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Wireless random-access networks and spectra of random graphs

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Wireless Random-Access Networks and Spectra of Random Graphs

Matteo Sfragara

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Wireless Random-Access Networks and Spectra of Random Graphs

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