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# Comment on "Absolute and Convective Instabilities in Nonlinear Systems"

When a spatially extended system goes unstable, the ensuing dynamics depends sensitively on whether the system is convectively unstable [in which case perturbations grow in time but are convected away fast enough that they die at each fixed position in the (lab) frame considered] or absolutely unstable (in which case there exists a perturbation and a location where the perturbation does not decay). The distinction between the two cases for infinitesimal disturbances is well understood; such a *linear* stability analysis captures most of the essential physics near a supercritical (continuous) bifurcation. Recently, Chomaz [1] studied the *nonlinear* convective (NLC) versus absolute (NLA) instability near a subcritical (discontinuous) bifurcation for a simple equation that derives from a free-energy-like (Lyapunov) function. The purpose of this Comment is to point out that the case studied by Chomaz is quite restrictive, since it relies on the existence of a unique front separating the basic state from the bifurcating state. In the general case there is a continuum of bifurcating states and an ensuing continuum of fronts, so the problem of *selection* must be faced. The situation was discussed earlier by two of us [2] in a general investigation of front and pulse propagation near subcritical bifurcations. The extension to systems not governed by a Lyapunov function is particularly relevant for the study of nonlinear stability of open hydrodynamic flows or of systems with traveling waves.

As a simple model for dynamics near a subcritical bifurcation, Chomaz [1] studied the real equation

$$\partial_t A + U_0 \partial_x A = \partial_x^2 A + \mu A + A^3 - A^5. \quad (1)$$

The nonlinear stability properties depend on the response to disturbances of *finite* extent and amplitude. For  $-\frac{1}{4} < \mu < 0$  Eq. (1) admits two homogeneous stable states,  $A_0 = 0$  and  $A_2 \neq 0$ . To study the nonlinear stability of the  $A_0$  state it suffices to consider a front solution joining the state  $A_2$  for  $x \rightarrow -\infty$  with the state  $A_0$  for  $x \rightarrow \infty$ , in the symmetrical ( $U_0 = 0$ ) frame where the  $U_0 \partial_x A$  term is absent. If the front speed  $v$  of this solution is negative, an isolated droplet of the  $A_2$  state in a background of the  $A_0$  state shrinks; hence the  $A_0$  state is stable. If  $v$  is positive,  $A_2$  droplets grow and the  $A_0$  state is (nonlinearly) unstable. Since for  $U_0 = 0$ , Eq. (1) is governed by a Lyapunov function [ $\partial_t A = -\delta \mathcal{L} / \delta A$ ,  $\mathcal{L} = \int dx \{(\partial_x A)^2/2 - \mu A^2/2 - A^4/4 + A^6/6\}$ ], the sign of  $v$  depends on the relative magnitude of  $\mathcal{L}(A_0)$  and  $\mathcal{L}(A_2)$ , and  $v = 0$  for  $\mu = \mu_M = -\frac{3}{16}$  where  $\mathcal{L}(A_0) = \mathcal{L}(A_2)$ . In the unstable domain  $\mu > \mu_M$  the instability in the  $U_0$  frame is convective (NLC) for  $v - U_0 < 0$ , and absolute (NLA) for  $v - U_0 \geq 0$ .

When a Hopf bifurcation to traveling waves occurs, the amplitude dynamics near a subcritical bifurcation can be modeled by an extension of (1), the complex Ginzburg-

Landau equation, which in the symmetrical ( $U_0 = 0$ ) frame reads

$$\begin{aligned} \partial_t A = & (1 + ic_1) \partial_x^2 A + \mu A + (1 + ic_3) A |A|^2 \\ & + (-1 + ic_5) A |A|^4. \end{aligned} \quad (2)$$

Here  $A$  is the complex valued amplitude, and the  $c$ 's are real parameters associated with the linear ( $c_1$ ) and nonlinear ( $c_3, c_5$ ) dispersion. Equation (2) cannot be derived from a Lyapunov function, and contrary to (1) has a continuum of bifurcating states.

The surprising finding of Ref. [2] is that the stability properties of the state  $A_0$  are largely determined by the existence or absence of an exact *nonlinear* front solution with speed  $v^\dagger(\mu, c_1, c_3, c_5)$  that increases for increasing  $\mu$  and is zero for  $\mu = \mu_3(c_1, c_3, c_5)$ . It is found [2] that either (a) this front solution exists and has positive  $v^\dagger$  for some range  $\mu > \mu_3$  with  $\mu_3 < 0$ ; (b) for all  $\mu < 0$  the front speed is negative (i.e.,  $\mu_3 > 0$ ); or (c) for  $\mu < 0$  no nonlinear front solution exists.

In case (a) the behavior for  $\mu > \mu_3$  is similar to that found in the real equation when  $\mu > \mu_M$ : The state  $A_0$  is unstable, and the instability is NLA for  $v^\dagger - U_0 \geq 0$  and NLC for  $v^\dagger - U_0 < 0$ . For  $\mu < \mu_3$ , on the other hand, typically stationary pulse solutions exist, over a range  $\mu_2 < \mu < \mu_3$ , so although  $v^\dagger < 0$ , the state  $A_0$  remains *unstable*. Since the pulse velocity is in general zero, the instability is NLC for any  $U_0 > 0$ . For  $\mu < \mu_2$  the state  $A_0$  is stable. In case (b) the pulse region extends up to  $\mu = 0$ , and for  $\mu > 0$  the stability properties are similar to those of a supercritical bifurcation with a front velocity  $v \propto \sqrt{\mu}$ . For case (c) less is known, but chaotically spreading front solutions as well as pulses have been found [2]. In some experiments [3], the latter structures help stabilize a system by absorbing small perturbations that are convected into them. It is an open question which regime is relevant for planar Poiseuille flow, where  $c_1 \approx 0.4$  and  $c_3 \approx 6$  [4] but  $c_5$  is not known.

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