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Summary

When folding a crease pattern we change a two-dimensional, flat material into an often complex three dimensional shape. Examples of such crease patterns have been known for centuries as 'origami'. Recently however, material scientists and physicists have discovered that some crease patterns have exotic material properties when folded. An example of this is the so called 'Miura-ori' pattern (see chapter 1), which has a negative Poisson's ratio in its folded configuration, which means it shrinks in multiple directions when squeezed. Such properties depend solely on the geometry of the crease pattern, and can be applied on the very small scale, such as in insect wings, and on the very large scale, such as in solar panels for space-satellites. In the first part of this thesis we therefore study the geometry of folding patterns. Specifically, we focus on crease patterns consisting entirely of four-vertices; these are points where four fold lines come together. A single four-vertex is the simplest example of a foldable crease pattern that can be folded without bending the material in between the folds, and has a remarkable property: despite its single degree of freedom, it has two distinct folding motions. We make use of this property, and show how to design arbitrarily large four-vertex crease patterns, which can fold into two or more shapes. This is in contrast to other design methods, which produce patterns that can only fold into one specific shape. In the second part of this thesis, we study single four-vertices, and show a robust method to obtain four-vertices with three energy minima, which correspond to three different stable folded configurations. This too is in contrast to other experimental methods, which can only generate bistable vertices or patterns.

Most crease patterns consisting of four-vertices are periodic, and are designed by copying a single unit cell, which allows for the creation of arbitrarily large crease patterns. These crease patterns satisfy the so called 'rigid-folding' condition, which says that the panels in between the fold lines –quadrilaterals in this case– are not allowed to deform during the folding process. Such a crease pattern can be considered as a mechanism, consisting of rigid quadrilateral panels connected by freely moving

hinges. In this work we show a combinatorial method capable of generating arbitrarily large *aperiodic* crease patterns, which also have multiple distinct folding motions. We do this by representing crease patterns by puzzle pieces; each of these puzzle pieces represents a minimal four-vertex folding pattern where the four corners of a quadrilateral are each occupied by a four-vertex. By using a set of vertices that are related to each other by symmetry, the crease pattern associated with each puzzle piece can fold into at least two different configurations. Furthermore, these puzzle pieces are designed such that two (or more) puzzle pieces that fit together, automatically lead to a rigidly foldable pattern. The resulting combinatorial fold patterns can be categorized into different classes, since some puzzle pieces can appear within one and the same pattern, and some can not. We count the number of crease patterns of a given size within each class, and the number of folding motions for each pattern, which varies per class.

We then focus on a special class, which has exactly two folding motions, regardless of the size of the pattern. In this class the shape of both folding motions can be tuned independently of each other, by choosing which puzzle pieces are at the top- and left-side of the pattern. We demonstrate this design strategy by laser cutting two plastic sheets with identical crease patterns, and folding them up into two different shapes. As far as we know, this is the first (experimental) demonstration of such a bipotent folding pattern.

In the second part of this thesis we study the behavior of single four-vertices. Specifically, we focus our attention on non-euclidean four-vertices. These are four-vertices which can not be folded from a flat piece of material, as the four angles between the four fold lines of the vertex add up to less (or more) than 360 degrees. Because of this, the two folding branches of the single four-vertex –which come together at the flat configuration for a euclidean four-vertex– are disconnected from each other by the rigid folding condition. This separation makes it impossible to switch between the two folding branches without bending the panels. However, it is still possible to switch from folding branches by forcing the vertex through the flat configuration. Effectively we therefore created an energy barrier between the two folding branches, since the material of the vertex has to be bent.

We show that this energy barrier can be harnessed to design four-vertices with three stable configurations by adding a single torsional spring to one of the folds. This spring ensures that two stable configurations are created on one of the two folding branches, which both correspond to a global energy minimum. On the other folding branch, we find a single stable configuration, corresponding to a local energy minimum. This minimum is stable, provided that the energy barrier mentioned above is high enough. We give an experimental demonstration of such a system by 3D-printing non-euclidean four-vertices of plastic, and placing a single torsional spring on one of the folds. We show that the resulting vertices indeed have three stable configurations. Finally, we compare the theoretical energy curves of the folding branches with experimental energy curves. We do this by testing the vertices in a torsion-tester, which can measure the torque and energy required to switch from one stable configuration to another.

The design strategies presented in this thesis can be applied by making use of new, computer-controlled manufacturing techniques, such as laser cutters and 3D-printers. These techniques can be applied to make patterns on the scale of centimeters, such as in this thesis, but are also applicable on the smaller scale of micro- and millimeters. This puts materials with completely novel functionalities within reach.