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Counting points on K3 surfaces and other arithmetic-geometric objects

Visse, H.D.

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prof.dr. Aad van der Vaart (voorzitter; Universiteit Leiden)
prof.dr. Bart de Smit (secretaris; Universiteit Leiden)
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dr. Damaris Schindler (Universiteit Utrecht)
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Nothing will come of nothing

Lear, KING LEAR, Scene 1.1, line 82

Note about the epigraphs: the spelling of names, quoted line numbers, line breaks, interpunction, and exact wording are taken from [Sha08].

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