



Universiteit
Leiden
The Netherlands

Zeta-values of arithmetic schemes at negative integers and Weil-étale cohomology

Beshenov, A.

Citation

Beshenov, A. (2018, December 10). *Zeta-values of arithmetic schemes at negative integers and Weil-étale cohomology*. Retrieved from <https://hdl.handle.net/1887/68224>

Version: Not Applicable (or Unknown)

License: [Licence agreement concerning inclusion of doctoral thesis in the Institutional Repository of the University of Leiden](#)

Downloaded from: <https://hdl.handle.net/1887/68224>

Note: To cite this publication please use the final published version (if applicable).

Cover Page



Universiteit Leiden



The handle <http://hdl.handle.net/1887/68224> holds various files of this Leiden University dissertation.

Author: Beshenov, A.

Title: Zeta-values of arithmetic schemes at negative integers and Weil-étale cohomology

Issue Date: 2018-12-10

Bibliography

- [AV1964] Michael Artin and Jean-Louis Verdier, *Seminar on étale cohomology of number fields*, Lecture notes prepared in connection with the seminars held at the summer institute on algebraic geometry. Whitney estate, Woods hole, Massachusetts. July 6 – July 31, 1964, American Mathematical Society, Providence, R.I., 1964.
<http://www.jmilne.org/math/Documents/woodshole3.pdf>
- [BBD1982] Alexander Beilinson, Joseph Bernstein, and Pierre Deligne, *Faisceaux pervers*, Analysis and topology on singular spaces, I (Luminy, 1981), Astérisque, vol. 100, Soc. Math. France, Paris, 1982, pp. 5–171. MR751966
<http://publications.ias.edu/sites/default/files/Faisceaux%20pervers.pdf>
- [Bei1984] Alexander Beilinson, *Higher regulators and values of L-functions*, Current problems in mathematics, Vol. 24, Itogi Nauki i Tekhniki, Akad. Nauk SSSR, Vsesoyuz. Inst. Nauchn. i Tekhn. Inform., Moscow, 1984, pp. 181–238. MR760999
<http://dx.doi.org/10.1007/BF02105861>
- [BHS2011] Ronald Brown, Philip J. Higgins, and Rafael Sivera, *Nonabelian algebraic topology. filtered spaces, crossed complexes, cubical homotopy groupoids*, EMS Tracts in Mathematics, vol. 15, European Mathematical Society (EMS), Zürich, 2011, With contributions by Christopher D. Wensley and Sergei V. Soloviev. MR2841564
<http://dx.doi.org/10.4171/083>
- [Blo1986a] Spencer Bloch, *Algebraic cycles and higher K-theory*, Adv. in Math. **61** (1986), no. 3, 267–304. MR852815
[http://dx.doi.org/10.1016/0001-8708\(86\)90081-2](http://dx.doi.org/10.1016/0001-8708(86)90081-2)
- [Blo1986b] ———, *Algebraic cycles and the Beilinson conjectures*, The Lefschetz centennial conference, Part I (Mexico City, 1984), Contemp.

- Math., vol. 58, Amer. Math. Soc., Providence, RI, 1986, pp. 65–79.
MR860404
<https://doi.org/10.1090/conm/058.1/860404>
- [Blo1994] ———, *The moving lemma for higher Chow groups*, J. Algebraic Geom. **3** (1994), no. 3, 537–568. MR1269719
- [Blo2005] ———, *Basic definition and properties of higher chow groups*, 2005, Unpublished. Available on the webpage of Spencer Bloch.
http://www.math.uchicago.edu/~bloch/cubical_chow.pdf
- [Bro1994] Kenneth S. Brown, *Cohomology of groups*, Graduate Texts in Mathematics, vol. 87, Springer-Verlag, New York, 1994, Corrected reprint of the 1982 original. MR1324339
- [BS2013] Bhargav Bhatt and Peter Scholze, *The pro-étale topology for schemes*, ArXiv e-prints (2013).
<http://arxiv.org/abs/1309.1198>
- [Cis2006] Denis-Charles Cisinski, *Les préfaisceaux comme modèles des types d'homotopie*, Astérisque (2006), no. 308, xxiv+390. MR2294028
<http://www.mathematik.uni-regensburg.de/cisinski/ast.pdf>
- [Con2007] Brian Conrad, *Deligne's notes on Nagata compactifications*, J. Ramanujan Math. Soc. **22** (2007), no. 3, 205–257. MR2356346
<http://math.stanford.edu/~conrad/papers/nagatafinal.pdf>
- [Con2009] ———, *Erratum for "Deligne's notes on Nagata compactifications"*, J. Ramanujan Math. Soc. **24** (2009), no. 4, 427–428. MR2599570
- [Del1971] Pierre Deligne, *Théorie de Hodge. II*, Inst. Hautes Études Sci. Publ. Math. (1971), no. 40, 5–57. MR0498551
http://www.numdam.org/item?id=PMIHES_1971__40__5_0
- [Elements] Euclid, *The thirteen books of the Elements*, Cambridge University Press, Cambridge, 1908, Translated with introduction and commentary by Sir Thomas L. Heath.
<http://archive.org/details/thirteenbookseu00heibgoog>
- [EM1953] Samuel Eilenberg and Saunders MacLane, *Acyclic models*, Amer. J. Math. **75** (1953), 189–199. MR0052766
<http://dx.doi.org/10.2307/2372628>
- [EV1988] Hélène Esnault and Eckart Viehweg, *Deligne-Beilinson cohomology*, Beilinson's conjectures on special values of L -functions, Perspect.

- Math., vol. 4, Academic Press, Boston, MA, 1988, pp. 43–91.
MR944991
[http://wwwmath.uni-muenster.de/u/schneider/publ/
beilinson-volume/](http://wwwmath.uni-muenster.de/u/schneider/publ/beilinson-volume/)
- [FM2016] Matthias Flach and Baptiste Morin, *Weil-étale cohomology and Zeta-values of proper regular arithmetic schemes*, ArXiv e-prints (2016).
<http://arxiv.org/abs/1605.01277>
- [Ful1998] William Fulton, *Intersection theory*, second ed., Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge. A Series of Modern Surveys in Mathematics [Results in Mathematics and Related Areas. 3rd Series. A Series of Modern Surveys in Mathematics], vol. 2, Springer-Verlag, Berlin, 1998. MR1644323
<http://dx.doi.org/10.1007/978-1-4612-1700-8>
- [Gei2004] Thomas Geisser, *Motivic cohomology over Dedekind rings*, Math. Z. **248** (2004), no. 4, 773–794. MR2103541
<http://dx.doi.org/10.1007/s00209-004-0680-x>
- [Gei2005] ———, *Motivic cohomology, K-theory and topological cyclic homology*, Handbook of K-theory. Vol. 1, 2, Springer, Berlin, 2005, pp. 193–234. MR2181824
<http://www.math.illinois.edu/K-theory/handbook/>
- [Gei2010] ———, *Duality via cycle complexes*, Ann. of Math. (2) **172** (2010), no. 2, 1095–1126. MR2680487
<http://dx.doi.org/10.4007/annals.2010.172.1095>
- [GJ2009] Paul G. Goerss and John F. Jardine, *Simplicial homotopy theory*, Modern Birkhäuser Classics, Birkhäuser Verlag, Basel, 2009, Reprint of the 1999 edition [MR1711612]. MR2840650
<http://dx.doi.org/10.1007/978-3-0346-0189-4>
- [GKZ1994] Israel M. Gel'fand, Mikhail M. Kapranov, and Andrei V. Zelevinsky, *Discriminants, resultants, and multidimensional determinants*, Mathematics: Theory & Applications, Birkhäuser Boston, Inc., Boston, MA, 1994. MR1264417
<http://dx.doi.org/10.1007/978-0-8176-4771-1>
- [Gon1995] Alexander Goncharov, *Chow polylogarithms and regulators*, Math. Res. Lett. **2** (1995), no. 1, 95–112. MR1312980
<http://dx.doi.org/10.4310/MRL.1995.v2.n1.a9>

- [Ive1986] Birger Iversen, *Cohomology of sheaves*, Universitext, Springer-Verlag, Berlin, 1986. MR842190
<https://doi.org/10.1007/978-3-642-82783-9>
- [Jan1988] Uwe Jannsen, *Deligne homology, Hodge-D-conjecture, and motives*, Beilinson's conjectures on special values of L-functions, *Perspect. Math.*, vol. 4, Academic Press, Boston, MA, 1988, pp. 305–372. MR944998
<http://wwwmath.uni-muenster.de/u/schneider/publ/beilinson-volume/>
- [Kat1994] Nicholas M. Katz, *Review of ℓ -adic cohomology*, 21–30. MR1265520
- [KLMS2006] Matt Kerr, James D. Lewis, and Stefan Müller-Stach, *The Abel-Jacobi map for higher Chow groups*, *Compos. Math.* **142** (2006), no. 2, 374–396. MR2218900
<http://arxiv.org/abs/math/0409116>
- [KM1976] Finn Faye Knudsen and David Mumford, *The projectivity of the moduli space of stable curves. I. Preliminaries on “det” and “Div”*, *Math. Scand.* **39** (1976), no. 1, 19–55. MR0437541
<http://mscand.dk/article/view/11642/9658>
- [Lev1994] Marc Levine, *Bloch's higher Chow groups revisited*, *Astérisque* (1994), no. 226, 10, 235–320, *K-theory* (Strasbourg, 1992). MR1317122
<http://www.uni-due.de/~bm0032/publ/HigherChowRevisit.pdf>
- [Lic2005] Stephen Lichtenbaum, *The Weil-étale topology on schemes over finite fields*, *Compos. Math.* **141** (2005), no. 3, 689–702. MR2135283
<http://dx.doi.org/10.1112/S0010437X04001150>
- [Lic2009a] ———, *Euler characteristics and special values of zeta-functions*, *Motives and algebraic cycles*, *Fields Inst. Commun.*, vol. 56, Amer. Math. Soc., Providence, RI, 2009, pp. 249–255. MR2562461
- [Lic2009b] ———, *The Weil-étale topology for number rings*, *Ann. of Math. (2)* **170** (2009), no. 2, 657–683. MR2552104
<http://dx.doi.org/10.4007/annals.2009.170.657>
- [Lur2006] Jacob Lurie, *Stable infinity categories*, *ArXiv e-prints* (2006).
<http://arxiv.org/abs/math/0608228>
- [LW1933] Solomon Lefschetz and J. H. C. Whitehead, *On analytical complexes*, *Trans. Amer. Math. Soc.* **35** (1933), no. 2, 510–517. MR1501698
<http://dx.doi.org/10.2307/1989779>

- [Mas1977] William S. Massey, *Algebraic topology: an introduction*, Springer-Verlag, New York-Heidelberg, 1977, Reprint of the 1967 edition, Graduate Texts in Mathematics, Vol. 56. MR0448331
- [May1992] J. Peter May, *Simplicial objects in algebraic topology*, Chicago Lectures in Mathematics, University of Chicago Press, Chicago, IL, 1992, Reprint of the 1967 original. MR1206474
- [May2001] J. P. May, *The additivity of traces in triangulated categories*, Adv. Math. **163** (2001), no. 1, 34–73. MR1867203
<https://doi.org/10.1006/aima.2001.1995>
- [Mil2006] J. S. Milne, *Arithmetic duality theorems*, second ed., BookSurge, LLC, Charleston, SC, 2006. MR2261462
<http://www.jmilne.org/math/Books/ADTnot.pdf>
- [MLM1994] Saunders Mac Lane and Ieke Moerdijk, *Sheaves in geometry and logic*, Universitext, Springer-Verlag, New York, 1994, A first introduction to topos theory, Corrected reprint of the 1992 edition. MR1300636
<http://dx.doi.org/10.1007/978-1-4612-0927-0>
- [Mor2014] Baptiste Morin, *Zeta functions of regular arithmetic schemes at $s = 0$* , Duke Math. J. **163** (2014), no. 7, 1263–1336. MR3205726
<http://arxiv.org/abs/1103.6061>
- [Nee1991] Amnon Neeman, *Some new axioms for triangulated categories*, J. Algebra **139** (1991), no. 1, 221–255. MR1106349
[https://doi.org/10.1016/0021-8693\(91\)90292-G](https://doi.org/10.1016/0021-8693(91)90292-G)
- [Ram2016] Niranjana Ramachandran, *Higher Euler characteristics: variations on a theme of Euler*, Homology Homotopy Appl. **18** (2016), no. 1, 231–246. MR3491851
<http://arxiv.org/abs/1509.05089v1>
- [Ser1965] Jean-Pierre Serre, *Zeta and L functions*, Arithmetical Algebraic Geometry (Proc. Conf. Purdue Univ., 1963), Harper & Row, New York, 1965, pp. 82–92. MR0194396
- [SGA 1] Alexander Grothendieck and Michèle Raynaud (eds.), *Séminaire de géométrie algébrique du Bois-Marie 1960–61 (SGA 4): Revêtements étales et groupe fondamental*, Lecture Notes in Mathematics, Vol. 224, Springer-Verlag, Berlin-New York, 1971. MR2017446
<http://arxiv.org/abs/math/0206203>

- [SGA 4] Michael Artin, Alexander Grothendieck, and Jean-Louis Verdier (eds.), *Séminaire de géométrie algébrique du Bois-Marie 1963–1964 (SGA 4): Théorie des topos et cohomologie étale des schémas*, Lecture Notes in Mathematics, Vol. 269, 270, 305, Springer-Verlag, Berlin-New York, 1972–73, Avec la collaboration de N. Bourbaki, P. Deligne et B. Saint-Donat. MR0354652
<http://fabrice.orgogozo.perso.math.cnrs.fr/SGA4/>
- [SGA 4 $\frac{1}{2}$] Pierre Deligne, *Cohomologie étale*, Lecture Notes in Mathematics, Vol. 569, Springer-Verlag, Berlin-New York, 1977, Séminaire de Géométrie Algébrique du Bois-Marie SGA 4 $\frac{1}{2}$, Avec la collaboration de J. F. Boutot, A. Grothendieck, L. Illusie et J. L. Verdier. MR0463174
<https://link.springer.com/book/10.1007%2FBFB0091516>
- [SGA 5] Alexander Grothendieck (ed.), *Séminaire de géométrie algébrique du Bois-Marie 1965–66 (SGA 5): Cohomologie ℓ -adique et fonctions L*, Lecture Notes in Mathematics, Vol. 589, Springer-Verlag, Berlin-New York, 1977, Avec la collaboration de I. Bucur, C. Houzel, L. Illusie, J.-P. Jouanolou et J.-P. Serre. MR0491704
<http://library.msri.org/books/sga/>
- [SGA 7] Alexander Grothendieck, Pierre Deligne, and Nicholas Katz (eds.), *Séminaire de géométrie algébrique du Bois-Marie 1967–69 (SGA 7): Groupes de monodromie en géométrie algébrique*, Lecture Notes in Mathematics, Vol. 288, 340, Springer-Verlag, Berlin-New York, 1972–73, Avec la collaboration de M. Raynaud et D.S. Rim. MR0354657
<http://library.msri.org/books/sga/>
- [Stacks] The Stacks Project Authors, *Stacks project*, 2017.
<http://stacks.math.columbia.edu>
- [Tôhoku] Alexander Grothendieck, *Sur quelques points d'algèbre homologique*, Tôhoku Math. J. (2) **9** (1957), 119–221. MR0102537
<http://projecteuclid.org/euclid.tmj/1178244839>
- [vdW1930] Bartel L. van der Waerden, *Topologische Begründung des Kalküls der abzählenden Geometrie*, Math. Ann. **102** (1930), no. 1, 337–362. MR1512581
<http://dx.doi.org/10.1007/BF01782350>
- [Ver1976] Jean-Louis Verdier, *Classe d'homologie associée à un cycle*, 101–151. Astérisque, No. 36–37. MR0447623

- [Verdier-thèse] ———, *Des catégories dérivées des catégories abéliennes*, Astérisque (1996), no. 239, xii+253 pp., With a preface by Luc Illusie, Edited and with a note by Georges Maltsiniotis. MR1453167
<http://webusers.imj-prg.fr/~georges.maltsiniotis/jlv.html>
- [Voi2002] Claire Voisin, *Hodge theory and complex algebraic geometry. I*, Cambridge Studies in Advanced Mathematics, vol. 76, Cambridge University Press, Cambridge, 2002, Translated from the French original by Leila Schneps. MR1967689
<http://dx.doi.org/10.1017/CB09780511615344>
- [Wei1994] Charles A. Weibel, *An introduction to homological algebra*, Cambridge Studies in Advanced Mathematics, vol. 38, Cambridge University Press, Cambridge, 1994. MR1269324
<http://dx.doi.org/10.1017/CB09781139644136>
- [Wei2017] Thomas Weißschuh, *A commutative regulator map into Deligne–Beilinson cohomology*, vol. 152, 2017. MR3608294
<https://doi.org/10.1007/s00229-016-0867-6>

Acknowledgements

This thesis was written under the framework of the ALGANT DOC program supported by the European Union, and I was very lucky to spend my PhD between Bordeaux and Leiden, both being places of great importance in number theory.

Above all, I am deeply grateful to my advisors, BAPTISTE MORIN and BAS EDIXHOVEN, who have taught me a lot of mathematics and showed exceptional kindness and patience. Their support means a lot to me, both mathematically and personally.

I thank STEPHEN LICHTENBAUM and NIRANJAN RAMACHANDRAN for being the referees and providing their invaluable feedback.

It is also important to mention the influence of MATTHIAS FLACH: I have learned many ideas that appear here from reading his joint paper with Baptiste Morin.

I would also like to thank YURI BILU for his support, his dedication to the ALGANT program over the years, and above all his influence on my number theoretical education.

BOAS EREZ introduced me to arithmetic geometry during my master studies in Bordeaux and also introduced me to Baptiste Morin.

I am also indebted to NIKOLAĀ MISHACHEV and DMITRIĀ PASECHNIK for their support during my early steps in mathematics.

On the personal side, I am grateful to GABRIEL CHICAS REYES for all the experiences shared together, to my brother DMITRIĀ BESHENOV, and to all of my friends and fellow PhD students from Milan, Bordeaux, and Leiden, particularly JAVIER ESTRADA, ANDREA GAGNA, MAURO MANTEGAZZA, CHLOE MARTINDALE, MAXIM MORNEV, FRANCES ODUMODU, GIULIO ORECCHIA, DIMA SHVETSOV, PAVEL SOLOMATIN, JOSÉ VILLANUEVA, and ROSA WINTER.

Agradezco a toda la familia LÓPEZ ARAVENA y particularmente a OLGA, ANGÉLICA “KIKI”, HÉCTOR “TITO”, CRISTIAN y PAOLO por su compañía durante mi estancia en Burdeos, todas las experiencias compartidas y por hacer más feliz mi vida de estudiante en Francia.

Y por último, pero no menos importante, les doy gracias a mis amigos salvadoreños: MARIO CARPIO, JORGE FLORES, DIANA HERRERA y YOCEMAN SIFONTES, y también al Ministro CARLOS CANJURA, NERYS FUNES y AMÉRICO HIDALGO por su apoyo en San Salvador donde terminé de redactar esta tesis.