

Graphing formulas by hand to promote symbol sense

Becoming friends with algebraic formulas



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Becoming friends with algebraic formulas

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