Expansions of quantum group invariants

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Propositions

appended to the dissertation

Expansions of quantum group invariants

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1. Let b^- be the Lie bialgebra over the ring $R_\epsilon = \mathbb{R}[\epsilon]/(\epsilon^2)$ constructed from the Lie bialgebra $\mathfrak{b}^- \subset sl_3$ of lower triangular matrices by multiplying the cobracket of $\mathfrak{b}^-$ with ϵ. Let b^+ be its dual Lie bialgebra. Then b^- and b^+ have a quantization over R_ϵ, denoted as $U_q(b^-)$ and $U_q(b^+)$ respectively. Moreover, $U_q(b^\pm)$ are QUE-dual. Here $q = e^{-\epsilon h}$, and $h \in R_\epsilon[[h]] =: P_\epsilon$. *(This thesis, section 1.4, p. 34-41)*

2. Let $B = \{z_1, \cdots, z_n\}$ and B^* its dual. Let W_B be the space of exponentials with generators B as defined in proposition 2.2.1 and let $w \in W_B$. Let $\zeta s \subset B^*$, and denote $\langle w \rangle_{\zeta s} = (w|_{\zeta \mapsto \zeta^* \forall \zeta \in \zeta s})|_{\zeta^* = 0 \forall \zeta \in \zeta s}$. Then $\langle w \rangle_{B^*}$ converges and can be written as $\langle w' \rangle_{B^*}$, such that w' has no quadratic terms $z_i z_j^*$ in the exponential. *(This thesis, section 2.2, p. 57-62)*

3. Let K be an oriented framed long knot, and let $Z_3^\epsilon(K)$ be the knot invariant corresponding to $U_q(sl_3^\epsilon)$. Then $Z_3^0(K)$ is the product of the inverse of the Alexander polynomial of K in the variables e^{hs_i} where s_i, $i \in \Phi$, are the elements of the subalgebra of non-semisimple elements of $U_q(sl_3^0)$ corresponding to the positive roots Φ. *(This thesis, section 3.3, p. 92-94)*

4. Let $U_{\tilde{q}}(\tilde{sl}_n^\epsilon)$ with $\tilde{q} = e^{\epsilon h}$, where ϵ is invertible, be the Hopf algebra over $\mathbb{R}(\epsilon)[[h]]$ generated topologically by $\{X_i^\pm, H_i^\pm\}_{i=1,\cdots,n-1}$ and the relations in theorem 4.1.3. Then there exists a set of algebra automorphisms T_i such that $T_i T_j T_i = T_j T_i T_j$ for all i, j. Moreover,
$$\Delta(T_i(X_j^\pm)) = \overline{\mathcal{R}}_i T_i \otimes T_i(\Delta(X_j^\pm))\overline{\mathcal{R}}_i^{-1},$$
where $\overline{\mathcal{R}}_i$ is the partial R-matrix as defined in section 4.3 of the thesis. *(This thesis, section 4.3, p. 112-124)*

5. Let $G = (V, E)$ be a complete directed graph with n vertices labeled by integers $1, .., n$. We do not allow loops, and between any two vertices there can be at most one edge with a given direction. This allows for two edges between any two vertices. Define a cycle $C = (V', E')$ as a connected subgraph of G with vertices $V' \subset 1, ..., n$ in which each vertex has degree precisely 2, with one incoming and one outgoing edge. Define the length of C as $|E'|$. Then 71.8 percent of the partitions of G in disjoint cycles does not contain any cycles of length 2, when n is sufficiently large.

6. In the situation of proposition 5, when G is a bipartite graph with $2n$ vertices where both groups of vertices are exactly size n, 63.2 percent of the partitions of G into disjoint cycles does not contain any cycles of length two, for sufficiently large n.

7. Define a tetrahedron as the polyhedron that is obtained through the attachment of two sticks s_1 and s_2 of length l_1 and l_2 with four edges, where there runs a straight line between all pairs of endpoints of the sticks, and the sticks are rotated 90 degrees with respect to each other. Call the tetrahedron perfect when the four sides are equal

and so $l = l_1 = l_2$, and call a tetrahedron almost perfect when there are two equal sides. Define the circumference of a line as twice the length of the line. Define the height h of the tetrahedron t as the line that is perpendicular to the plane P_t where both sticks s_1 and s_2 run parallel to. Define the circumference of a tetrahedron t at height h as the circumference of the section of the tetrahedron that is parallel to P_t. Then the circumference of a perfect tetrahedron is $2l$ at all heights, and the space of perfect tetrahedron is equal to the space of cylinders of minimal height $(\pi/2)r$, where r is the radius of the cylinder.

8. Let (a, b) be an ellipse with circumference $C(a, b)$, where a (resp. b) is the distance between the (resp. co-) vertex of the ellipse. Define a topless cylinder as a cylinder with the part above a section removed, where the section does not intersect the edge of the cylinder and the sum of the highest and lowest intersection point with heights h resp. h′ obeys $\frac{1}{2}C(\sqrt{(\frac{(h-h')}{2})^2 + r^2}, r) \leq (h' + h)$.
   In the situation of proposition 7, there is a one to one correspondence between almost perfect tetrahedra and topless cylinders.

9. The worldwide abandonment of the legal form Ltd. and its international counterparts will lead to a more uniform distribution of wealth.

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