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Breaking of ensemble equivalence for complex networks

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Citation

Roccaverde, A. (2018, December 5). *Breaking of ensemble equivalence for complex networks*. Retrieved from <https://hdl.handle.net/1887/67095>

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Breaking of Ensemble Equivalence for Complex Networks

Andrea Roccaverde

Breaking of Ensemble Equivalence for Complex Networks

Proefschrift

ter verkrijging van
de graad van Doctor aan de Universiteit Leiden,
op gezag van Rector Magnificus prof. mr. C. J. J. M. Stolker,
volgens besluit van het College voor Promoties
te verdedigen op woensdag 5 december 2018
klokke 15.00 uur

door

Andrea Roccaverde
geboren te Modena
in 1990

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Contents

1	Introduction	9
§1.1	Gibbs Ensembles	10
§1.2	Equivalence of Ensembles	11
§1.3	Definition of Ensemble Equivalence	12
§1.3.1	Measure equivalence	13
§1.4	Statistical Ensembles for Complex Networks	14
§1.4.1	Microcanonical and Canonical Ensemble for Complex Networks	15
§1.4.2	α_n -Equivalence of Ensembles	16
§1.5	Summary of Chapter 2	17
§1.6	Summary of Chapter 3	18
§1.7	Summary of Chapter 4	19
§1.8	Summary of Chapter 5	19
§1.9	Summary of Chapter 6	20
§1.10	Development of the chapters	20
§1.11	Conclusions and Open Problems	21
2	Ensemble Nonequivalence in Random Graphs with Modular Structure	25
§2.1	Introduction and main results	26
§2.1.1	Background and outline	26
§2.1.2	Microcanonical ensemble, canonical ensemble, relative entropy	28
§2.1.3	Main Theorems (Theorems 2.1.1-2.1.10)	30
§2.2	Discussion	37
§2.2.1	General considerations	37
§2.2.2	Special cases of empirical relevance	41
§2.3	Proofs of Theorems 2.1.1-2.1.10	48
§2.3.1	Proof of Theorem 2.1.1	48
§2.3.2	Proof of Theorem 2.1.4	49
§2.3.3	Proof of Theorem 2.1.5	50
§2.3.4	Proof of Theorem 2.1.6	52
§2.3.5	Proof of Theorem 2.1.7	54
§2.3.6	Proof of Theorem 2.1.8	55
§2.3.7	Proof of Theorem 2.1.9	58
§2.3.8	Proof of Theorem 2.1.10	60

3	Covariance structure behind breaking of ensemble equivalence in random graphs	65
§3.1	Introduction and main results	66
§3.1.1	Background and outline	66
§3.1.2	Constraint on the degree sequence	67
§3.1.3	Relevant regimes	68
§3.1.4	Linking ensemble nonequivalence to the canonical covariances .	69
§3.1.5	Discussion and outline	71
§3.2	Proof of the Main Theorem	75
§3.2.1	Preparatory lemmas	75
§3.2.2	Proof (Theorem 3.1.5)	77
§A	Appendix	79
§B	Appendix	80
4	Is Breaking of Ensemble Equivalence Monotone in the Number of Constraints?	85
§4.1	Introduction and main results	86
§4.1.1	Background	86
§4.1.2	Constraint on the full degree sequence	87
§4.1.3	Constraint on the partial degree sequence	88
§4.1.4	Linking ensemble nonequivalence to the canonical covariances .	90
§4.1.5	Discussion	92
§4.2	Proof of the Main Theorem	95
§4.2.1	Preparatory lemmas	95
§4.2.2	Proof (Theorem 4.1.4)	98
§A	Appendix	98
§B	Appendix	100
5	Ensemble Equivalence for dense graphs	103
§5.1	Introduction	104
§5.1.1	Background and motivation	104
§5.1.2	Relevant literature	105
§5.1.3	Outline	105
§5.2	Key notions	106
§5.2.1	Microcanonical ensemble, canonical ensemble, relative entropy	106
§5.2.2	Graphons	108
§5.2.3	Large deviation principle for the Erdős-Rényi random graph . .	109
§5.3	Variational characterisation of ensemble equivalence	111
§5.3.1	Subgraph counts	111
§5.3.2	From graphs to graphons	112
§5.3.3	Variational formula for specific relative entropy	113
§5.4	Main theorem	115
§5.5	Choice of the tuning parameter	117
§5.5.1	Tuning parameter for fixed n	118
§5.5.2	Tuning parameter for $n \rightarrow \infty$	120

§5.6	Proof of the Main Theorem 5.4.1	123
§5.6.1	Proof of (I)(a) (Triangle model $T_2^* \geq \frac{1}{8}$)	123
§5.6.2	Proof of (I)(b) ($T_2^* = 0$)	123
§5.6.3	Proof of (II)(a) (Edge-Triangle model $T_2^* = T_1^{*3}$)	124
§5.6.4	Proof of (II)(b) ($T_2^* \neq T_1^{*3}$ and $T_2^* \geq \frac{1}{8}$)	124
§5.6.5	Proof of (II)(c) ($T_2^* \neq T_1^{*3}$, $0 < T_1^* \leq \frac{1}{2}$ and $0 < T_2^{*3} < \frac{1}{8}$)	125
§5.6.6	Proof of (II)(d) ((T_1^*, T_2^*) on the scallopy curve)	126
§5.6.7	Proof of (II)(e) ($0 < T_1^* \leq \frac{1}{2}$ and $T_2^* = 0$)	127
§5.6.8	Proof of (III) (Star model $T[j]^* \geq 0$)	127
§A	Appendix	128
6	Breaking of Ensemble Equivalence for Perturbed Erdős-Rényi Random Graphs	133
§6.1	Introduction	134
§6.2	Definitions and preliminaries	135
§6.3	Theorems	137
§6.4	Proofs of Theorems 6.3.1-6.3.3	142
§6.4.1	Proof of Theorem 6.3.1	142
§6.4.2	Proof of Theorem 6.3.2	144
§6.4.3	Proof of Theorem 6.3.3	145
§6.5	Proofs of Propositions 6.3.5-6.3.7	146
§6.5.1	Proof of Proposition 6.3.5	147
§6.5.2	Proof of Lemma 6.5.1 and Lemma 6.5.2	154
	Bibliography	162
	Samenvatting	169
	Acknowledgements	171
	Curriculum Vitae	173

