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**In memoriam Kobus Oosterhoff (1933--2015): statistics as both a purely mathematical activity and an applied science**

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**In Memoriam** Kobus Oosterhoff (1933–2015)

# Statistics as both a purely mathematical activity and an applied science

On 27 May 2015 Kobus Oosterhoff passed away at the age of 82. Kobus was employed at the Mathematisch Centrum in Amsterdam from 1961 to 1969, at the Roman Catholic University of Nijmegen from 1970 to 1974, and then as professor in Mathematical Statistics at the Vrije Universiteit Amsterdam from 1975 until his retirement in 1996. In this obituary Piet Groeneboom, Jan van Mill and Aad van der Vaart look back on his life and work.

Kobus (officially: Jacobus) Oosterhoff was born on 7 May 1933 in Leeuwarden, the capital of the province of Friesland in the north of the Netherlands. He was the eldest of three sons of Wijbo Jacobus Oosterhoff and Gerarda Elisabeth Johanna Oosterhoff-Hoefer Wijehen. The Second World War broke out during his early school days, but Kobus escaped famine and other war miseries, as his father had sent him to his uncle's farm, while himself continuing his practice as a lawyer in Leeuwarden.

After the war Kobus attended the Stedelijk Gymnasium in Leeuwarden (a top level high school with a curriculum that includes ancient Greek and Latin), where he graduated in 1951 in the science ('beta') track. He next started studies in geography at the University of Amsterdam, in line with a passion for traveling and collecting minerals that he maintained throughout his life. However, he switched to mathematics, in which he obtained his 'kandidaatsdiploma' in 1959 and his 'doctoraal-

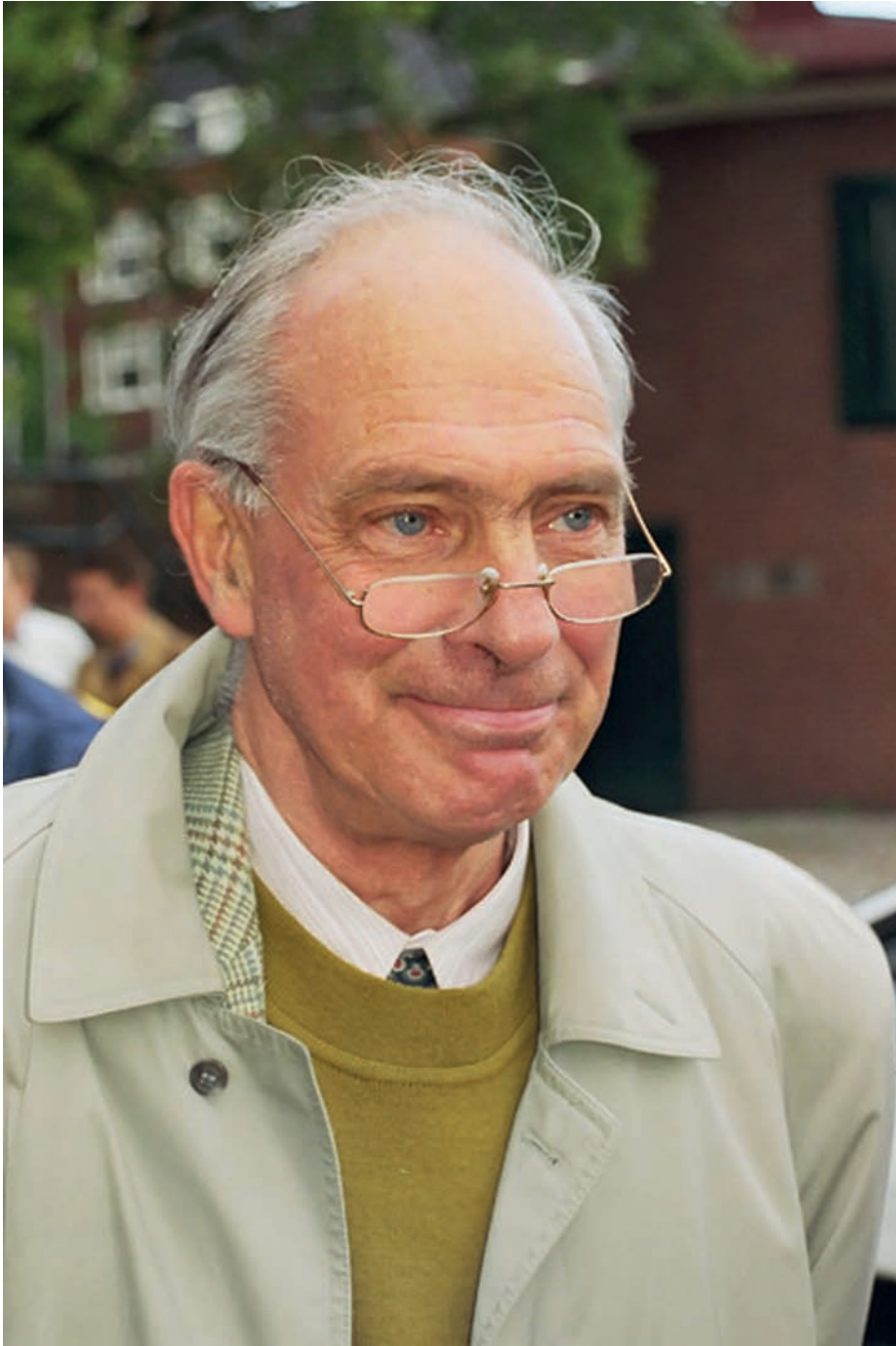
diploma' (comparable to a masters) in 1963. His favorite lecturer was the topologist J. de Groot, who seems to have deeply influenced Kobus outlook on mathematics. During his student days he married Käthe Hötte, with whom he was to share the rest of his life and to have two sons, Wijbo and Rutger; his eldest was born before his graduation.

## *Career*

From 1961 on Kobus was employed at the Mathematisch Centrum in Amsterdam (now CWI), as an assistant until he obtained his master in 1963, then as a 'medewerker', and from 1967 to 1969 as the deputy-director ('sous-chef') of the department of Mathematical Statistics. The head of this department was J. Hemelrijk, and the Mathematisch Centrum was a thriving place for both applied and mathematical statistics. Kobus was involved in consulting projects and also worked on a PhD thesis. In his thesis Kobus acknowledges the regular

contact with Hemelrijk and the encouragement received from him, but he did his thesis under the direction of Willem van Zwet who, one year younger than Kobus, had been a professor at Leiden University since 1965. Kobus became Willem's first PhD student, defending his dissertation *Combination of One-sided Test Statistics* on 26 June 1969 at Leiden University.

After his graduation Kobus spent a year as a visiting assistant professor at the University of Oregon in Eugene, where he became friends with Don Truax (who was professor of Statistics there) and his family. In 1970 he was appointed as 'lector' at the (then) Roman Catholic University of Nijmegen, and then in 1975 as full professor in Mathematical Statistics at the Vrije Universiteit Amsterdam, a position he held until his retirement in 1996. It is ironic that Kobus switched from a catholic to a protestant university, being not at all religious himself. Nowadays both universities have a general outlook, but this was different in the 1970s. The Vrije Universiteit had to grant Kobus official (and rare) dispensation from endorsing the Christian objectives of the university, and demanded him to write a statement that he would respect the protestant character of the university.



Kobus Oosterhoff in 1998

Photo: Andrew S. Tanenbaum

Kobus always kept to this promise, which was made easy by the fact that his political ideas were in tune with Christian ethics and his colleagues in the mathematics department were liberal, even if in majority adhering to the religious character of the university. On occasions such as a PhD defense, where the chair person had to pronounce a prayer, Kobus would have himself replaced for this part of the ceremony.

The Mathematisch Centrum in Amsterdam remained a meeting place for statisticians in the Netherlands, and Kobus

returned to the Centrum as an advisor in 1971, a position he kept until 1981. In this capacity he became PhD advisor to Richard Gill and Piet Groeneboom, who were appointed at the Centrum and both obtained their PhD in 1979. They were the second and third of eight doctoral students under Kobus guidance, all defending at the Vrije Universiteit. The first student was Wilbert Kallenberg, who had started with Kobus in Nijmegen and defended in Amsterdam

in 1977. Kobus enjoyed working with students, and was an encouraging teacher. Although he left his students free to pursue whatever subject they chose, he read every word of their dissertations, and his preference for exact and clear expression made him want to rewrite every sentence. Later he more than once jokingly claimed that perhaps Richard Gill had taught him survival analysis, but certainly he had taught Richard (a native speaker of course) to write correct English. A typical feedback might start with saying “You make people work so hard. Why don’t you give more explanation!”, but would quickly turn into positive comments. The Mathematisch Centrum also organised courses, usually one week long, for mathematicians, also from Belgium. Kobus was involved in two such courses: one on contiguity and efficiency of rank statistics, and one on efficiency of tests and large deviations (with Piet Groeneboom and Rob Potharst as lecturers). These were important topics of research at the time; the second was the topic of the theses of three of Kobus students.

#### Music

Kobus played the violin in his youth, and was interested in music, both classical music and jazz. Piet knows this because Kobus told him, but his sons do not remember having heard him play. Piet still remembers the following events, which he considers somehow typical for his relationship with Kobus. During the big ISI congress in Amsterdam in 1985 a ‘gathering of friends’ (Kobus, Henry Daniels, Ronald Pyke, Jon Wellner and Piet) was planned at Piet’s house in the town Utrecht, which is about an hour’s drive from Amsterdam. It was a day of extreme rainfall, which made driving difficult. The plan was that Kobus would take some of the friends in his car and Piet the remaining persons, and that they would meet at the first gasoline station on the highway from Amsterdam to Utrecht, after which Kobus would follow Piet to his house in Utrecht. Piet waited for a long time at the first gasoline station, but did not see his PhD supervisor appear. He reluctantly got out of the car into the heavy rain to call Kobus’ wife from the station (pre mobile phone time!), but she also had absolutely no idea where her husband could be. What had happened was that Kobus had taken a second entrance to the highway, missing the first gasoline station.

Anyway, the group gathered at the second gasoline station (close to Utrecht), and everybody finally arrived at Piet's house. Ronald Pyke then said: "Henry and Piet, why don't you play something for us?" And Henry Daniels immediately set himself at the piano, after which it was decided that Händel's famous and beautiful D major sonata for violin and continuo would be played. Henry's favourite instrument was a British invention, the concertina (a kind of accordeon), but he also was an excellent pianist. Piet met Kobus again the next morning and Kobus told him: "I did not exactly have high expectations of how this was going to be with Henry and you, but to my relief it was actually pretty nice, I enjoyed it!"

#### *A modest man*

Kobus suffered from asthmatic bronchitis, which he kept under control by regularly sniffing from a small inhalator. For the rest he was a strong man, and would never complain about health issues. He loved rowing and walking in the mountains. He also loved to cook and enjoyed fine dining. He took his wife and sons at least once a year to a Michelin star(s) restaurant, and when taking out scientific guests for dinner, it was usually Kobus who knew the best restaurants. One everyday memory of Kobus is that he always carried a black purse, in which he kept his money and also the inhalator. At a statistical meeting in Palermo, Sicily, this was snatched away by boys on a passing scooter, but this did nothing to change this habit. One also remembers his interest in Chinese porcelain, especially the soft blue-green qingbai, made during the Song and Yuan dynasties, of which he had a collection in a display cabinet in his bedroom, shown to special house guests.

Kobus was a modest man, probably too modest. He once observed that many hard working individuals end up failing repeatedly in life, through their inability to accept their own shortcomings. It is much easier, Kobus said, to know yourself and take it from there. Kobus was honest with others too, and of the highest integrity. Although he was aware that being honest is not the same as being right, he was sometimes direct in giving his honest opinions. He generously supported the young assistant professors in statistics (first Wilbert Kallenberg and later Aad van der Vaart and Math-

isca de Gunst), enabling them to travel and to develop in teaching and research, and pointing out less obvious details of academic life.

#### *'Algemene Statistiek'*

At the time of his appointment at the Vrije Universiteit in 1975, the department of mathematics there was on its way to finding a balance between theoretical and applied mathematics. There had been recent appointments in applied analysis and numerical mathematics, and computer science was developing. Although he also taught pure mathematical statistics courses, some advanced, Kobus naturally weighed in on the applied side. A large proportion of students would conclude their studies with an internship in industry on some statistical project. A course in applied data analysis, using state-of-the-art computing with the GLIM package was unique in the Netherlands. The lecture notes of Kobus undergraduate course, named 'Algemene Statistiek' to reach out also to non-mathematics students, became the basis for a Dutch language book that appeared in the Epsilon series in 2013. The richness of statistics as both a purely mathematical activity and an applied science was central to Kobus thinking, as is also expressed in his farewell lecture ('Wiskunde of niet', 1996; the title is also a pun on his own initial choice not to study mathematics, perhaps, as he recalls, in response to his high school rector's opinion that mathematics and Greek and Latin have much in common: eminent value and total absence of usefulness).

#### *Administrator*

Soon after his appointment at the Vrije Universiteit it became clear that Kobus had strong administrative skills. He served several times as chairman of the Subfaculteit Wiskunde, and later became dean of the Faculty of Mathematics and Computer Science. He was an effective leader, respected and trusted not only by the mathematicians, but also by the computer scientists. When in 2002 the Division of Mathematics and Computer Science was split into two independent departments, Kobus (emeritus professor by then) drafted the financial plan. Although nobody understood the figures, everybody agreed within days, also the computer scientists. Given the strained relationship between the two parties, mostly caused by financial disputes, this

was one of the best compliments to him ever given.

Kobus was full of ideas, especially on new education plans. He was dean when in 1990 the study program *Bedrijfs-wiskunde en Informatica* (now Business Analytics) was founded at the Vrije Universiteit. It became a great success, although co-founder Gerke Nieuwland once jokingly complained that Kobus together with Bert Kersten had turned his concept of a modern information-technology-based mathematics into mere applied statistics. In later years Kobus was thinking about how mathematics could be integrated with life sciences in order to attract more students, and worked out the financial basis to the successful exchange program with Eastern-European universities.

Kobus retired early at age 62 to make room for the younger generation, after having first put pressure on other professors to do so too. In the years to follow he remained involved and became a source of information for newly appointed and much younger administrators, and also for the board of the newly established Faculty of Science (comprising mathematics, computer science, physics and chemistry). At one time Kobus produced a financial formula that the dean did not understand, but as the 'Oosterhoff formula' became a matter of heated discussion during board meetings of the Faculty of Science.

Kobus was also active as an administrator outside the Vrije Universiteit. He was president of the Netherlands Statistical Society from 1976–1979 and associate editor of the *Annals of Statistics* in the 1980s. During his term as treasurer of the Bernoulli Society in the 1990s he helped establish the journal *Bernoulli*, now among the best journals in statistics. He was the main advisor when the board of the Koninklijk Wiskundig Genootschap took over the Epsilon publishing house. Kobus wisdom and responsibility were highly valued in such administrative matters. Kobus was also involved in statistical education in Indonesia and in setting up a program in 'Bedrijfs-wiskunde en Informatica' in Potchefstroom, South Africa, after the end of apartheid.

Kobus was elected as a member of the International Statistical Institute in 1975 and received the Commemorative Medal of the Faculty of Mathematics and Physics of Charles University, Prague, in 1988.

He truly was a man of many talents.

**Work**

Kobus research output was modest, but spans his career and includes significant contributions, mostly to statistical testing theory.

*Statistical decision theory*

His first major paper was joint with W.R. van Zwet (see [14]) and was published in the *Annals of Mathematical Statistics*, the premier journal in the area (nowadays split into the *Annals of Statistics* and the *Annals of Probability*). In the reprint of his thesis [10], finished two years later, Kobus was to write “The present study is a continuation of previous work ([14] in our references) by professor W.R. van Zwet and the author”. The paper and the main chapter of the thesis deal with a problem in *statistical decision theory* given a  $k$ -dimensional Gaussian vector  $T$  with mean vector  $\mu$  and unit covariance matrix, test the null hypothesis  $H_0: \mu = 0$  versus the alternative  $H_1: \mu > 0$ , where the inequality on  $\mu = (\mu_1, \dots, \mu_k)$  is understood coordinatewise. There is no ‘best’ solution to this problem, because the most powerful level  $\alpha$  test for testing  $H_0$  versus  $H_1: \mu = \mu_1$ , given by the Neyman-Pearson lemma, depends on  $\mu_1$ . To overcome this Oosterhoff and van Zwet undertook to find a *most stringent* test: one that minimizes the maximum short-coming to the Neyman-Pearson test. If  $\beta^+(\mu_1)$  is the power of the Neyman-Pearson test, this means to find for a given level  $\alpha \in (0, 1)$  the measurable set (critical region)  $K \subset \mathbb{R}^k$  with  $\Pr_{\mu=0}(T \in K) = \alpha$  that minimizes

$$\sup_{\mu > 0} [\beta^+(\mu) - \beta_K(\mu)],$$

$$\beta_K(\mu) = \Pr_{\mu}(T \in K).$$

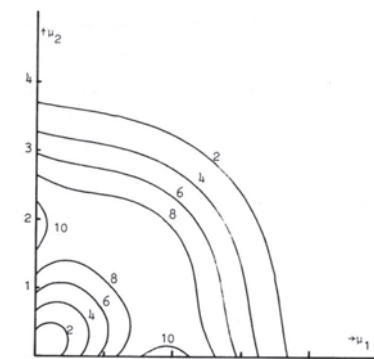


Fig. 2.5.3. Lines of equal shortcoming of the exponential procedure (in percents); size  $\alpha = .05$  and  $k = 2$ .

It turns out that the solution depends strongly on the value of  $\alpha$ . One highlight in the paper is that for  $\alpha = 0.05$  a region of the form

$$K = \{(t_1, t_2): e^{r(\alpha)t_1} + e^{r(\alpha)t_2} \geq c(\alpha)\}$$

gives a most stringent test. Such an exponential combination of tests was and is unusual. Its optimality results from the fact that it is Bayes relative to a certain prior that is supported at two points of maximal shortcoming. In his dissertation Kobus manages to extend a number of key results to dimension  $k > 2$ , but solves the full problem only up to a conjecture (see p.73), which is presumably still open in 2016. For  $\alpha$  decreasing to zero the least favourable prior will have more support points, and analytical solutions are difficult. Kobus proves that in this case the maximal short-coming of the likelihood ratio test tends to zero, and other tests with the same property are essentially equivalent to this test. Two other chapters are on the combination of  $t$ -tests, and on multinomial tests, and the thesis also contains numerical investigations. The two panels in Figure 1 are reprinted from Figures 2.5.3 and 2.5.5, and Kobus thankfully acknowledges the programmer of the ‘plotter of the EL-X1’. The thesis makes a solid impression and gained Kobus a cum laude PhD.

The Gaussian testing problem arises as a limit when the sample size tends to infinity. Another joint paper with Willem van Zwet (see [12]) is concerned with *contiguity*, a concept introduced by Lucien le Cam, which can be used to make this connection mathematically rigorous. The paper gives necessary and sufficient conditions for connecting contiguity of product measures

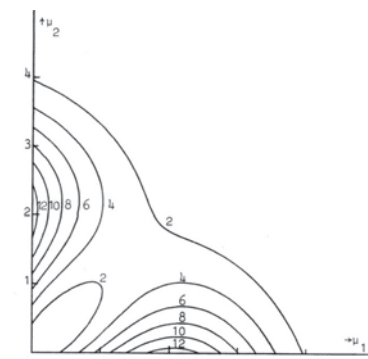


Fig. 2.5.5. Lines of equal shortcoming of FISHER's test (in percents); size  $\alpha = .05$  and  $k = 2$ .

Figure 1 Contour plot of shortcoming as a function of the alternative  $\mu_1 \in \mathbb{R}^2$  of the minimax exponential combination test (left) and Fisher's combination test (right).

to local asymptotic normality and the Hellinger distance. It is known for its clarity of results and exposition.

*Large deviations and efficiencies*

Testing problems with level tending to zero were also the motivation of the theses of Kobus' students Wilbert Kallenberg, Piet Groeneboom and Arnold Kester, on *large deviations* and Bahadur efficiency and deficiency; Kobus published several joint papers with Piet on this subject [6–8]. The *relative efficiency* of a sequence of test statistics  $\{T_n^{(2)}\}$  with respect to another such sequence  $\{T_n^{(1)}\}$  at given  $0 < \alpha < \beta < 1$  and a parameter  $\theta$  from the alternative hypothesis is defined as

$$\text{eff}(T_n^{(2)}, T_n^{(1)}; \alpha, \beta, \theta) = \frac{N^{(1)}(\alpha, \beta, \theta)}{N^{(2)}(\alpha, \beta, \theta)},$$

if  $N^{(j)}(\alpha, \beta, \theta)$  is the minimal sample size such that the test has level  $\alpha$  (probability of falsely rejecting the null hypothesis), and power at  $\theta$  at least  $\beta$  (probability of correctly rejecting the null hypothesis). A large value indicates that  $\{T_n^{(2)}\}$  is superior to  $\{T_n^{(1)}\}$ , since fewer observations are needed to achieve the same power. The relative efficiency is usually difficult to determine exactly, but one can simplify by taking a limit, sending  $\alpha$  to 0,  $\beta$  to 1, or letting  $\theta$  approach the null hypothesis, making the testing problem harder in each case. Letting  $\alpha$  tend to 0, meanwhile keeping  $\beta$  and  $\theta$  fixed, yields the *Bahadur efficiency*, which is directly connected to large deviation theory. Because the alternative is kept fixed, the  $p$ -values  $L_n^{(j)}$  of the tests (defined as the probability of a more extreme value of the test statistic than observed calculated under the assumption that the null hypothesis is true) usually tend to zero exponentially fast in the number of observations, almost surely under the alternative, and  $-(2/n) \log L_n^{(j)}$  tends to a positive limit, called the *Bahadur slope*. Then the Bahadur efficiency is the ratio of the Bahadur slopes: under the probability measure defined by the alternative  $\theta$ , almost surely:

$$\lim_{\alpha \downarrow 0} \text{eff}(T_n^{(2)}, T_n^{(1)}; \alpha, \beta, \theta) \stackrel{\text{a.s.}}{=} \lim_{n \rightarrow \infty} \frac{\log L_n^{(2)}}{\log L_n^{(1)}}.$$

Many test statistics can be written as a functional  $T(\mathbb{P}_n)$  of the empirical distribution  $\mathbb{P}_n$  of the data points. This is the uniform distribution on the data points:

$\mathbb{P}_n(B)$  is the fraction of data points falling in a set  $B$ . For such test statistics calculation of the Bahadur slope comes down to establishing limits of the type or more generally  $\lim_{n \rightarrow \infty} n^{-1} \log \Pr\{\mathbb{P}_n \in \Omega\}$ , for a set  $\Omega$ , with the data points a random sample from the distribution  $P$  determined by the alternative  $\theta$ . *Sanov's theorem* (see [13] which has, however, some difficulties, a version on Polish Spaces can be deduced from [2]) guarantees the existence of such a limit for Polish sample spaces and sets  $\Omega$  such that  $K(\text{int}(\Omega), P) = K(\text{cl}(\Omega), P)$ . Here  $K(\Omega, P)$  is the infimum over  $Q \in \Omega$  of the Kullback–Leibler divergence  $K(Q, P)$  between the probability measures  $Q$  and  $P$ , and  $\text{int}(\Omega)$  and  $\text{cl}(\Omega)$  the interior and closure of the set  $\Omega$  with respect to the weak topology on the set of all Borel probability measures (which is the topology generated by all maps  $Q \mapsto \int f dQ$ , for bounded continuous functions  $f$ ). The limit is then given by

$$\lim_{n \rightarrow \infty} n^{-1} \log \Pr\{\mathbb{P}_n \in \Omega\} = -K(\Omega, P).$$

In [8] this was improved to sample spaces that are just Hausdorff and the topology on the set of probability measures generated by all maps  $Q \mapsto Q(B)$ , for Borel sets  $B$ , called  $\tau$  topology, which seems to be the right topology for the Sanov theorem, although it is non-metrizable. Because

this is stronger than the weak topology, the result is obtained for a wider class of sets  $\Omega$ , which is essential to its application to statistical functionals. The  $\tau$  topology has later been used in the more general probabilistic theory on large deviations as well (see, e.g., [1]), but has some peculiar properties. For example, in contrast with the weak topology a sequence of discrete distributions cannot converge to a continuous non-atomic distribution, but a net of discrete distributions can.

#### Goodness-of-fit tests

Towards the end of his career Kobus concentrated on *goodness-of-fit tests*. Such tests have the purpose of investigating whether a dataset can be a sample from a given probability distribution. A *chi square statistic* is a certain quadratic form in the vector of counts of the numbers of observations belonging to the sets of a given partition of the sample space. It compares this to the expected number if the null hypothesis were true. Such statistics are popular, but are sensitive to the choice of the partition, which is up to the user. Kobus (with co-authors Wilbert Kallenberg and Bert Schriever, see [9]) studied this in the large sample setting, allowing the size of the partition to tend to infinity when the number of observations increases indefinitely. One finding was that for con-

tamination alternatives the partition size should remain finite if the expectation of the quotient of the null and contamination densities is finite for some  $r > \frac{4}{3}$  and tend to infinity if this norm is infinite for some  $r < \frac{4}{3}$ . This precise criterion contradicts the naive idea that the partitions should always be enriched when more and more observations become available.

Another problem connected to goodness-of-fit testing is that the null hypothesis typically consists of more than one distribution, making it necessary to estimate the expected null counts. Kobus (with co-authors, see [5]) studied this problem for general statistics that compare the empirical distribution of the data to its expected value under the null hypothesis, finding that non-robust estimators of location and scale of the null distribution can improve the power for heavy-tailed alternatives. With the same co-authors Kobus also studied power approximations for a class of tests encompassing the likelihood ratio and chi square tests (see [3, 4]), with a main finding that the usual non-central chi square approximation can be much improved, in particular for the likelihood ratio test.

Kobus published his last paper in 1994 [11], two years before his retirement: a comparison of the merits of trimmed means versus the median. ☛

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