Wireless random-access networks and spectra of random graphs

Sfragara, M.

Author: Sfragara, M.
Title: Wireless random-access networks and spectra of random-graphs
Issue Date: 2020-10-28

Summary

This thesis is divided into two parts. In Part I we study metastability properties of queue-based random-access protocols for wireless networks. The network is modeled as a bipartite graph whose edges represent interference constraints. In Part II we study spectra of inhomogeneous Erdős-Rényi random graphs. We focus in particular on the limiting spectral distribution of the adjacency and Laplacian matrices and on the largest eigenvalue of the adjacency matrix.

Part I

Random-access protocols have been introduced in the context of wireless networks with the aim to avoid collisions between ongoing transmissions of wireless signals. The main idea behind these protocols is to associate with each device a random clock that determines when the device attempts to transmit. Hence, devices decide autonomously when to start a transmission using only local information. We consider random-access networks where each node represents a server with a queue. The nodes can be either active or inactive: a node deactivates at unit rate, while it activates at a rate that depends on its queue length, provided none of its neighbors is active. Wireless random-access networks are known to exhibit metastability effects. In a regime where the activation rates become large, the stationary distribution of the joint activity process concentrates on states where the maximum number of nodes is active, with extremely slow transitions between them. Individual nodes may experience prolonged periods of starvation, resulting in severe build-up of queues and long delays. A deeper understanding of metastability properties is important for designing mechanisms to improve the network performance. We model the network as a bipartite graph and, in the limit as the queues become large, we study the transition time between the two states where one half of the network is active and the other half is inactive.

In Chapter 2 we consider complete bipartite graphs. We compare the transition time of an internal model in which the activation rates depend on the current queue lengths with that of an external model in which the activation rates depend on the current mean queue lengths. We define two perturbed models with externally driven activation rates that sandwich the queue lengths of the internal model and its transition time. We show with the help of coupling that with high probability the mean transition time and its distribution for the internal model are asymptotically the same as for the external model. The law of the transition time divided by its mean is exponential, truncated polynomial or deterministic, depending on the activation rate functions.

In Chapter 3 we consider arbitrary bipartite graphs. We decompose the transition into a succession of transitions on complete bipartite subgraphs. This succession

depends in a delicate manner on the full structure of the graph. We formulate a greedy algorithm to analyze the most likely transition paths between dominant states. By combining our results for complete bipartite graphs with a detailed analysis of the algorithm, we determine the mean transition time and its distribution along each path. Depending on the activation rate functions, we again identify three regimes of behavior.

In Chapter 4 we consider dynamic bipartite graphs. In order to try to capture the effects of user mobility in wireless networks, we analyze dynamic interference graphs where the edges are allowed to appear and disappear over time. A node can activate either when its neighbors are simultaneously inactive or when the edges connecting it with its neighbors disappear. Interpolation between these two situations gives rise to different scenarios and interesting behavior. We identify how the order of the mean transition time depends on the speed of the dynamics.

Part II

Eigenvalues play a central role in our understanding of graphs. Spectral graph theory studies the properties of eigenvalues and eigenvectors of the associated adjacency and Laplacian matrices. The eigenvalues of the adjacency matrix carry information about topological features of the graph, such as connectivity and subgraph counts. The eigenvalues of the Laplacian matrix carry information about random walks on the graph and allow us to analyze approximation algorithms. The standard Erdős-Rényi random graph model, which is the most basic model in random graph theory, is formed by connecting each pair of vertices with a certain fixed probability, independently of each other. The spectra of both the adjacency and Laplacian matrices are well studied and well understood. We consider inhomogeneous Erdős-Rényi random graphs, where each pair of vertices is connected with a certain probability that is not necessarily the same for all pairs, but still independently of each other.

In Chapter 5 we consider inhomogenerous Erdős-Rényi random graphs in the non-dense non-sparse regime, where the degrees of the vertices diverge sublinearly with the size of the graph. We study the limiting behavior of the emprical spectral distributions of the adjacency and Laplacian matrices. When the connection probabilities have a multiplicative structure, we give an explicit description of the scaling limits using tools from free probability theory. Furthermore, we apply our results to constrained random graphs, Chung-Lu random graphs and social networks.

In Chapter 6 we consider inhomogenerous Erdős-Rényi random graphs in the dense regime, where the degrees of the vertices are proportional to the size of the graph. Using the theory of graphons, we derive a large deviation principle for the largest eigenvalue and analyze the associated rate function in detail. Indeed, the structure of the graph conditional on a large deviation can be expressed in terms of a variational problem involving graphons. When the connection probabilities have a multiplicative structure, we analyze the variational formula in order to identify the scaling properties of the rate function.