Néron models in high dimension: Nodal curves, Jacobians and tame base change

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Stellingen

behorend bij het proefschrift getiteld:
"Néron models in higher dimension:
nodal curves, Jacobians and tame base-change".

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1 - Over a regular and excellent base of dimension greater than 1, Néron models of nodal curves and their Jacobians exist much more often without conditions of separatedness. (For curves, Theorem 7.48 of this thesis; and for Jacobians, Corollary 6.13 of Models of Jacobians of Curves, together with David Holmes, Samouil Molcho and Giulio Orecchia.)

2 - Let R be a strictly local, noetherian and excellent unique factorization domain. Put $K = \operatorname{Frac} R$, and let X_K be the elliptic K-curve given in affine coordinates by the equation

$$y^2 = (x-1)(x^2 - \Delta)$$

for some non-invertible $\Delta \in R$. Let n be the number of prime factors of Δ in R. Then, the graphs appearing as dual graphs of the closed fiber of a nodal S-model of X_K are precisely those consisting of a cycle of length at most n, with trees attached to it.

3 - Let S be a regular and strictly local scheme, and $X' \to X$ a morphism of nodal models of a smooth curve over the generic point of S. Let $\bar{e}$ (resp. $\bar{e}'$) be the scheme-theoretical closure of the unit section in $\mathrm{Pic}_{X/S}$ (resp. $\mathrm{Pic}_{X'/S}$). Then, the natural map

$$\mathrm{Pic}_{X/S} / \bar{e} \to \mathrm{Pic}_{X'/S} / \bar{e}' \tag{1}$$

is an open immersion. If, in addition, $X' \to X$ restricts to an isomorphism above the complement of the closed subset of X consisting of the singular points that are disconnecting in their fiber, then (1) is an isomorphism. (Proof of Proposition 6.19 of this thesis)

4 - Let S be a regular, integral and excellent scheme and X/S a nodal curve with smooth generic fiber. Then, the category whose objects are morphisms $Y \to X$ where Y is a smooth S-algebraic space, and whose morphisms are the appropriate commutative triangles, has a terminal object $N \to X$. (Follows from Proposition 7.10 of the thesis.)

5 - Let n be an integer not divisible by 2, 3 or 5, and P be a platonic solid. Then, the number of different ways to color the faces of P (up to rotation) with n colors is a multiple of n^2.

6 - If one wishes to defend their PhD at an age comprised between a square and a cube, the only two options are 0 and 26, no matter how much progress medicine makes.

7 - Let P be a finite set (of voters) of cardinality ≥ 12, and $\mathcal{A}$ a set (of candidates) with three elements A, B and C. Let $O_{\mathcal{A}}$ be the set of total orders on $\mathcal{A}$, and suppose we have a map $f\colon P \to O_{\mathcal{A}}$ (that we think of as taking a voter to their order of preference), such that $P_1 := f^{-1}(C < A < B)$ has cardinal $\lceil \frac{\#P}{2} \rceil - 1$; $P_2 := f^{-1}(B < A < C)$ has cardinal $\lfloor \frac{\#P}{2} \rfloor - 2$, and the three elements of $P_3 := P \backslash (P_1 \cup P_2)$ are all sent to $B < C < A$.

Then, a strict majority of voters prefer A to B and a strict majority of voters prefer A to C. Yet, if a presidential election with these conditions takes place in Iceland (respectively, France), and if each voter votes for their favourite candidate, then B will win (respectively C will win). However, if the voters have accurate polls at their disposal, and if each voter is willing to lie in order to get the best possible result relatively to their personal preference, then A wins in both Iceland and France.

8 - An important part of the job of a mathematician consists in guessing accurately enough which statements the reader or interlocutor already accepts as true.

9 - A principle seemingly as consensual as "be nice to others", if examined rationally, has consequences that go far beyond intuition. Yet it is consensual for a reason.