Zeta-values of arithmetic schemes at negative integers and Weil-étale cohomology

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Abstract

This work is dedicated to interpreting in cohomological terms the special values of zeta functions of arithmetic schemes.

This is a part of the program envisioned and started by Stephen Lichtenbaum (see e.g. Ann. of Math. vol. 170, 2009), and the conjectural underlying cohomology theory is known as Weil-étale cohomology. Later on Baptiste Morin and Matthias Flach gave a construction of Weil-étale cohomology using Bloch's cycle complex and stated a precise conjecture for the special values of proper regular arithmetic schemes at any integer argument $s = n$. The goal is to extend the above mentioned result and conjecture to special values of arbitrary arithmetic schemes (possible singular or non-proper) while restricting to the case $n < 0$.

Following the ideas of Flach and Morin, the Weil-étale complexes are defined for $n < 0$ for arbitrary arithmetic schemes, under standard conjectures about finite generation of motivic cohomology. Then it is stated as a conjecture how these complexes are related to the special values. For proper and regular schemes, this conjecture is equivalent to the conjecture of Flach and Morin, which also corresponds to the Tamagawa number conjecture.

We prove that the conjecture stated in this work is compatible with the decomposition of an arbitrary scheme into an open subscheme and its closed complement. We also show that this conjecture for an arithmetic scheme X at $s = n$ is equivalent to the conjecture for $\mathbb{A}^r_X$ at $s = n - r$, for any $r \geq 0$. It follows that, taking as an input the schemes for which the conjecture is known, it is possible to construct new schemes, possibly singular or non-proper, for which the conjecture holds as well. This is the main unconditional outcome of the machinery developed in this thesis.