{1}{6M_{\rm Pl}^2}\delta T^i_i, \tag{1.31}$$

$$\frac{1}{a^2}\partial^i\partial_j\left(\psi - \phi\right) = \frac{1}{M_{\rm Pl}^2}\delta T^i_j, \quad i \neq j. \tag{1.32}$$

These equations are more conveniently studied in the spatial Fourier space, i.e. using the spatial Fourier components of the corresponding variables. Our convention for Fourier decomposition for a field $\varphi(x)$ is

$$\varphi(x) = \int \mathrm{d}^3k\varphi_{\mathbf{k}}e^{i\mathbf{k}\cdot\mathbf{r}}, \tag{1.33}$$

where $\mathbf{k}$ is the spatial Fourier wavenumber and $\mathbf{r}$ is the spatial real-space coordinate.

Now, going to Fourier space and combining Eqs. (1.29) and (1.30) we obtain the Poisson equation

$$\frac{k^2}{a^2}\psi = \frac{1}{2M_{\rm Pl}^2}\left(3H(1+w)\bar{\rho}v - \delta\rho\right), \tag{1.34}$$

where v is the scalar sector of the matter velocity, i.e. $v^i \equiv \partial^i v$.

Additionally, for matter sources which have $\delta T^i_j = 0$ we have an important relation

$$\phi = \psi. \tag{1.35}$$

For simplicity in our analysis we will consider only the modes which are very deep inside the Hubble horizon, i.e. $k^2/a^2 \gg H^2$. Additionally, we will be considering the so-called *quasistatic regime*, where one assumes that the cosmological variables can change only at the time scales close to the order of the Hubble rate, i.e $H^2\delta\varphi \sim H\delta\dot{\varphi} \sim \delta\ddot{\varphi}$. In this approximation we have

$$\frac{2k^2}{a^2}\psi = -\frac{1}{M_{\rm Pl}^2}\delta\rho \tag{1.36}$$

Besides the Einstein equations an extra information is contained in the perturbed conservation equations. The $\nu = 0$ and $\nu = i$ components of the continuity equation $\nabla_\mu T^\mu{}_\nu = 0$ in sub-horizon limit, during dark matter domination, yield

$$\delta' + \theta = 0, \tag{1.37}$$

$$\theta' + \mathcal{H}\theta - k^2\left(\phi + c_s^2\delta\right) = 0, \tag{1.38}$$

where we have now started to use the conformal time, related to the cosmic time through $a\mathrm{d}t = \mathrm{d}\tau$, and primes denote derivatives with respect to conformal time. Additionally, we have defined $\theta \equiv \partial_i v^i$ and $\delta \equiv (\rho(x) - \bar{\rho}(t))/\bar{\rho}(t)$. From these two equations we then obtain the master equation for linear structure formation

$$\delta'' + \mathcal{H}\delta' + \left(c_s^2k^2 - \frac{3}{2}\mathcal{H}^2\right)\delta = 0. \tag{1.39}$$

The perturbations will experience a growing force by gravity, but the growth will be slowed down by the non-zero sound speed (i.e. by pressure). For cold dark matter the sound speed is negligible, $c_s^2k^2 \ll \mathcal{H}^2$, and the perturbations δ will grow as $\sim t^{2/3}$.

1.2 OBSERVATIONS

In the past decades several types of observations have become sufficiently robust and now serve as the basis for our current understanding of the cosmological standard model. Let us here briefly discuss the main of these cosmological observables (see, e.g., [10]). Before doing that, however, it is important to mention that various distance definitions are used for interpreting different cosmological observations. Distances between two points in FLRW spacetime are in fact not uniquely defined, so let us start by defining various useful distances and give the relationships among them.

- **Comoving distance.** The *comoving distance* from us to an object at a given redshift z is given by

  $$D_{\rm com} \equiv \frac{c}{a_0 H_0} \int_0^z \frac{d\tilde{z}}{E(\tilde{z})}, \tag{1.40}$$

  where H_0 is the value of the Hubble rate at present time, and $E(z) \equiv H(z)/H_0$.

- **Luminosity distance.** For a source with an absolute luminosity $\mathcal{L}$, observed to have a flux $\mathcal{F}$ on our detectors, we can define the so-called *luminosity distance* to the source

  $$D_{\rm lum}^2 \equiv \frac{\mathcal{L}}{4\pi\mathcal{F}}. \tag{1.41}$$

- **Angular diameter distance.** For an object of proper size (in the direction perpendicular to the line of observation) $\Delta\ell$, observed to subtend an angle $\Delta\theta$, we can define the so called *angular diameter distance* to the object as

  $$D_{\rm ang} \equiv \frac{\Delta\ell}{\Delta\theta}. \tag{1.42}$$

For a spatially flat universe the luminosity distance is related to the comoving cosmic distance by $D_{\mathrm{lum}} = (1+z)D_{\mathrm{com}}$. This expression is rather generic and holds for almost any cosmology. It should, however, be kept in mind that it will be violated in a theory where the photon number is not conserved, for example, due to mixing of photons with some hidden sector. Additionally, the luminosity distance is related to the angular diameter distance by $D_{\mathrm{lum}} = (1+z)^2 D_{\mathrm{ang}}$.

After this prelude we can start discussing the main cosmological observations.

Supernovae Type Ia. Perhaps the best-known cosmological constraints are from *Supernovae.* The luminosities (or the absolute magnitudes) of these objects are known to be highly correlated with the widths of their light-curves. This fact allows for an accurate determination of the absolute magnitude, given the light-curve observation of a supernova. A key relation for cosmological purposes is the relation between the *distance modulus* μ (the difference between the apparent and absolute magnitudes) and the luminosity distance

$$\mu = 5 \log D_{\mathrm{lum}}/10\mathrm{pc}, \tag{1.43}$$

Having the distance modulus measurements of supernovae, one then can measure the luminosity distance, and hence constrain a particular cosmological model.

Cosmic Microwave Background. The Cosmic Microwave Background (CMB) is one of the major sources of information in cosmology. As we mentioned earlier, after inflation the universe was filled with a hot photon-baryon plasma. The baryons tend to cluster through gravitational attraction, but the photonic pressure stops this clustering. As a result the cosmic plasma experiences acoustic oscillations. When the universe cools down sufficiently the photons decouple from baryons and start to free-stream through the universe. This decoupling happens at redshift $z_{\mathrm{dec}} \approx 1090$ and is known as the *decoupling* or *recombination* era. The free-streaming photons make up the CMB sky. The fluctuations of the photon temperature are sensitive to the density perturbations of the relevant energy components at the decoupling era, their velocities and the gravitational potentials. These fluctuations, measured as a function of direction $\hat{n}$, can be decomposed in spherical harmonics as

$$\frac{\delta T(\hat{n})}{T} = \sum_{\ell} \sum_{m} a_{\ell m} Y_{\ell m}(\hat{n}), \tag{1.44}$$

where T is the average temperature of the CMB, $a_{\ell m}$'s are the corresponding angular modes and $Y_{\ell m}(\hat{n})$'s denote the spherical harmonics.

For each mode ℓ, the variance of the $a_{\ell m}$ modes is known as the angular power spectrum, given by

$$C_{\ell} = \frac{1}{2\ell + 1} \sum_{m} \langle |a_{\ell m}|^2 \rangle \tag{1.45}$$

A relatively simple information in the CMB angular power spectrum is encoded in the scale of acoustic oscillations. The effective sound speed of the photon-baryon fluid c_{s} determines this scale through

$$r_{\mathrm{s}}(z_{\mathrm{dec}}) = \int_{z_{\mathrm{dec}}}^{\infty} \mathrm{d}\tilde{z} \frac{c_{\mathrm{s}}(\tilde{z})}{H(\tilde{z})}. \tag{1.46}$$

The associated angular scale $\theta_{\mathrm{ang}}(z_{\mathrm{dec}})$ and the angular diameter distance to the acoustic scale $D_{\mathrm{ang}}(z_{\mathrm{dec}})$ are related with each other by

$$(1 + z_{\mathrm{dec}}) D_{\mathrm{ang}}(z_{\mathrm{dec}}) = \frac{r_{\mathrm{s}}(z_{\mathrm{dec}})}{\theta_{\mathrm{ang}}(z_{\mathrm{dec}})}. \tag{1.47}$$

Baryon Acoustic Oscillations (BAO). The acoustic oscillations mentioned in the context of CMB affect not only photons but also baryons. Similarly to the acoustic scale in Eq. (1.46), there is a similar scale for baryons, imprinted during the so-called drag epoch, taking place at $z_{\mathrm{drag}} \approx 1020$, when baryons are decoupled from photons.

One expects an enhanced galaxy population at the scales of cosmic structure separated by $r_{\rm s}(z_{\rm drag})$. The corresponding angular scale at a particular redshift z then serves as a useful probe for the cosmic background. The relevant geometric expression is similar to (1.47) and is given by

$$(1+z)\theta_{\rm s}(z) = \frac{r_{\rm s}(z_{\rm drag})}{D_{\rm ang}(z)}. \tag{1.48}$$

Growth of structure An instrumental quantity often employed in LSS studies is the growth rate f, defined as

$$f \equiv \frac{{\rm d}\ln\delta}{{\rm d}\ln a}. \tag{1.49}$$

There is a useful fitting formula for this quantity, given by $f = \Omega_{\rm m}^{\gamma}$, where the power is constant in ΛCDM and is approximately equal to $\gamma \sim 0.55$. An observed deviation from this value will be a smoking gun evidence for beyond ΛCDM physics.

Weak lensing. One of the striking predictions of any modern theory of gravity is the light deflection by massive sources. Cosmologists have come up with a beautiful idea which exploits the gravitational lensing for measuring the properties of the large scale structure. When a light from a galaxy is travelling trough the LSS, it gets slightly distorted. The distortions of this light can be characterized by the gradient $\partial\theta^i_{\rm source}/\partial\theta^j$, where θ^j is the angle under which we observe the given light ray, while $\theta^i_{\rm source}$ is the unaltered (unlensed) angle, and the indices (i, j) label two directions on the sky. In a theory of gravity (not necessarily GR) this gradient is given by

$$\frac{\partial\theta^i_{\rm source}}{\partial\theta^j} - \delta_{ij} \equiv \int_0^{r_{\rm source}} {\rm d}\tilde{r} \left(1 - \frac{\tilde{r}}{r_{\rm source}}\right) \tilde{r}(\phi - \psi)_{,ij}, \tag{1.50}$$

where $r_{\rm source}$ denotes the comoving distance to the considered galaxy.

This matrix is conventionally written as

$$\frac{\partial\theta^i_{\rm s}}{\partial\theta^j} - \delta_{ij} \equiv \begin{pmatrix} -\kappa_{\rm wl} - \gamma_1 & -\gamma_2 \\ -\gamma_2 & -\kappa_{\rm wl} + \gamma_1 \end{pmatrix}. \tag{1.51}$$

It can be shown that the so-called *convergence* $\kappa_{\rm wl}$ describes the overall magnification of the sources, while the components of the *shear* γ_1 and γ_2 describe its distortions. The measurements of these quantities and their cross-correlations provide valuable cosmological information.

1.3 THE INFLATIONARY PARADIGM

In the previous sections of this introduction we have presented the main ideas of the cosmological standard model. In that discussion we have taken the observed large-scale homogeneity and isotropy of the universe, as well as the small value of the spatial curvature, as granted. They are, however, rather unnatural in the standard FLRW universe with a sequence of radiation and matter dominated epochs. This has motivated the birth of the inflationary paradigm.

Let us start our discussion from the so-called *flatness problem*. In a decelerating universe[1] the absolute value of the curvature contribution in Eq. (1.16) increases, because its denominator $aH = \dot{a}$ decreases, unless the curvature of the universe is exactly zero. The observed spatial flatness then suggests that in the past the universe has experienced a phase of accelerated expansion, known as *cosmic inflation* (see e.g. [10] for a pedagogical introduction to inflation).

Another striking issue with the standard cosmological picture is the overall homogeneity of CMB. It can be estimated that the CMB patches of more than ~ 1 degree apart never would have time to communicate with each other starting from the time of infinitely small universe (the *Big Bang*)

1 Notice that both radiation- and matter-dominated epochs are necessarily decelerating because the second Friedmann equation Eq. (1.7) shows that $\ddot{a} < 0$ for any equation of state satisfying $1 + 3w > 0$.

to the time of recombination [10][2]. While the CMB photons were, in fact, in causal contact after the last scattering, the entire idea of CMB suggests that they shouldn't interact, hence they cannot thermalize after decoupling.

The crucial quantity for our discussion here is the *comoving particle horizon*, defined as

$$d_{\mathrm{H,com}} \equiv \int_0^a \frac{\mathrm{d}\tilde{a}}{\tilde{a}} \frac{1}{\tilde{a}H(\tilde{a})}, \tag{1.52}$$

which measures the maximum distance the light could have travelled in FLRW spacetime between times characterized by scale factors 0 and a. It is instructive to rewrite $d_{\mathrm{H,com}}$ in terms of the *comoving Hubble radius* $(aH)^{-1}$ as

$$d_{\mathrm{H,com}} = \int_1^{\ln a} \frac{\mathrm{d}\ln\tilde{a}}{\tilde{a}H(\tilde{a})}. \tag{1.53}$$

The last expression suggests a solution to the horizon problem. If we could have en epoch during which $(aH)^{-1}$ is increasing towards the past, then $d_{\mathrm{H,com}}$ could be made larger. What we are seeking for is a mechanism which would make $d_{\mathrm{H,com}}$ much larger than $(aH)^{-1}$ during the standard expansion. This is precisely the idea of inflation; make the comoving Hubble radius larger in the past, so that the entire observable CMB would have been in causal contact at some point in the past.

It is easy to notice that achieving such a regime does not only resolve the issue with the horizon, but also resolves the flatness problem. Indeed, the condition $\mathrm{d}(aH)^{-1}/\mathrm{d}t < 0$ implies that $\ddot{a} > 0$ and hence, using the second Friedmann equation, that $w < -1/3$, which is exactly the condition for the flat universe to be an attractor under cosmic evolution.

The most common dynamical realization of inflation is through a canonically normalized scalar field φ with a potential $V(\varphi)$. The homogeneous

2 There is, however, a substantial assumption here. In such arguments we assume that the classical picture of spacetime holds till the very Big Bang. From quantum gravity perspective this might be seen as an oversimplification.

and isotropic equation of motion (i.e. taking $\varphi(x)$ to be a function of time only) of such a field is given by

$$\ddot{\varphi} + 3H\dot{\varphi} + V(\varphi)_{,\varphi} = 0, \tag{1.54}$$

where $V(\varphi)_{,\varphi} \equiv \partial V(\varphi)/\partial\varphi$.

Additionally, the energy and momentum of the scalar field can be shown to be

$$\rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + V(\varphi), \tag{1.55}$$

$$p_\varphi = \frac{1}{2}\dot{\varphi}^2 - V(\varphi), \tag{1.56}$$

respectively.

In order this field to be able to successfully drive the inflationary dynamics, we need the equation of state of the scalar field to be close enough to -1, which is the case of the shallow potentials. Additionally, in order to have long enough inflation, we need the above condition to be satisfied for long enough period. These two conditions are formally written as $\epsilon \equiv -\dot{H}/H^2 \ll 1$ and $\eta \equiv \dot{\epsilon}/H\epsilon \ll 1$. The inflationary stage should be followed by the stage of hot FLRW expansion, which means that the inflationary stage must end eventually. This additionally means that inflation cannot be realized via a cosmological constant, because in that case the universe would have no physical clock specifying when the inflation should end.

While the primary goal of inflationary scenario was to solve the abovementioned horizon and flatness problems, it turned out that it can do much more than that. In fact, inflation is a beautiful mechanism which transfers the quantum fluctuations of the inflationary scalar field and the spacetime metric to the classical seeds of the large scale structure. Particularly, inflation predicts that the power of primordial fluctuations in matter density, which later seeds the LSS formation, should be nearly scale invariant, with a slight tilt characterized by a slope of $n_s - 1$, where n_s is typically referred to as *scalar spectral index*. Additionally, inflation also predicts presence of primordial gravitational waves, again from the initial quantum fluctuations of the metric. The amount of produced primordial gravitational waves are typically characterized by the ratio of powers in tensor and scalar fluctuations, referred to as the *tensor to scalar ratio*, and denoted by r.

The idea of inflation is summarized in Fig. 1.1. The decreasing comoving Hubble radius $(Ha)^{-1}$ makes the particle horizon at the epoch of CMB formation larger compared to the value in the standard, non-inflationary scenario. This additionally resolves the flatness problem. Moreover, and perhaps more importantly, inflation is an elegant mechanism for generating the observed large scale structure from the primordial quantum fluctuations of the inflaton field and the spacetime metric.

1.4 BEYOND THE STANDARD MODEL: DYNAMICAL DARK ENERGY AND MODIFIED GRAVITY

Dynamical dark energy Similarly to inflation, one might naturally think that the present-day accelerated expansion of the universe is not caused by a cosmological constant but rather by a slowly rolling scalar field. Such a scenario is known as dynamical dark energy (also referred to as *quintessence*) scenario [11, 12]. Unlike the inflationary epoch, the late-universe acceleration does not need to end, hence the cosmological constant explanation is a completely viable one from this point of view. However, one might argue that a dynamical scenario is a more elegant explanation for the accelerated expansion, and it is one of the motivations to study such models in detail. There is, of course, also a strong theoretical motivation to do this. It turns out that the low-energy, effective descriptions of the potentially fundamental theories contain scalar degrees of freedom. This means that if the future probes detect a deviation from the ΛCDM scenario, we can potentially

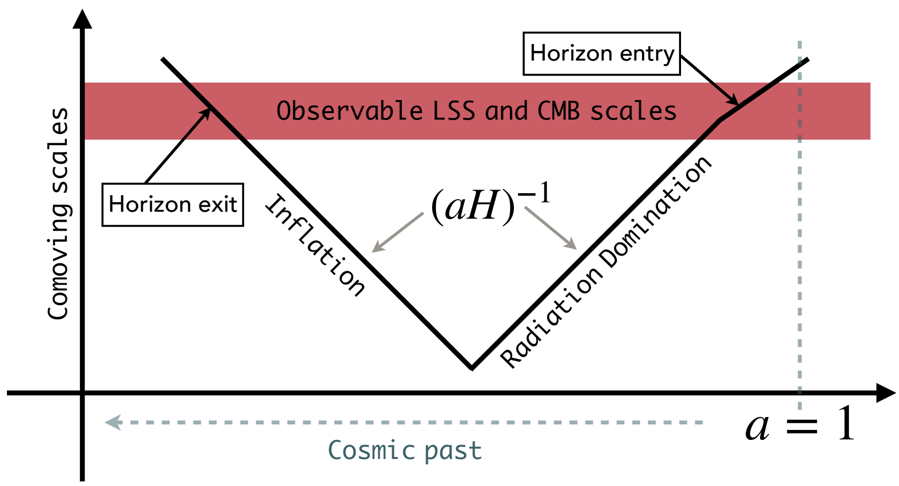

Figure 1.1: The idea of inflation is to modify the expansion history of the universe in such a way that the comoving Hubble radius is decreasing before the standard expansion regime starts.

learn about the fundamental theories. In practice this argument, of course, is more complicated, as the observational consequences of, for example, string theory so far are rather ambiguous. An interesting development in this direction was suggested in [13]. The authors have *conjectured* that the scalar field potential for all consistent theories should satisfy the constraint

$$\frac{|\nabla_\phi V|}{V} \geq c \,, \qquad c \sim 1 \,. \tag{1.57}$$

This conjecture is in contrast to the string theory landscape scenario [14–19] (see Ref. [20] for a brief review of related ideas), where it is considered that string theory describes an enormous number of metastable de-Sitter vacua.

There is no consensus about the theoretical validity of Eq. (1.57) in the string theory community (see [21] for a review). Moreover, the use of the conjecture in its current shape for cosmological phenomenology is still rather ambiguous. Indeed, as the conjecture does not specify the value of the constant c, it is difficult to confront it with phenomenological studies. The way forward in this situation is to study the phenomenological implications of the models presented in [13] which have been served as the primary support for the conjecture. These models are given in terms of concrete potentials and therefore their precise phenomenologies can be worked out. The main result of such an investigation in [22] is that all these considered models are incompatible with cosmological data. This, perhaps, is difficult to interpret as a very strong observational challenge for Eq. (1.57) because, again, in the latter the imprecise nature of c makes it impossible to draw decisive, quantitative conclusions.

Even if not making the connection to any fundamental theory, the dynamical dark energy scenario is still very interesting to study. One of the interesting motivations to study such alternatives is the so-called *coincidence problem*, which is based on the question of why is the universe starting to accelerate exactly at the present time, and not, say, much later in the future. In this context let us discuss a particularly appealing feature, namely the presence of the so called *scaling fixed points* in the phase space of quintessence models. The model that we will study has a simple exponential potential of the form $V(\phi) = V_0 e^{\lambda\phi}$. For $\lambda^2 > 3(w_{\rm B} + 1)$, with $w_{\rm B}$ being the equation of state for the background fluid (e.g. dark matter or radiation), the universe enters a scaling regime where the scalar field mimics the evolution of the background fluid, with $w_{\rm DE} = w_{\rm B}$; the dark energy density parameter takes the form $\Omega_{\rm DE} = 3(w_{\rm B} + 1)/\lambda^2$ (see [23] for a review). This scaling property is illustrated in the left panel of Fig. 1.2 for a sufficiently large value of λ

(chosen to be $\sqrt{750}$ in this example), where we have shown the evolution of the quintessence energy density compared to that of dark matter. The figure shows that the scalar field, after some oscillations, quickly follows the background and one can achieve a scaling solution during matter domination in this example. Te horizontal axis here is $N \equiv \ln a$, with $N = 0$ corresponding to the present time. Such scaling solutions may indeed provide a solution to the coincidence problem. Even though these are very interesting features, the obvious problem, of course, is that a single-exponential potential has a constant slope, and therefore, once the scaling regime is switched on it never ends, so there is no dark energy domination.

A particular extension of the considered model is the following two-exponential potential

$$V(\phi) = V_1 e^{\lambda_1 \phi} + V_2 e^{\lambda_2 \phi} \,. \tag{1.58}$$

The phenomenological merit of this double exponential model is that under certain conditions the scaling solution can gracefully exit to the desired accelerating phase at late times. This transition can be obtained if $\lambda_1^2 > 3(w_B + 1)$ and $\lambda_2^2 < 3(w_B + 1)$ in the potential (1.58). At early times, the potential is dominated by the $e^{\lambda_1 \phi}$ term, for which the scalar field follows the equation of state of radiation and/or matter, hence scaling solutions. Later in the evolution of the universe, the $e^{\lambda_2 \phi}$ term dominates, for which the evolution is not of the scaling form and the late-time attractor is the scalar field dominated solution (with $\Omega_{DE} = 1$). In this scenario, the asymptotic value of the dark energy equation of state is $w_{DE} = -1 + \lambda_2^2/3$, providing viable cosmologies, just as for the single exponential with $\lambda^2 < 3(w_B + 1)$. The right panel of Fig. 1.2 shows an example of this so-called *scaling freezing* scenario with the double-exponential potential, where the transition from the scaling evolution to the scalar field dominated evolution has been depicted.

Modifications of gravity. The *dynamical dark energy scenario* mentioned above is one interesting way to go beyond the standard ΛCDM scenario.

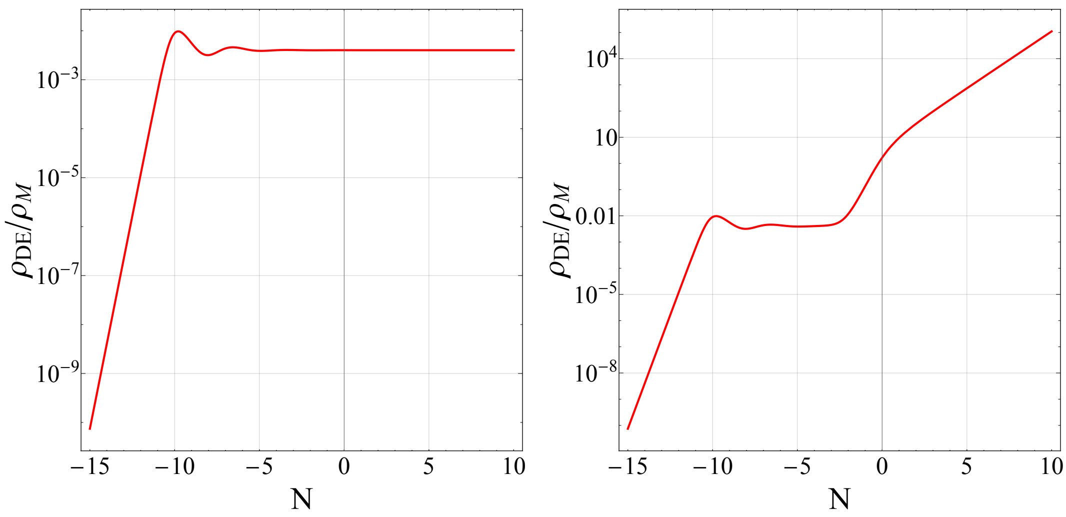

Figure 1.2: The ratio of the dark energy density $\rho_{\rm DE}$ to that of matter $\rho_{\rm M}$ as a function of N. *The left panel* demonstrates the scaling solutions of a single-exponential model $V(\phi) = V_0 e^{\lambda\phi}$ with $\lambda^2 > 3(w_{\rm B}+1)$, while *the right panel* is for $V(\phi) = V_1 e^{\lambda_1\phi} + V_2 e^{\lambda_2\phi}$ with $\lambda_1^2 > 3(w_{\rm B}+1)$ and $\lambda_2^2 < 3(w_{\rm B}+1)$.

There is, however, another exciting prospect. As we mentioned earlier, gravity is the most relevant interaction at the cosmological scales. This means that cosmology is an ideal playground for testing the underlying theory of gravity. In order to effectively study the limitations of GR at cosmological scales, one needs to consider its viable modifications.

GR, in fact, is the *unique* theory of interacting, massless, spin-2 field (see [24] for a proof). This immediately suggests that in order to construct an alternative to GR one can either consider a massive extension of the latter, or add extra dynamical degrees of freedom, such as additional scalar field(s).

In the second class of modifications a particularly well-studied and understood class is the scalar-tensor gravity, where the dynamics of GR is extended with a scalar field. It turns out, however, that many a-priori valid modifications should be actually discarded based on theoretical arguments.

A particular problem for a given theory is the presence of unstable solutions. Commonly discussed types of instabilities are the so-called *ghost* and *gradient instabilities*.

Let us start by discussing the gradient instability. It basically originates from a wrong sign gradient term in the Lagrangian of the theory. For the simplest possible example let us consider a scalar field theory in Minkowski spacetime which has a wrong sign spatial gradient term. The equation of motion for the scalar field φ of such a theory in Fourier space is simply given by

$$\ddot{\varphi}_k - k^2\varphi = 0, \tag{1.59}$$

where k is the absolute value of the spatial Fourier wavenumber. Note that in a healthy theory the second term would have been with an opposite sign. The solutions of this equation scale as $\varphi_k(t) \sim e^{\pm kt}$, the growing part of which leads to a *gradient instability*. The characteristic timescale of the instability scales with the wavenumber as $1/k$.

Another widely encountered type of pathology is the ghost instability. To understand ghosts it is enough to consider the following, non-gravitational toy example for two scalar fields χ and φ

$$\mathcal{L} = \frac{1}{2}\partial^\mu\chi\partial_\mu\chi - \frac{1}{2}\partial^\mu\varphi\partial_\mu\varphi + V(\chi,\varphi), \tag{1.60}$$

where the potential $V(\chi,\varphi)$ is given by

$$V(\chi,\varphi) = -\frac{1}{2}m_\chi^2\chi^2 - \frac{1}{2}m_\varphi^2\varphi^2 + \lambda\chi^2\varphi^2, \tag{1.61}$$

with m_χ and m_φ being the masses of the fields and λ a positive constant.

Note, in particularly, that the two fields have opposite sign kinetic terms. This is precisely what we mean by a ghost degree of freedom - a field with a wrong sign kinetic term. As the χ field has a negative energy, the vacuum state can decay into χ and φ particles and the rate of this decay is in fact infinite [25, 26] (assuming the considered theory is valid up to arbitrarily high energies). This means that the presence of ghosts makes the theory highly undesirable. Ghost fields are typically present in theories whose equations of motion contain higher than second order time derivatives [27, 28]. This fact is one of the main locomotives for constructing alternatives to GR. One of the most well studied class of theories is in fact the Horndeski theory - the theory of a single scalar field coupled to gravity in such a way that the resulting equations of motion are second order in time [29, 30]. This last requirement ensures the absence of ghosts.

Horndeski theory is a generalization of *scalar-tensor* theories known since a long time ago. One of the first examples is the *Brans-Dicke theory* [31], the main idea of which is to promote the gravitational constant to a dynamical field. In the so called *Jordan frame* (which means that the matter fields are minimally coupled to the metric $g_{\mu\nu}$), the Brans-Dicke theory has the following action

$$S = \frac{M_{\rm Pl}^2}{2}\int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}\varphi R - \frac{\omega_{\rm BD}}{2\varphi}\nabla^\mu\varphi\nabla_\mu\varphi - V(\varphi)\right] + S_{\rm m}(g_{\mu\nu},\Psi_i), \quad (1.62)$$

where $\omega_{\rm BD}$ is a constant. The GR limit of this theory is recovered in the limit of infinitely large Brans-Dicke parameter $\omega_{\rm BD}$.

From the point of view of cosmology this theory is an interesting example of modified gravity because we can clearly see the effects of the additional scalar degree of freedom on large scale structure. Particularly, the Poisson equation (which in GR is given by Eq. (1.34)) in this theory, in the quasistatic limit, is given by

$$\frac{k^2}{a^2}\phi = -4\pi G\mu(a,k)\rho_{\rm m}\delta, \quad (1.63)$$

where

$$\mu(a,k) \equiv \frac{M_{\rm Pl}}{\bar\varphi}\frac{2(2+\omega_{\rm BD}) + (\bar\varphi/M_{\rm Pl})m^2a^2/k^2}{3+2\omega_{\rm BD} + (\bar\varphi/M_{\rm Pl})m^2a^2/k^2}, \quad (1.64)$$

with $\bar{\varphi}$ being the homogeneous background sector of the scalar field φ, and m being the mass of the scalar field. As we see, contrary to GR, here the gravitational strength, which controls the effectiveness of dark matter clustering, is a function of scale and time.

Additionally, the relation between the two gravitational potentials (which in GR is given by the simple identity Eq. (1.35)) in the quasistatic limit is given by:

$$\eta(a,k) \equiv \frac{\phi}{\psi} = \frac{2(1+\omega_{\mathrm{BD}}) + m^2\bar{\varphi}a^2/k^2}{2(2+\omega_{\mathrm{BD}}) + m^2\bar{\varphi}a^2/k^2}. \tag{1.65}$$

Interestingly, the functional forms of these two new fucntions $\mu(a,k)$ and $\eta(a,k)$ are generic for the entire class of Horndeski gravity [32]. Particularly, these can be written as

$$\mu(a,k) = h_1(a)\frac{1+k^2h_5(a)}{1+k^2h_3(a)} \tag{1.66}$$

$$\eta(a,k) = h_2(a)\frac{1+k^2h_4(a)}{1+k^2h_5(a)}, \tag{1.67}$$

where h_i are functions of background only, and their form is model-specific.

Horndeski gravity is expected to be constrained by several high-precision large-scale structure surveys. However, the recent detection of the gravitational waves originating from a pair of merging neutron stars and the simultaneous detection of their electromagnetic counterpart, the LIGO event GW170817 [33] and its counterpart GRB 170817A [34], have already cut a large portion out from the Horndeski Lagrangian. This has been achieved through the strong bounds imposed on the speed of gravitational waves (which is constrained to be very close to the speed of light in vacuum); see [35] for a recent review on the topic. Note, however, that the mentioned bound on the speed of gravitational waves is strictly valid only at the scales of LIGO events, which is $k \sim \mathcal{O}(10-100)$ Hz. Horndeski gravity, on the other hand, in the cosmological context is typically used at the scale of present-time cosmic expansion rate, H_0, which is about 20 orders of magnitude smaller than the LIGO scale. This means that for interpreting the LIGO bounds one might need to include corrections to the considered theories, which can then naturally bring the speed of gravitational waves in these theories to be very close to the speed of light.

Let us conclude this section by mentioning that while the Horndeski-type general approach to Modified Gravity is very fruitful, it still misses some important classes of theories. Among these modifications to gravity, the bimetric theory of ghost-free, massive gravity is of particular interest. It stands out especially because of the strong theoretical restrictions on the possibilities for constructing a healthy theory of this type. Indeed, historically it has proven to be difficult to invent a healthy theory of massive, spin-two field beyond the linear regime. The linearised theory has been known for a long time [36], while at the fully nonlinear level the theory has been discovered only recently by constructing the ghost-free theory of massive gravity [37–46]. This development has also naturally led to the healthy theory of interacting, spin-2 fields, i.e. the theory of ghost-free, massive bigravity [47]; see Refs. [48–50] for reviews.

Over the past decade, there has been a substantial effort directed towards understanding the cosmological behaviour of bimetric models, both theoretically and observationally. Particularly, it has been shown that bigravity admits FLRW cosmologies which perfectly agree with cosmological observations at the background level (see Ref. [51, 52] for reviews). At the level of linear perturbations the cosmological solutions have been shown to suffer from either ghost or gradient instabilities, although the latter can be pushed back to arbitrarily early times by imposing a hierarchy between the parameters of the theory [53]. It is also conjectured [54] that the gradient instability might be cured at the nonlinear level due to the presence of the Vainshtein screening mechanism (see later in this chapter) in the theory.

The version of the bimetric theory studied in all these works is the so-called singly-coupled scenario, where the matter sector is assumed to couple to only one of the two metrics (spin-2 fields). The metric directly coupled to matter is called physical metric, and the other spin-2 field, called reference metric, affects the matter sector only indirectly and through its interaction with the physical metric.

1.5 SCREENING MECHANISMS IN MODIFIED GRAVITY

One of the most well-understood properties of modified gravity theories is that there is an extra (often referred to as a "fifth") force in addition to the standard Newtonian force. To understand this effect let us study the following, quite generic coupling of the matter fields to the scalar field sector:

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\rm Pl}^2}{2} R - \frac{1}{2} \nabla^\mu \varphi \nabla_\mu \varphi - V(\varphi) \right] + S_{\rm m}(\tilde{g}_{\mu\nu}, \Psi_i), \quad (1.68)$$

where

$$\tilde{g}_{\mu\nu} \equiv A(\varphi) g_{\mu\nu}. \quad (1.69)$$

A central equation here is the geodesic equation for a non-relativistic test particle in the Newtonian limit:

$$\ddot{x}^i + \Gamma^i_{00} = -\frac{d\ln A}{d\varphi} \nabla^i \varphi, \quad (1.70)$$

where Γ denotes the Christoffel symbol, and x^i are the spatial coordinates of the considered test particle. This equation motivates us to interpret the right hand side as a fifth force.

This then leads to a problem – gravity is very well tested at small scales, for example in Solar System, and no fifth forces have been detected [55]. The obvious question then is how to reconcile the modifications of gravity with the local tests. There are several interesting proposals which allow for the fifth force to be *screened* in an environment-dependent manner. For demonstrating the main idea behind the common screening mechanisms let us consider the field equation of motion of the theory given in Eq. (1.68)

$$\Box\varphi = V(\varphi)_{,\varphi} - \frac{\mathrm{d}\ln A}{\mathrm{d}\varphi}\mathrm{Tr}\left[T^{\mu\nu}\right], \tag{1.71}$$

where $T^{\mu\nu}$ is the Einstein frame metric, $\Box \equiv \nabla^{\mu}\nabla_{\mu}$ is the d'Alambert operator, and the trace is taken with $g_{\mu\nu}$. For a non-relativistic matter sector, such as cold dark matter, $\mathrm{Tr}\left[T^{\mu\nu}\right] = -\rho$, with ρ being the matter density.

This motivates us to define $V^{\mathrm{eff}}(\varphi;\rho) \equiv V(\varphi) + \rho\ln A(\varphi)$; an effective potential which reacts to the matter density of the ambient space. Let us discuss two qualitatively different choices for the $A(\varphi)$ function and the potential $V(\varphi)$:

- $V(\varphi) = \frac{\Lambda^{n+4}}{\varphi^{n}}$, $A(\varphi) = e^{\varphi/M_{\mathrm{c}}}$,
- $V(\varphi) = -\frac{1}{2}\mu^2\varphi^2 + \frac{\lambda}{4}\varphi^4$, $A(\varphi) = 1 + \frac{\varphi^2}{2M_{\mathrm{s}}^2}$,

where Λ (not to be confused with the cosmological constant), μ, λ, M_{c} and M_{s} are constants.

The first of these choices is in the class of *Chameleon screening mechanisms* [56], the idea of which is to enhance the effective mass of the scalar field, hence rendering the corresponding fifth force to be a short-range one; see the left panel of Fig. 1.3. The second choice corresponds to the *Symmetron mechanism,* [57] the idea of which is to suppress the coupling of the scalar field to the matter; see the right panel of Fig. 1.3.

Another important mechanism is the Vainshtein mechanism [58, 59], which relies on the non-linearities of the scalar field induced due to higher order derivative self-couplings, such us $\mathcal{L} \supset\sim \partial_{\mu}\varphi\partial^{\mu}\varphi\Box\varphi$. Vainshtein mechanism is central for several interesting theories, including massive gravity/bigravity discussed above.

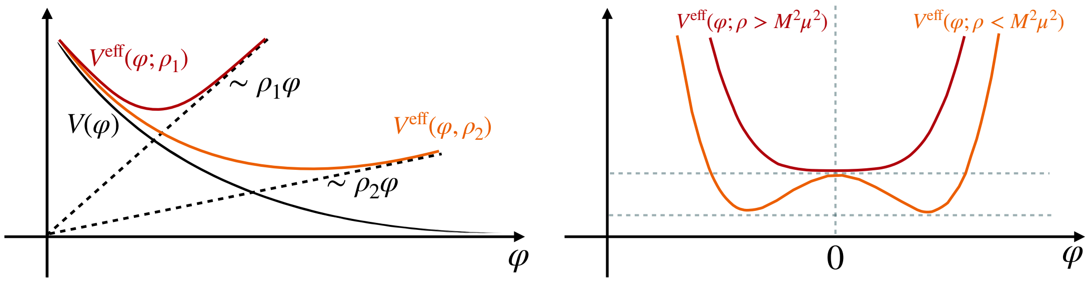

Figure 1.3: *Left panel:* Demonstration of the Chameleon screening mechanism *Right panel:* Demonstration of the Symmetron screening mechanism. See the text for details.

1.6 THE ERA OF PRECISION COSMOLOGY

Before summarizing the content of this thesis let us present a comment on how fast the presion of cosmological observations grows. The quality of modern cosmological datasets posits very high standards in front of cosmological model building initiatives. As a striking demonstration of this let us examine Fig. 1.4, which shows the current observational constraints on inflationary models by the CMB data given by the Planck collaboration [60] alongside with the same constraints from a decade-old WMAP collaboration [61]. We see that many interesting models, e.g. the polynomial inflationary models with potential ϕ^k with $k = 2, 4/3, 1, 2/3$, are now disfavored or ruled out by date. All these models were inside the 95% sweet spot of the data in 2009 provided by the WMAP collaboration, as one can see in Fig. 1.4, while they are now either outside or close to the boundary of the 95% confidence region of the Planck 2018 data.

This demonstrates that in the theoretical analysis of the data, it is no longer possible to perform a *parametric, order of magnitude analysis* as it was

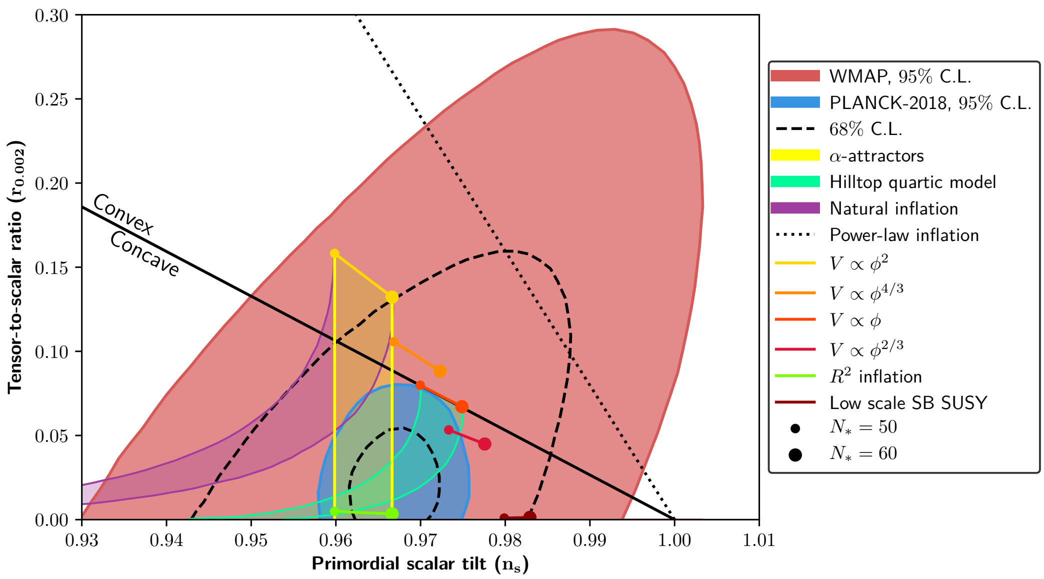

Figure 1.4: Evolution of precision in inflationary parameters over a decade, from WMAP [61] to Planck [60]. The reconstructed Planck constraints correspond to the combination TT,TE,EE+lensing+BK14+BAO provided in [60]. One can look, for example, at the area between $n_s = 0.95$ and $n_s = 0.98$. Although both of these values were inside the 68% contour back in 2009, they are now strongly disfavored with more than 95% confidence.

normal in the past, especially in string theory phenomenology. The same concerns such expressions as "parametrically small", or "parametrically large". We can see examples in Fig. 1.4 showing that reducing the bound on r from ~ 0.08 to ~ 0.04 has made various theoretical ideas either supported or ruled out by the precision data in cosmology.

Similarly, the constraints on dark energy become more and more precise. For example, 15 years ago the constraints on the parameter λ in the exponential potential $e^{\pm\lambda\phi}$ for quintessence, allowed $\lambda = 1.6$ [62, 63]. Meanwhile, in the discussion of the quintessence models supporting the recent swampland conjecture [13] it was necessary to pay full attention to a small difference between numbers such as $\lambda < 1$ and $\lambda < 1.4$. Indeed, models with $\lambda > 1$ are ruled out by cosmological observations with more than 99.7% confidence, whereas the condition $\lambda < 1$ is not satisfied by the string theory models of [13].

1.7 THIS THESIS

- Chapter 2 is dedicated to a study of a new class of inflationary models known as cosmological *α-attractors*. We promote these models towards a unified framework describing both inflation and dark energy. We construct and study several phenomenologically rich models which are compatible with current observations. In the simplest models, with vanishing cosmological constant Λ, one has the tensor to scalar ratio $r = \frac{12\alpha}{N^2}$, with N being the number of e-folds till the end of inflation, and the asymptotic equation of state of dark energy $w = -1 + \frac{2}{9\alpha}$. For example, for a theoretically interesting model given by $\alpha = 7/3$ one finds $r \sim 10^{-2}$ and the asymptotic equation of state is $w \sim -0.9$. Future observations, including large-scale structure surveys as well as Cosmic Microwave Background B-mode polarization experiments will test these, as well as more general models presented here. We also discuss the gravitational reheating in models of quintessential inflation and argue that its investigation may be interesting from the point of view of inflationary cosmology. Such models require a much greater number of e-folds, and therefore predict a spectral index n_s that can exceed the value in more conventional models of inflationary α-attractors by about 0.006. This suggests a way to distinguish the conventional inflationary models from the models of quintessential inflation, even if the latter predict $w = -1$. This chapter is based on Ref. [64].

- The topic of Chapter 3 is the theory of massive bigravity, where one has two dynamical tensor degrees of freedom. We consider an interesting extension where both of the metrics are coupled to the matter sector, which is known as the *doubly-coupled bigravity*. The main aim of this chapter is the study of gravitational-wave propagation in this theory. We demonstrate that the bounds on the speed of gravitational waves imposed by the recent detection of gravitational waves emitted by a pair of merging neutron stars and their electromagnetic counterpart, events GW170817 and GRB170817A, strongly limit the viable solution space of the doubly-coupled models. We have shown that these bounds either force the two metrics to be proportional at the background level or the models to become singly-coupled (i.e. only one of the metrics to be coupled to the matter sector). The mentioned proportional background solutions are particularly interesting. Indeed, it is shown that they provide stable cosmological solutions with phenomenologies equivalent to that of ΛCDM at the background level and at the level of linear perturbations. The nonlinearities, on the other hand, are expected to show deviations from ΛCDM. This chapter is based on Ref. [65].

- In Chapter 4 we study the first cosmological implications of a novel massive gravity theory, recently proposed by Chamseddine and Mukhanov, known as the *mimetic theory of massive gravity*. This is a theory of ghost-free massive gravity, which additionally contains a so-called *mimetic dark matter* component. In an echo of other modified gravity theories, there are self-accelerating solutions which contain a ghost instability. In the ghost-free region of parameter space, the effect of the graviton mass on the cosmic expansion history amounts to an effective negative cosmological constant, a radiation component, and a negative curvature term. This allows us to place constraints on the model parameters—particularly the graviton mass—by insisting that the effective radiation and curvature terms be within observational bounds. The late-time acceleration must be accounted for by a separate positive cosmological constant or other dark energy sector. We impose further constraints at the level of perturbations by demanding linear stability. We comment on the possibility of distinguishing this theory from ΛCDM with current and future large-scale structure surveys. This chapter is based on Ref. [66].

- The final Chapter 5 is dedicated to the study of the effects of screening mechanisms in modified gravity on the dynamics of the spherical collapse of dark matter. In particular, we investigate the splashback scale in *Symmetron modified gravity.* The splashback radius $r_{\rm sp}$ has been identified in cosmological N-body simulations as an important scale associated with gravitational collapse and the phase-space distribution of recently accreted material. We employ a semi-analytical approach, namely the self-similar spherical collapse framework, to study the spherical collapse of dark matter halos in Symmetron gravity. We provide, for the first time, insights into how the phenomenology of splashback is affected by modified gravity. The Symmetron is a scalar-tensor theory which exhibits a screening mechanism whereby higher-density regions are screened from the effects of a fifth force. In this model, we find that, as over-densities grow over cosmic time, the inner region becomes heavily screened. In particular, we identify a sector of the parameter space for which material currently sitting at the splashback radius $r_{\rm sp}$, during its collapse has followed the formation of this screened region. As a result, we find that for this part of the parameter space the splashback radius is maximally affected by the Symmetron force and we predict changes in $r_{\rm sp}$ up to around 10% compared to its General Relativity value. Because this margin is within the precision of present splashback experiments, we expect this feature to soon provide constraints for Symmetron gravity on previously unexplored scales. This chapter is based on Ref. [67].

Other papers of the author include Refs. [22, 68–75].