Hunting for new physics in the primordial Universe

Wang, D.-G.

Author: Wang, D.-G.
Title: Hunting for new physics in the primordial Universe
Issue date: 2020-08-27

1 Introduction

At the dawn of time,
who was there to transmit the 'Dao'?
When the shape had not been stabilized,
by which means they could be explored?

The darkness and light are in chaos;
who could dive into the mysteries?
For this unique existence from nothing,
how to identify its pheno?

Inquiries of Heaven
Qu Yuan
(c. 340–278 BC)

In the past one hundred years, our understanding of the Universe has been tremendously improved. Cosmology, shifting away from speculative tones of philosophy and theology, has become a solid scientific subject, which can be analysed quantitatively and tested via precise experiments. The current research reveals that our Universe originated from the Big Bang 13.8 billion years ago and keeps expanding from then on. Furthermore not only the evolution history of the Universe, but also the matter distribution within it, can be described by scientific methods. With the latest advances, it has also become possible for the human being to give a natural and simple explanation for the origin of the Universe.

In the history of modern cosmology, the developments of fundamental physics played an important role. At the beginning, General Relativity reformulated our view of spacetime and provided the mathematical framework for describing the expansion of the Universe, which initiated the modern advance of cosmology. Later on, the hot Big Bang theory was constructed with the help of nuclear physics, thermodynamics and statistical physics. More significantly, the recent developments in *Quantum Field Theory* (QFT) have renovated our understanding for the origin of the Universe. The most important progress in this direction is the proposal of *cosmic inflation* as a possible consequence of QFT at extremely high energy scales of the primordial Universe [1–6]. This theory, positing an exponentially expanding phase at the very beginning of the Universe, successfully explains the very fine-tuned initial conditions of the hot Big Bang cosmology. Moreover, during inflation vacuum fluctuations of quantum fields are expected to generate seeds for galaxy formation, which explains the origin of cosmic structures. Thus there has been great theoretical interest in the inflation scenario in the past several decades.

On the other hand, the last century has also witnessed a rapid revolution in astronomy, which greatly changed the situation of observational cosmology. Now we have more and more data coming from various cosmological observations, which lead us into the era of precision cosmology. In particular, measures of temperature fluctuations in the cosmic microwave background (CMB) have provided a clean window for looking into the primordial perturbations generated in the very early Universe. As being tested by the CMB data more and more precisely [7, 8], cosmic inflation has been established as the leading paradigm of primordial cosmology. Furthermore, the upcoming experiments of large scale structure (LSS) surveys are expected to reveal more information about the early Universe in the near future.

Despite of the phenomenological success of inflation, it has also been realised for many years that there are some theoretical challenges, as it is notoriously difficult to embed inflation in more fundamental theories. Meanwhile, the idea of *Effective Field Theory* (EFT) provides a constructive approach, where the underlying microscopic details become irrelevant and an effective description may demonstrate the interesting physics in a model-independent way. Because of the high energy scale during inflation, typically we expect there would be some new physics, which may leave imprints in the primordial perturbations and become testable in cosmological observations. This line of thinking leads us to take inflation as a natural laboratory for probing fundamental physics at extremely high energy scales.

This thesis is a contribution to the hunting for new physics in the primordial Universe. One goal here is to trace observable effects in theoretically consistent theories of inflation. In particular I will dive into inflation with curved field spaces, which are generally expected in high energy physics theories. On the other hand, there will be also phenomenological studies directly motivated by observations. Here the main focus goes to one particularly important observable – primordial non-Gaussianity, which is expected to be a powerful tool for testing new physics effects.

The outline of the introduction is organized as follows. In Section 1.1, I will briefly review how new discoveries in fundamental physics have reformulated our understanding of the very early Universe. In particular, I will first introduce inflationary cosmology, including its historical origin, current status, and possible issues. Next, I will introduce the developments of cosmological perturbation theory and connections with astronomical observations. Section 1.2 turns to test new fundamental physics through primordial cosmology and focuses on two major frameworks. Firstly there will be a bird's eye review of multi-field inflation with a focus on the effects of curved field spaces. Next, I will present the idea of EFT and its applications in inflationary cosmology. After that there will be discussions on the current status and future directions. In the end the structure of the thesis will be outlined in Section 1.3. In this introduction, we use natural units with $\hbar = c = 1$ and set the Planck mass as $M_{\rm pl} \equiv (8\pi G)^{-1/2}$.

1.1 From fundamental physics to primordial cosmology

The advance of modern cosmology started from the theory of General Relativity which was proposed by Albert Einstein in 1915 [9]. This gravity theory suggests we live in a curved spacetime, while for our Universe on the largest scales, the geometry can be described by a flat, homogeneous and isotropic Robertson-Walker metric

$$ds^2 = -dt^2 + a(t)^2 \left[dr^2 + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right)\right] \ , \tag{1.1}$$

where $a(t)$ is the scale factor reflecting the expansion or contraction of the space. In 1927, Lemaître proposed the recession behaviour of galaxies in an expanding Universe [10], which was confirmed by Hubble's observation two years later [11]. After that the expansion of the Universe is established, and the expansion rate is characterized by the Hubble parameter $H \equiv \dot{a}/a$, where the dot denotes derivative with respect to the cosmic time t. Then from the Einstein field equation, one can derive the following Friedmann equations [10, 12]

$$3H^2 M_{\rm pl}^2 = \rho, \tag{1.2}$$

$$M_{\rm pl}^2 \dot{H} = -\frac{1}{2}(\rho + p) \ . \tag{1.3}$$

This setup, also known as the Friedmann-Lemaître-Robertson-Walker (FLRW) model, provides a quantitative description for the background dynamics of the Universe.

One particularly important solution of the Friedmann equations is the *de Sitter spacetime*, which was proposed by Willem de Sitter in 1917 [13, 14]. There an unconventional matter content with negative pressure $p = -\rho$ drives the Universe to expand exponentially $a(t) \propto e^{Ht}$. As we will discuss later, a potential-dominated vacuum energy in QFT may lead to this type of accelerating expansion, which plays an important role in the modern studies of primordial cosmology.

With information of the density ρ and pressure p of the matter components, we can reconstruct the expansion history of our Universe. Later *the hot Big Bang theory* proposed that our observable Universe originated from a small patch with a hot and dense state. Since the energy scales are extremely high in the early Universe, elementary particles are unbounded and nuclear physics effects become dominant. After the first several minutes of the Big Bang, free neutrons and protons formed the nuclei of light elements, which is called the *Big Bang Nucleosynthesis* (BBN) [15]. As the Universe expands and cools, nuclei and electrons were combined to form neutral atoms around 380000 years later. As a result, photons can travel freely through space from then on, and some of them remain in today's Universe. This relic radiation forms the so-called called *cosmic microwave background* (CMB).

The time evolution of the Universe is shown in Fig. 1.1 with energy scales. Both BBN and CMB, as the landmark predictions of the hot Big Bang theory, have been confirmed by observational experiments. In particular, astronomers precisely measured the light element abundance of BBN and the temperature of black body radiation from the CMB, which provides strong supporting evidence of the Big Bang origin of the Universe.

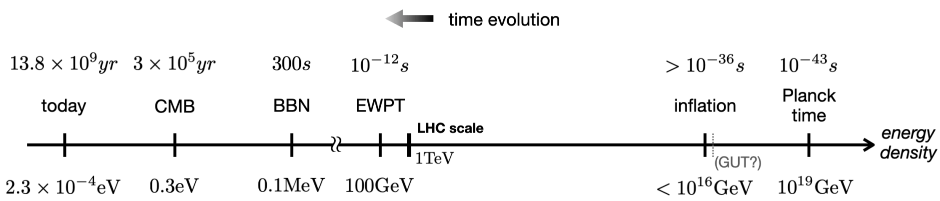

Figure 1.1: The evolution history of the Universe with energy scales. (EWPT, LHC and inflation will be discussed in the following sections.)

In spite of these successes, some unsolved puzzles remain in the hot Big Bang theory. The most famous one is the so-called *horizon problem*. In the Big Bang cosmology, the casually connected area of the Universe was much smaller than the physical size of today's observable Universe in the early times. This means that the homogeneous matter distribution we observe today were not correlated at the very beginning. Thus it is confusing why different casually disconnected areas shared the same properties during the Big Bang.

Another puzzle is the flatness problem, which questions why the spatial curvature of our Universe is negligible today. Since it is more natural to have a spatially curved Universe after the Big Bang expansion, the current observations of a zero spatial curvature may need fine tuning of the initial conditions. In addition, although the Universe is homogeneous and isotropic on the largest scales, there are also cosmic structures such as clusters, galaxies and stars. Their origin and distribution remain unexplained. In essence, these puzzles are all related the initial conditions of the Universe, which indicates that there may be an earlier phase before the hot Big Bang.

Meanwhile, if we go further back in time, and consider even earlier stages of the Universe, the energy density becomes much higher than the scale of particle colliders on earth, and the size of the observable Universe enters the unknown microscopic regime. According to our understanding of elementary particles in the subatomic world, quantum fields are expected to play an important role in this extreme environment. This consideration led to studies on cosmic phase transitions and later the development of inflationary cosmology, which will be elaborated on in the following sections.

1.1.1 Cosmic inflation

Inflation was first introduced to solve the puzzles of the Big Bang cosmology in early 1980s by Alan Guth [1]. It assumes that around the initial 10^{-36} seconds, the Universe undergoes a quasi-de Sitter expansion. This exponentially accelerating period expands the Universe at least e^{60} times larger, and thus dilutes away the unwanted relics and possible spatial curvature at the very beginning. Meanwhile, as a result of the rapid accelerating expansion, the observable Universe today all comes from one casually connected region at the beginning of inflation. Therefore by the end of inflation, a flat, homogeneous and isotropic initial condition was naturally given for the following Big Bang expansion.

Historically inflation was also proposed as one possible consequence of QFT in the very early Universe. Here we will first review the early developments of inflation theory from the perspective of spontaneous symmetry breaking and cosmic phase transitions, and then move to its standard scenario and remaining issues.

1.1.1.1 Cosmic phase transition and inflation

Spontaneous symmetry breaking (SSB) is one of the most profound concepts in modern QFT. It corresponds to the situation where the theory obeys a certain symmetry, but the system in the lowest-energy vacuum state does not respect the same symmetry. One simple example is the breaking of a $U(1)$ symmetry in the following theory of a complex scalar field

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi^*\partial^\mu\phi - \lambda(\phi^*\phi - v^2)^2 \tag{1.4}$$

where the potential has a Mexican hat form as shown in Fig. 1.2. As the minima of the potential are located at $|\phi| = v$, there are an infinite number of vacua in this theory. For each vacuum state, for instance $\phi = v$, it is no longer invariant under the $U(1)$ symmetry. Meanwhile it is convenient to parametrize the complex field as $\phi = \rho e^{i\theta}$. Then we find the radial field ρ is massive, and the angular direction θ turns out to be massless. This massless mode, or the angular direction in the circle of the minima, is called a Goldstone field, which is naturally associated with a shift symmetry $\theta \to \theta + const.$.

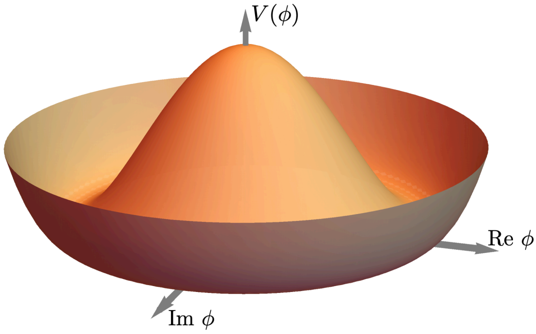

Figure 1.2: Mexican hat potential and SSB.

The concept of SSB played a central role in the construction of the Standard Model of particle physics, where a Higgs field is introduced to spontaneously break the gauge symmetry $SU(2) \times U(1)$ of the electroweak theory and unify the electromagnetic and weak interactions. Soon after, it was realized that the $SU(2) \times U(1)$ symmetry should be restored in the early Universe. As the energy is higher in the earlier stages, the finite-temperature effects in QFT will change the form of the Mexican hat potential, and the minimum is expected to become the point at the origin which respects the symmetry. Therefore, when the temperature goes down as the Universe expands, there should be a phase transition process in the early stage which evolves from the symmetric phase to the broken phase.

In the Standard Model of particle physics, the electroweak phase transition (EWPT) is not expected to dramatically change the course of the Big Bang expansion. However, there may be significant consequences if we consider another SSB process for a larger symmetry group which can unify the electroweak theory with the strong interaction with $SU(3)$ symmetry as well. This hypothetical theory, called Grand Unified Theory (GUT), although has not been verified in experiments, may lead to a drastic phase transition in the very early Universe.

The first proposal of cosmic inflation was realized in the first-order phase transition of a GUT theory. There at the beginning the scalar field, which is also called the inflaton, is supposed to be at the local minimum $\phi = 0$. This is a false vacuum as shown in the left panel of Fig. 1.3. As a consequence, the nonzero vacuum energy with $\rho_{\text{vac}} \simeq V$ and negative pressure $p_{\text{vac}} \simeq -V$ becomes dominant, which drives the de Sitter expansion of the Universe. Then as the Universe inflates, vacuum decay happens and the inflaton field will move to the true minimum at the bottom of the potential via quantum tunnelling, initiating the following hot Big Bang expansion.

This first scenario was later named as “old inflation”. However, this idea does not work, since a graceful exit from inflation is not provided here. As the vacuum decay leads to bubbles of the true vacuum in the Universe, the de Sitter expansion stops inside the bubbles, but still keeps going elsewhere. As a result, bubbles may never collide with each other to end inflation in the whole Universe.

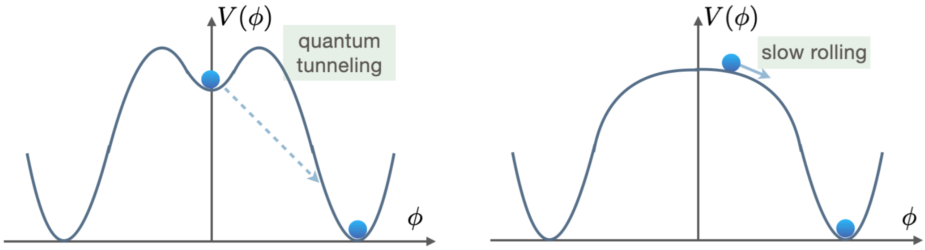

Figure 1.3: “Old inflation” (left) and “new inflation” (right) on cross-section profiles of the Mexican-hat-type potentials.

Soon after, the graceful exit problem was solved in the “new inflation” scenario proposed by Andrei Linde [2]. Instead of the first-order phase transition process with bubble nucleation in the old inflation, here a continuous GUT phase transition is considered. As shown in the right panel of Fig. 1.3, the scalar field rolls on a Coleman-Weinberg potential without barriers. Due to the effects of Hubble friction, the field velocity turns out to be quite slow, and the energy is dominated by the potential. As a result, a quasi-de Sitter expansion is provided and inflation gracefully ends in the whole Universe as the inflaton slowly rolls to the true minimum of the potential.

1.1.1.2 Slow-roll inflation

After the idea of “slow-roll” was proposed, physicists found it is not necessary to stick to GUT phase transitions, and many other models have shown similar behaviour but are less constrained than the new inflation. In general, they can be simply described by the following action with Einstein gravity and a canonically normailzed inflaton field

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}\mathbf{R} - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right] , \tag{1.5}$$

where the potential should satisfy the following slow-roll conditions

$$\epsilon_V \equiv \frac{M_{\rm pl}^2}{2}\left(\frac{\partial_\phi V}{V}\right)^2 \ll 1 \ , \qquad |\eta_V| \equiv M_{\rm pl}^2 \frac{|\partial_{\phi\phi} V|}{V} \ll 1 \ . \tag{1.6}$$

As a result, a quasi-de Sitter expansion is supposed to be driven by the vacuum energy of the scalar field. This requirement, saying that the inflaton potential should be sufficiently flat, can be equivalently parameterized by using the Hubble slow-roll parameters as

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \ll 1, \qquad \eta \equiv \frac{\dot{\epsilon}}{\epsilon H} \ll 1 \ . \tag{1.7}$$

It turns out that this slow-roll dynamics is an attractor in the phase space $(\phi, \dot{\phi})$, where non-slow-roll conditions will converge to it rapidly. Here the smallness of the first slow-roll parameter ϵ ensures that the kinetic energy of the inflaton is much smaller than the potential, such that accelerating expansion can happen, while $\eta \ll 1$ guarantees inflation will not end prematurely. This class of models are named as *single field slow-roll inflation*. Since current observations are in favor of these simplest models, they are regarded as the standard scenario of inflation.

In recent years, more accurate CMB data indicates a hierarchy between slow-roll parameters $\epsilon \ll \eta$. This has led to stronger constraints on slow-roll potentials, where inflating on a concave plateau is more favoured by observations. As a result, a subclass of slow-roll models with plateau-like potentials have been extensively investigated lately. Famous examples here include Starobinsky inflation [3], Higgs inflation [16] and α-attractors [17, 18].

1.1.2 Possible issues

From the phenomenological perspective, single field slow-roll scenario is very successful. Meanwhile its theoretical construction relies on the ultraviolet (UV) physics at higher energy scales. If we seriously look into these inflation models in a consistent UV theory, typically they turn out to be problematic [19]. Here I list several difficulties faced by the UV-completion of inflation and also some generic lessons that we can learn from these theoretical challenges.

1.1.2.1 η-problem

The most well-known challenge for single field slow-roll models is the so-called η-problem [20]. As the slow-roll parameter η is related to the mass of the inflaton field via $\eta_V \simeq m_\phi^2/H^2$, the second condition in (1.6) can also be seen as a requirement that the Hubble parameter during inflation should be much larger than the mass scale of the inflaton field. This is fine if we only treat the scalar field at the classical level. However, in a complete theory with quantum fields, the effects of radiative corrections should also be taken into account. If we do so, the inflaton mass receives contributions from loop diagrams which are typically around or larger than the Hubble scale $\Delta m_\phi \gtrsim H^2$. As a result of this large correction, the second slow-roll condition will be violated

$$\Delta\eta_V \simeq \frac{\Delta m_\phi^2}{H^2} \gtrsim 1, \tag{1.8}$$

thus the inflaton potential can no longer stay sufficiently flat to sustain long enough inflation.

Generally speaking, this is a problem for most of the slow-roll models. As the characteristic energy scale during inflation is given by the Hubble parameter, we expect there is always a hierarchy between H and m_ϕ in the system, which may generically spoil the flatness of the slow-roll potential. Therefore the η-problem can be seen as the hierarchy problem of inflation and is essentially the same as the one of the Higgs field in particle physics, where with a UV cutoff for new physics, the radiative correction to the Higgs mass is expected to be much larger than the measured value in Large Hadron Collider (LHC) [1].

In order to solve this problem and avoid a certain level of fine-tuning, usually it becomes necessary to impose some symmetries for the inflaton field. A concrete example is realized in natural inflation [21], where an approximate shift symmetry of a pseudo-Goldstone field is introduced to protect the flatness of the potential. We shall elaborate on this issue in Section 1.2.2.3 where its connections with SSB and implications for the effective theory will be discussed.

1.1.2.2 Swampland conjectures

If we care about the embedding of inflation within quantum gravity, there have been some speculative criteria called swampland conjectures which might tell whether a low-energy effective theory of inflation can emerge from

[1] According to the modern understanding of QFT, the hierarchy problem can be universal for scalar fields. In general spinor and gauge fields are protected by chiral symmetries and gauge symmetries respectively, thus will not be affected by quantum corrections, but this is not generically the case for scalar field theories.

UV-complete theories or not. Although rigorous proofs are still missing, these conjectures may give some hints about theoretical constructions of inflation models. Here are two famous examples:

- *Swampland distance conjecture.* To ensure the validity of the effective description, the field excursion distance during inflation is conjectured to be smaller than the Planck scale [22], *i.e.*

  $$\Delta\phi \lesssim c M_{\rm pl} , \tag{1.9}$$

  where c is a $\mathcal{O}(1)$ constant. This is also related to the well-studied Weak Gravity Conjecture [23] which has got indications from various perspectives recently. If it is generally true, many inflation models with super-Planckian field excursion ($\Delta\phi > M_{\rm pl}$) might be problematic.

- *Swampland de Sitter conjecture.* This one supposes scalar field potentials in consistent effective theories ought to satisfy the following condition [24]

  $$\left|\frac{\nabla V}{V}\right| \gtrsim \frac{1}{M_{\rm pl}}\mathcal{O}(1) . \tag{1.10}$$

  Basically it means the slope of the potentials should be quite steep. Compared with the distance conjecture, the de Sitter conjecture is in tension with all the single field inflation models, but it is also controversial. So far there is still no solid supporting evidence from quantum gravity theories.

1.1.2.3 Other challenges and alternatives to inflation

Besides the issues above, some other theoretical considerations may challenge inflationary cosmology as a whole. For instance, it has been shown that in inflationary spacetimes geodesics are incomplete towards the past direction, thus one expects a cosmic singularity [25]. On the other hand, the physical wavelength of some quantum fluctuations during inflation can be much smaller than the Planck length (trans-Planckian) at the beginning, thus one may worry if the analysis of perturbation is valid [26]. These issues, though not fatal, motivate us to also consider alternative paradigms for the primordial Universe.

Bouncing cosmologies provide a simple solution for the singularity problem and the trans-Planckian problem [27, 28]. In this class of paradigms the Universe was contracting at the beginning, and then transited to the Big

Bang expansion through a bouncing phase. A particular example is called *matter bounce cosmology* [29–31], where the contracting phase is dominated by the pressureless matter. In Chapter 6, I will study the distinguishable predictions from matter bounce models and also look into their difficulties.

1.1.3 Cosmological perturbation theory

Besides inflation, cosmological perturbation theory is another great achievement of primordial cosmology in the past several decades [32–34]. There a natural and elegant explanation is given for the origin of inhomogeneous structures in the Universe.

For the analysis of a perturbation mode k in a cosmic background, one particularly important length scale is the Hubble radius H^{-1}, which is also the curvature scale of the FLRW spacetime. Since for the de Sitter Universe, the Hubble radius is the size of the so-called event horizon as well, we also refer it as the horizon scale in this thesis. When the physical wavelength of this mode is larger than this scale, it is called "superhorizon" ($a/k \gg H^{-1}$); correspondingly "subhorizon" refers to the regime where perturbations have shorter wavelengths than this curvature scale ($a/k \ll H^{-1}$).

According to the cosmological perturbation theory, microscopic quantum fluctuations during inflation were stretched outside of the horizon by the rapid accelerating expansion (which is called horizon-exit), and then the primordial perturbations were generated. After inflation, these tiny inhomogeneities re-entered the horizon, which led to the anisotropies of temperature fluctuations in the CMB, and also provided the seeds for macroscopic objects like galaxies and clusters. We briefly introduce the basic formulation here with the standard results of single field slow-roll inflation.

1.1.3.1 Primordial perturbations

In the primordial Universe, there are two types of metric perturbations that are relevant to today's cosmological observations. The first one is the curvature perturbation $\mathcal{R}(t, \mathbf{x})$, which is the scalar component in the perturbed metric; while the second one are the tensor modes, also known as the primordial gravitational waves $h_{ij}(t, \mathbf{x})$.

During inflation, the curvature perturbation is generated by the quantum fluctuations of the inflaton field $\delta\phi$. At the beginning these fluctuations are in the subhorizon regime and do not feel the spacetime curvature, thus they are well described by the vacuum state as in the flat spacetime, which is called the Bunch-Davies initial condition. As inflation stretches these fluctuations exponentially, their physical wavelengths become superhorizon, and a quantum-to-classical transition happens. After this horizon-exit process, the inflaton fluctuations acquire a typical size of $\delta\phi = H/(2\pi)$. This sources the generation of primordial curvature perturbation on superhorizon scales with a frozen amplitude $\mathcal{R} = \frac{H}{\dot{\phi}}\delta\phi$.

More specifically, one may begin with perturbing the action (1.5) of slow-roll inflation, and then derive the quadratic action of scalar perturbations

$$S_2 = \int d^4 x a^3 \epsilon \left[\dot{\mathcal{R}}^2 - \frac{1}{a^2}(\partial_i \mathcal{R})^2\right] . \tag{1.11}$$

As we see, there is no mass term for $\mathcal{R}$, which indicates its conservation on superhorizon scales. It is more convenient to work with the Fourier mode in momentum space. After canonical quantization and solving the linear equation of motion, we derive the following approximate solution for the mode function in the de Sitter limit

$$\mathcal{R}_k = \frac{H}{\sqrt{4\epsilon k^3}}(1 + ik\tau)e^{-ik\tau}, \tag{1.12}$$

where τ is the conformal time defined by $d\tau \equiv dt/a(t)$. This provides a good description for both the Bunch-Davies vacuum state $\mathcal{R}_k \sim 1/\sqrt{k}$ inside the horizon $(-k\tau \gg 1)$ and the superhorizon evolution $\mathcal{R}_k \sim k^{-3/2}$ $(-k\tau \ll 1)$.

A similar story goes for the generation of primordial gravitational waves: during inflation tensor fluctuations are also stretched to macroscopic scales and freeze after horizon-exit. But here the tensor perturbations $h_{ij}(t, \mathbf{x})$ come from the vacuum of gravitons, instead of the inflaton fluctuations. We may also write down its quadratic action

$$S_2 = \frac{M_{\rm pl}^2}{4} \int d^4 x \; a^3 \left[\dot{h_{ij}}^2 - \frac{1}{a^2}(\partial_k h_{ij})^2\right] , \tag{1.13}$$

while the solution of the mode function follows as

$$h_{ij} = \frac{iH}{\sqrt{2k^3}}(1 + ik\tau)e^{-ik\tau} e_{ij} \tag{1.14}$$

where e_{ij} is the polarization tensor. As a result, a stochastic background of gravitational waves is expected after inflation.

1.1.3.2 Cosmological observables from the primordial Universe

The statistics of primordial perturbations is well captured by the correlation functions, which are also the major observational targets in today's cosmological experiments. Here we list some of the important observables for the primordial cosmology.

- *Primordial power spectrum.* The power spectra of perturbations, which are the Fourier transformation of two-point correlation functions, are the leading observables for the primordial cosmology. Again we use single field slow-roll inflation as an example, then with the solution (1.12) for the curvature perturbation, the prediction for its power spectrum at the end of inflation becomes

$$P_{\mathcal{R}}(k) \equiv \frac{k^3}{2\pi^2} |\mathcal{R}_k|^2 = \frac{H^2}{8\pi^2 M_{\rm pl}^2 \epsilon} , \tag{1.15}$$

where H and ϵ ought to be evaluated at the time of horizon-exit τ_* of the k-mode given by $k = aH$ (or $-k\tau_* = 1$ equivalently). This spectrum is nearly scale-invariant, but taking into account the evolution of H and ϵ during inflation, we find it has a slight red tilt with less power on smaller scales. Usually a spectral index is defined to describe this mild scale-dependence

$$n_s - 1 \equiv \frac{d \ln P_{\mathcal{R}}}{d \ln k} = -2\epsilon - \eta, \tag{1.16}$$

where the definitions of the Hubble slow-roll parameters (1.7) have been used to get the single field result. The prediction of a nearly scale-invariant power spectrum from single field slow-roll inflation is in agreement with the current CMB observations [7].

Similarly we can define the power spectrum of primordial gravitational waves, and the single field slow-roll prediction follows directly

$$P_t(k) \equiv \frac{k^3}{2\pi^2} \left|h^*_{ij}(k) h^{ij}(k)\right|^2 = \frac{2H^2}{\pi^2 M_{\rm pl}^2} . \tag{1.17}$$

This stochastic background of gravitational waves has not been detected yet, which is the major target for the on-going and future observational measurements on the B-mode polarization of the CMB and also other gravitational wave experiments. Usually the tensor-to-scalar ratio is defined to represent the amplitude of this spectrum

$$r \equiv \frac{P_t}{P_{\mathcal{R}}} , \tag{1.18}$$

while its latest constraint comes from the Planck satellite $r < 0.064$ [7]. Also this power spectrum has a slight red tilt, and we define the tensor spectral index as

$$n_t \equiv \frac{d\ln P_t}{d\ln k}, \tag{1.19}$$

which can be shown to equal -2ϵ in single field slow-roll models.

- *Primordial non-Gaussianity.* Power spectra can capture the properties of Gaussian statistics successfully, on the other hand, a wealth of interesting information may exist in the non-Gaussian statistics [35–38]. Thus after measuring the power spectrum, a lot of efforts have been carried out in the studies of primordial non-Gaussianities. Usually we look into higher order correlation functions for possible deviations from Gaussian distribution. One major observable here is the primordial bispctrum of curvature perturbations, which is the Fourier transform of the three-point correlation function

$$\langle \mathcal{R}_{\mathbf{k_1}}\mathcal{R}_{\mathbf{k_2}}\mathcal{R}_{\mathbf{k_3}}\rangle \equiv (2\pi)^3\delta^{(3)}(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})B_{\mathcal{R}}(k_1,k_2,k_3)\ . \tag{1.20}$$

The three momenta must add up to zero by translation invariance and therefore they form a triangle. As we can see here, the bispectrum can have many possible shapes as functions of three momenta and also overall sizes. Usually a shape function $\mathcal{S}(k_1,k_2,k_3)$ is introduced to represent the various templates, and for each shape there is a nonlinear parameter $f_{\rm NL}$ describing the size of the non-Gaussian signal

$$B_{\mathcal{R}}(k_1,k_2,k_3) = \frac{18}{5} f_{\rm NL}\mathcal{S}(k_1,k_2,k_3)P_{\mathcal{R}}^2. \tag{1.21}$$

The typical templates include local, equilateral and folded shapes, while the size of the local non-Gaussianity has got the tightest observational constraint from CMB observations $f_{\rm NL}^{\rm local} = -0.9 \pm 5.1$.

Besides the shape function, another informative channel is the triangle configurations of momenta, such as the squeezed limit with $k_1 \ll k_2 = k_3$. The behaviours of the bispectrum at different triangle limits are supposed to encode information about various physical effects during inflation. One famous example is *Maldacena's consistency relation of single field inflation*, where the squeezed bispectrum is associated with the scalar spectral index as follows [39, 40]

$$\lim_{k_1\ll k_2=k_3} B_{\mathcal{R}}(k_1,k_2,k_3) = \frac{(2\pi)^4}{4k_1^3k_3^3}P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_3)\frac{d\ln P_{\mathcal{R}}(k_3)}{d\ln k_3} \tag{1.22}$$

Since this result is caused by the gravitational interactions during inflation, it is expected to be the minimal amount of non-Gaussianity generated, which is known as the gravitational floor. This relation was supposed to be valid for all the single field inflation models with Bunch-Davies vacuum. However, later a counterexample was constructed in non-attractor inflation [41–43]. Chapter 5 shall carefully reexamine the violation of the consistency relation in this class of non-standard models.

Although it is difficult to detect in observations, the rich phenomenology of primordial non-Gaussianity makes it a very powerful tool to probe new physics in the primordial Universe. In this thesis, besides Chapter 5, Chapter 4 will study the imprints on the squeezed bispectrum from inflationary models with more complicated internal field spaces, while Chapter 6 will investigate the non-Gaussian signals from one alternative scenario to inflation.

- *Other future opportunities.* Current CMB observations show that primordial curvature perturbations on very large scales are nearly scale-invariant and Gaussian, which is consistent with the single-field slow-roll scenario. Meanwhile there are still many possibilities to deviate from these standard predictions. One is called primordial features, which correspond to deviations from the nearly scale-invariance of the power spectrum, such as oscillating wiggles. Another possibility is called anomalies, which correspond to possible deviations from the statistical isotropy of the primordial perturbations, such as hemispherical asymmetry and cold spot indicated by the latest CMB data. These observables, which will be further constrained by future CMB and LSS experiments, provide opportunities for testing inflation from different perspectives.

  Finally, one recent topic is primordial black holes (PBHs) which are hypothetical objects formed in the early Universe. Since they may originate from enhanced curvature perturbations with wavelengths much shorter than the cosmological scales today, one can also probe the small-scale power spectrum via the tighter and tighter observational constraints on PBHs [44]. In order to generate PBHs which are of observational interest, it is also important to investigate natural mechanisms of amplifying curvature perturbations during inflation. One attempt in this direction is the proposal of *sound speed resonance*, where the small scale perturbations are efficiently enhanced by parametric resonance effects during inflation [45, 46]. It remains an open question if this idea can be naturally realized in consistent theories.

1.2 From primordial cosmology to fundamental physics

High energy physics aims to explore the fundamental laws of the microscopic world, which contributes to the greatest ambition of theoretical physics – a *theory of everything*, bringing together quantum mechanics and general relativity. From the experimental perspective, the traditional approach to high energy physics is using colliders to search for new particles. However the energy scale that can be reached by LHC is around 1TeV which is far below the Planck scale where quantum gravity effects are expected to dominate. At the current stage, it also becomes more and more difficult to achieve higher energy through particle collider experiments.

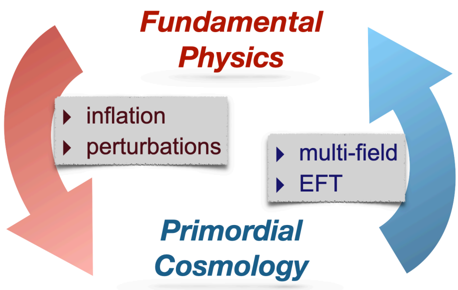

Figure 1.4: The interplay between fundamental physics and primordial cosmology.

Meanwhile, cosmology provides us a unique chance to peek into the new physics effects. In the last section, we have seen how fundamental physics tremendously changed the status of cosmology and helped with the development of inflationary cosmology. Notably inflation may also provide the highest energy scales in our Universe which can be probed via experiments. In this sense, cosmic inflation can be seen as a natural high energy laboratory for testing fundamental theories. In the following I shall elaborate on two major frameworks for this purpose.

1.2.1 Multi-field inflation in a nutshell

Although single field slow-roll models play the leading role in inflationary cosmology, we should also notice that in high energy theories typically there are multiple fields. This leads to studies on multi-field inflation. One particularly interesting question is whether other fields would lead to observable consequences. Furthermore, these additional fields and their interactions with the inflaton are typically associated with particles, field space and fundamental symmetries during inflation. In this sense, multi-field inflation provides powerful techniques for us to study the effects of these interesting physics.

Meanwhile, another motivation for studying multi-field inflation comes from the theoretical challenges of single field models in more fundamental theories. As we discussed in Section 1.1.2, these unsolved issues may put constraints on the UV completion of single field inflation. While the η-problem may be solved by assuming internal symmetries, it is still less clear how to avoid the issues from swampland conjectures. Here multi-field models may provide a natural solution [47, 48]. For instance, if the inflaton trajectory is turning in a multi-dimensional field space, its geodesic distance can remain sub-Planckian while the excursion range is larger than $M_{\rm pl}$, and inflating on a steep potential would also become possible because of the centrifugal force.

In the following I will approach multi-field inflation via the covariant formalism [49–53] and classify representative models into different regimes. After that there will be general discussions on a recent topic in this direction – inflation with a curved field manifold.

1.2.1.1 The covariant formalism

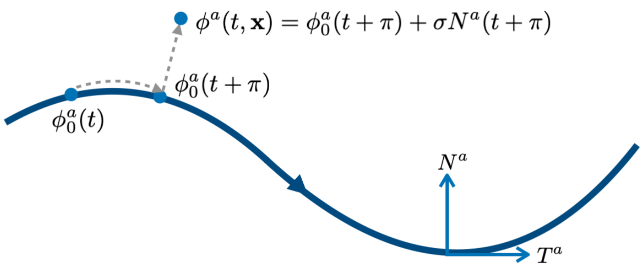

Figure 1.5: A generic inflaton trajectory with tangent and normal vectors in a two-dimensional field space, and the corresponding perturbations π and σ along and orthogonal to the trajectory respectively.

The starting point for many multi-field models is the following action with a set of scalar fields ϕ^a and Einstein gravity

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}\mathbf{R} - \frac{1}{2}G_{ab}(\phi)g^{\mu\nu}\partial_\mu\phi^a\partial_\nu\phi^b - V(\phi)\right] , \tag{1.23}$$

where $G_{ab}(\phi)$ is an internal field space metric. In a flat FLRW Universe, the background equations of this system can be written as

$$D_t\dot{\phi}_0^a + 3H\dot{\phi}_0^a + V^a = 0 , \qquad 3H^2 = \frac{1}{2}\dot{\phi}_0{}^2 + V , \tag{1.24}$$

where $D_t \equiv \dot{\phi}^a\nabla_a$ is the field space covariant derivative with respect to cosmic time and $V_a = \nabla_a V$ is the gradient of the potential. Notice that the latin field indices are manipulated with the internal space metric G_{ab}, e.g. $V^a = G^{ab}V_b$. The rolling motion of the inflaton forms a trajectory in the multi-dimensional field space. Here let us consider the two-field case as a simple example. As shown in Fig. 1.5, at each point of the trajectory we can define the tangent and normal unit vectors

$$T^a \equiv \frac{\dot{\phi}^a}{\dot{\phi}_0}, \qquad N_a \equiv \sqrt{\det G}\,\epsilon_{ab}T^b , \tag{1.25}$$

where $\dot{\phi}_0 \equiv \sqrt{G_{ab}\dot{\phi}_0^a\dot{\phi}_0^b}$ is the proper inflaton field velocity, and ϵ_{ab} is the Levi-Civita antisymmetric symbol with $\epsilon_{12} = 1$. Now we can define one particularly important parameter – the turning rate Ω of the trajectory as:

$$\Omega \equiv -N_a D_t T^a \tag{1.26}$$

Projecting the field equations of motion along the tangent and normal directions, we get respectively

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 + V_T = 0 , \tag{1.27}$$

$$V_N = \dot{\phi}_0\Omega , \tag{1.28}$$

where $V_T = T^aV_a$ and $V_N = N^aV_a$. The second equation shows the balancing between the centrifugal force and the gradient of the potential in the normal direction. As we see here, if $\Omega = 0$, which corresponds to the situation the trajectory is a geodesic in the field space, the field dynamics simply returns to the single field case. Thus for multi-field behaviour, one major difference from the single field one is a nonzero turning rate (or non-geodesic motion equivalently [52]).

Now let us turn to the analysis of perturbations. There are two types of scalar perturbations in multi-field inflation: the adiabatic perturbation along the trajectory and the isocurvature perturbation σ orthogonal to the trajectory. As shown in Fig. 1.5, the deviation along $\phi_0^a(t)$ can be parametrized by a fluctuation in time π, which corresponds to the curvature perturbation through $\mathcal{R} = H\pi$. At the linear order, we can decompose the field perturbations as

$$\delta\phi^a = \delta\phi_{\|} T^a + \sigma N^a \ , \tag{1.29}$$

where $\delta\phi_{\|} = \dot{\phi}_0\pi$. Then the quadratic action in terms of curvature perturbation $\mathcal{R}$ and σ can be derived as

$$S_2 = \int d^4x a^3 \left[\epsilon \left(\dot{\mathcal{R}} - \frac{2\Omega}{\sqrt{2\epsilon}}\sigma \right)^2 - \frac{\epsilon}{a^2}(\partial_i\mathcal{R})^2 + \frac{1}{2}\left(\dot{\sigma}^2 - \frac{1}{a^2}(\partial_i\sigma)^2 \right) - \frac{1}{2}\mu^2\sigma^2 \right] \ , \tag{1.30}$$

Here let us look into two important terms in this action. The first one is the derivative interaction term $\dot{\mathcal{R}}\sigma$. As we see, this term means that when the trajectory has a nonzero turning rate, the two perturbation modes are coupled to each other. As a result, there is conversion from isocurvature to curvature perturbations on superhorizon scales. This is the key feature of multi-field effects. On the other hand, generally the isocurvature modes have a mass μ^2. It can be expressed as

$$\mu^2 = V_{NN} + \epsilon\mathbb{R}H^2 + 3\Omega^2 \ , \tag{1.31}$$

where $V_{NN} = N^a N^b \nabla_a \nabla_b V$ and $\mathbb{R}$ is the Ricci scalar of the field space. Thus there are three different contributions: the Hessian of the potential in the normal direction, the field space curvature and the turning effect. This mass term, which provides a new scale during inflation, plays an important role in the studies of multi-field models. Based on the size of μ^2 relative to the Hubble scale, most of the models in the literature can be classified into three different regimes.

- $\mu \ll H$. This is the regime with light isocurvature fields. Typically this class of models can generate local type non-Gaussiantiy. Many previous works, such as the curvaton scenario [54, 55], focus on the situation where curvature and isocurvature perturbations are decoupled during inflation while the remaining isocurvature modes convert into adiabatic ones in post-inflation stages. Another well-studied situation is the slow-roll slow-turn models with $\Omega \ll H$ where $\mathcal{R}$ and σ are weakly coupled.

- $\mu \sim \mathcal{O}(H)$. This is the so-called quasi-single field regime with massive isocurvature fields [56–58]. Usually with a nonzero Ω, the primordial non-Gaussianity here has an intermediate shape between local and equilateral configurations, and the squeezed limit of the bispectrum also demonstrates a rich phenomenology related to the new mass scale. This regime has been extensively investigated in the framework of cosmological collider physics [59] in the past several years. We shall give more discussion about this direction in Section 1.2.3.
- $\mu \gg H$. This corresponds to the heavy field regime. In general, the heavy field here can be integrated out and then one may get a single-field effective theory with a reduced sound speed of the inflaton [52, 60–62]. The non-Gaussianity here has an equilateral shape as the result of the sound speed effect.

1.2.1.2 The recent revival of interest: curved field space

For inflation model building, most of the previous efforts were focused on the potential of the scalar fields, as it plays the central role of driving inflation. Meanwhile, it is worth noticing that besides the potential, there is another free function in the action (1.23) – the field space metric G_{ab}. If we consider inflation models from various UV theories, such as string theory, supergravity or nonlinear sigma model (NLSM)[2], typically they result in a metric function with curved geometry. One particularly interesting question is the role of this internal field space in inflation models.

This direction has drawn a lot of attention in the recent research of multi-field inflation. For instance, it is shown that for models with a negatively curved field manifold, a tachyonic instability caused by the field space curvature may deflect the background trajectory near the end of inflation, which is known as *geometrical destabilization* [63–67]. In another scenario called *hyperinflation* [68–72], as a consequence of the hyperbolic geometry, the multi-field evolution demonstrates a nontrival attractor behaviour on a steep potential. The first part of this thesis mainly focuses on two classes of models with curved field manifold. Here are some brief discussions.

The first one is the *inflationary α-attractors* [17, 18, 73–77]. Originated from the so-called Kahler potential of supergravity theories, the hyperbolic geometry plays a significant role in this class of models. There the inflaton potential is stretched to be of the plateau-like form by the effects of the

[2] See Section 1.2.2.3 for discussions about the realization of a curved field manifold in a NLSM.

curved field space, and as a result various single-field models yield universal predictions on the spectral index and tensor-to-scalar ratio which are favoured by the latest CMB observations. As the hyperbolic space has two-dimensions, α-attractors are multi-field models in principle. Chapter 2 will present the first two-field analysis and further demonstrate the role of the curved field space there.

The second class is called the *ultralight isocurvature scenario* [78–82]. These models explore one particular corner of multi-field inflation, where the additional field is massless and vigorously interacts with the inflaton. There the isocurvature modes freeze after horizon-exit, and source the growth of curvature perturbations on superhorizon scales. As a result, the final curvature perturbations at the end of inflation are mainly contributed by the isocurvature sourcing effects, and one still recovers a single-field like phenomenology. Chapter 3 will present a specific realization of this scenario – shift-symmetric orbital inflation.

After these models, one may wonder if there is any unified description of inflationary curved field space. Lately there have been studies of the multi-field background attractor behaviour within various models [83–86], but the perturbation analysis is still unclear. Just like in many multi-field models, predictions from these ones with curved field space usually are also model-dependent. And it remains a difficult question to figure out the observational signatures of the field space geometry. Chapter 4 provides an attempt toward this investigation, where the role of the internal manifold on the additional massive field is considered. For the further study, a more systematic and comprehensive approach to different curved field space scenarios may rely on the effective field theory of inflation.

1.2.2 Effective field theory approaches

In the theoretical studies of inflation, the top-down approach aims to build consistent models from UV-complete theories. However, it turns out to be quite difficult as we discussed in Section 1.1.2, and the model predictions may depend on specific constructions. Besides that, our current understanding of inflation has already taught us a lot about its consistent realizations in general. Thus one can also take the bottom-up approach, and the idea of effective field theory (EFT) provides a constructive framework, where a low energy theory may describe interesting physics in a model-independent way. Here I will introduce the concept of EFT and then discuss its applications in inflationary cosmology.

1.2.2.1 The philosophy of EFT

The physics of natural phenomena comes with many scales. The idea of EFT indicates that there are different theories describing the physics at distinct scales respectively, and for the physics of a certain scale, the EFT may not need information from other scales. In this sense, an EFT will be sufficient for describing a low-energy system after we integrate out (coarse-grain) the high energy physics (microscopic details). Moreover, operators in the EFT can be uniquely determined by fundamental symmetries at play. Therefore EFT provides a constructive approach of parametrizing our ignorance at UV scales and studying the low-energy physics with limited information.

In modern physics, there are many interesting and successful EFTs, with or without UV-completions, such as the Fermi theory of weak interactions, the chiral perturbation theory of pions and the low energy description of superconductivity. As an explicit and simple example, here let us look into *spontaneous symmetry breaking* (SSB) again and show how the low-energy physics in the broken phase can be described by an EFT of Goldstone fields.

Consider a set of scalar fields Σ whose action is invariant under a global symmetry G. The corresponding group transformation is $\Sigma \to e^{i\theta^A T^A}\Sigma$, where T^A are the generators of G and θ^A are parameters. If the vacuum of the theory is located at a nonzero value $\langle\Sigma\rangle = \bar{\Sigma}$ which is invariant under the transformation of a subgroup H, but changes under other remaining symmetry transformations, then this vacuum state spontaneously breaks the symmetry of G to the subgroup H. These remaining group elements of the broken symmetry (denoted by the generators t^a) form the so-called *coset* – G/H.

In general the symmetry breaking pattern is more complicated than the simplest $U(1)$ breaking example discussed at the beginning of Section 1.1.1.1. But the Mexican hat potential may also help us gain some intuition here. Suppose a UV toy model for this SSB is described by the following linear sigma model Lagrangian

$$\mathcal{L} = -\frac{1}{2}\partial\Sigma^\dagger\partial\Sigma - \lambda\left(\Sigma^\dagger\Sigma - f^2\right)^2 \ . \tag{1.32}$$

where f is the symmetry breaking scale[3] associated with the vacuum expectation value (vev) $\bar{\Sigma}$. In this case the coset space G/H corresponds to the vacua defined by $\Sigma^\dagger\Sigma = f^2$ in which the massless Goldstone fields

[3] In the case of a pseudo-Golstone field whose shift symmetry is explicitly broken, f is usually known as the axion decay constant.

live. There is also a "radial" direction which is supposed to be heavy and represent high energy physics in the system.

Now let us look at the low-energy state around the vev $\bar{\Sigma}$. Without losing generality, the scalar fields can be conveniently parameterized as

$$\Sigma = (\bar{\Sigma} + \sigma) U(\pi), \quad \text{with} \ \ U(\pi) = e^{i t^a \pi^a} \ . \tag{1.33}$$

where π^a are Goldstone fields living in the coset space and σ represents fluctuations along the heavy "radial" direction. Since at low energies the heavy physics related to σ becomes irrelevant, the unitary matrix-valued field $U(\pi)$ contains all the information of Goldstones. Since the mass terms are forbidden, with only derivative terms, the leading order Lagrangian of the Goldstones' EFT follows as

$$\mathcal{L}^0_{\text{eff}} = -\frac{f^2}{4} \text{Tr}[\partial_\mu U^\dagger \partial^\mu U] \ . \tag{1.34}$$

This is also well known as a *non-linear sigma model* (NLSM) which has a *curved* target manifold. We can further expand the Lagrangian in terms of π^a

$$\mathcal{L}^0_{\text{eff}} = -\frac{f^2}{2} \partial_\mu \pi^a \partial^\mu \pi^a + \frac{f^2}{6} \partial_\mu \pi^a \partial^\mu \pi^b \left(\pi^a \pi^b - \delta^{ab} \pi^c \pi^c \right) + \ldots \tag{1.35}$$

It is impressive to notice that we are only guided by the symmetry breaking pattern to derive the low-energy interacting Lagrangian (1.35) for Goldstones, while knowledge about the heavy radial modes is not used. In other words, it does not matter whether the UV physics is described by the toy model in (1.32) or other setup, while an EFT from symmetry argument provides good descriptions for Goldstone fields at low energy.

A concrete example of the above EFT formalism is the chiral perturbation theory, where pions turn out to be the pseudo-Goldstone fields of the SSB in quantum chromodynamics (QCD). Even if we do not know anything about QCD, the low-energy pions' interactions are well determined by the symmetry breaking pattern there. See Ref. [87] for more detailed discussion.

This approach of EFT is known as the Callan-Coleman-Wess-Zumino (CCWZ) coset construction [88, 89]. Through this example, we see low-energy physics can be mostly insensitive to the underlying microscopic details, while symmetries strongly constrain the form of the EFT and a model-independent description becomes possible.

1.2.2.2 EFTs in cosmology

When we apply EFT in the studies of inflationary cosmology, there can be different approaches. Here we mainly discusses two major ones.

- *EFT of background fields.* From the effective theory point of view, the action (1.5) of slow-roll models can be seen as the leading order approximation below some cutoff scales. In this sense higher order non-renormalizable operators, though suppressed, may also appear and show possible imprints of UV physics. The EFT of background fields provide a systematic way to study these perturbative corrections to the single-field slow-roll scenario [90].

  The key guideline for writing down these EFT operators is symmetry. As we discussed in Section 1.1.2.1, in order to avoid the η-problem, the inflaton field is expected to be protected by an approximate shift symmetry, thus the inflaton should appear with derivatives in the EFT operators. As a result, the leading extension in the single field scenario minimally coupled to gravity is to add a dimension-8 operator

  $$\frac{(\partial\phi)^2(\partial\phi)^2}{\Lambda_k^4}. \tag{1.36}$$

  This leads to the so-called *k-essence* theory where the kinetic term has a non-standard form[4] [91, 92]. In order to ensure the perturbativity of the EFT expansion, here this cutoff scale should satisfy the condition $\Lambda_k > \dot{\phi}$.

  If we consider operators with coupling to an additional field σ, again due to the symmetry argument, the leading order operator that we can write down is expected to be a dimension-5 one [93]

  $$\frac{(\partial\phi)^2\sigma}{\Lambda_s}. \tag{1.37}$$

  This operator, which is linear in σ, can also be seen as an equivalent description of non-geodesic trajectories in field space. Meanwhile the next-to-leading order correction is a dimension-6 operator, whose effects have been neglected in previous studies. Chapter 4 will give a more detailed discussion on this topic.

[4] In a general form the k-essence Lagrangian can be written as $P(X,\phi)$ with $X = (\partial\phi)^2$.

- *EFT of fluctuations.* Another approach to EFT of inflation is to put aside background theories and directly look into perturbations' behaviour. The starting point here is the observation that the background evolution in cosmology usually breaks the time translation symmetry, and the resulting Goldstone field is identified with the adiabatic perturbation. In this approach, one can write down all the possible operators of perturbations with the remaining spatial translation symmetry, thus it provides the most general description of single field inflation [94]. For instance, this EFT approach can also describe strongly coupled models, such as DBI inflation [95, 96], which are beyond the perturbative EFT expansion of the background field approach. However, in this framework one cannot trace the background information of inflation which may contain some important new physics, such as the internal symmetries of the inflaton field. Meanwhile a multi-field extension of this EFT has been investigated in Ref. [97]. But it remains an open question how to implement the recently discovered multi-field models with curved field manifold [98].

1.2.2.3 Inflation in coset space: a new type of EFT

As we discussed in Section 1.1.2.1, in order to avoid the η-problem, *symmetries are expected to play an important role for inflation.* In many fundamental constructions, the inflaton candidate is a pseudo-Goldstone boson protected by an internal symmetry. One direct consequence is that inflation is related to some spontaneous symmetry breaking process, and the inflaton evolves in a coset space defined by the symmetry breaking pattern[5].

Natural inflation provides the first and the simplest realization of this idea, in which the inflaton is a so-called axion associated with the breaking of a U(1) symmetry, and a soft explicit symmetry breaking generates a slow-roll potential for the axion field [21]. However, later it was realized that there a super-Planckian axion decay constant makes the effective description invalid [99]. Meanwhile in high energy theories, it is more natural to have more complicated symmetry breaking patterns, where the non-abelian coset spaces G/H are curved, and multiple (pseudo-)Goldstone fields will be involved besides the inflaton.

There are many interesting questions to be explored in this direction.

[5]This differs from the original proposals from phase transition discussed in Section 1.1.1.1. From the example of Mexican hat potential, inflation models around early 1980s correspond to the radial field, while inflation in coset space concerns the angular directions associated with the Goldstones.

For instance, the behaviour of the running inflaton in a general coset space G/H may have implications for inflation. This line of thinking was pioneered by the studies of *spontaneous symmetry probing* [100], where as one Goldstone field rolls in a non-abelian coset, the not-running Goldstones are found to become massive. This can be seen by considering a time-dependent configuration $\pi^1 = ct$ in the example of EFT with the broken symmetry in Section 1.2.2. After some algebra the effective Lagrangian (1.35) yields

$$\mathcal{L}^0_{\text{eff}} = -\frac{f^2}{2}\left[\partial_\mu \pi^1 \partial^\mu \pi^1 + \partial_\mu \pi^i \partial^\mu \pi^i + c^2 \pi^i \pi^i\right] + ... \tag{1.38}$$

where π^i are the Goldstones transverse to the running π^1 and they acquire masses. The implications of this effect on inflation will be explored in Chapter 4. In addition, with the CCWZ coset construction, a new type of EFT of inflation is expected from symmetry breaking patterns. Differing from the background and fluctuations EFTs, this one may be able to systematically track spontaneously broken internal symmetries during inflation. Inflation in coset space may also link a wide range of topics, such as curved field space, UV realizations and phenomenologies. This thesis will briefly touch some of the above topics, while more systematic investigation remains for future work [101].

1.2.3 Hunting for new physics in the primordial Universe

With multi-field inflation and EFT as powerful frameworks, now we can move forward to investigate new physics effects in the primordial Universe. To achieve this purpose, *model-independence* is also important. In the literature, there has been a large menu of inflation models with various phenomenologies. Generic conclusions will be impossible if predictions rely on some specific models. Therefore for testing new physics, one particular difficulty is to figure out the relation between theories and observable imprints, independent of models.

One interesting attempt in this direction is the *cosmological collider physics* program [59], which searches for observational signals of heavy particles during inflation in primordial non-Gaussianity. Not relying on a specific model, it is found that the squeezed limit of the primordial bispectrum contains information about these extra particles: there the oscillation pattern is uniquely determined by their masses; while the angular dependence of the bispectrum measures the spin. Although the signals usually are quite small, if detected, they would provide a clean channel and reveal a lot information about the possible new particles in the extreme environment of inflation. Lately this has been further extended to the proposal of cosmological bootstrap, which provides a systematic formalism for calculating inflationary correlation functions from symmetries [102].

Meanwhile it is interesting to notice one major difference between QFTs in particle physics and cosmology. The former usually has a static vacuum expectation value, while in cosmology field configurations are typically time-dependent. For instance, during inflation there is an excursion of the inflaton in the field space because of the slow-roll dynamics. This indicates that besides adopting the traditional strategy of collider physics, there may be novel approaches which are more suitable for searching for new physics in cosmology. For instance, with the excursion trajectory of the inflaton field, we may be able to probe properties related to the internal spaces, such as their geometries and underlying fundamental symmetries. There have been some pioneer works in this direction [103, 104], but systematic understanding is still unclear, which deserves future investigation.

1.3 The outline of the thesis

This thesis consists of two parts. Part I mainly focuses on multi-field inflation with curved field spaces, while in Part II I investigate the phenomenology of primordial non-Gaussianity in both inflation models and alternatives to inflation.

- **Part I. Excursion in curved field spaces.**

  Chapter 2 performs a study on multi-field α-attractors. α-attractors are a class of inflation models characterised by a hyperbolic field space, which have multiple fields involved in general. We present the first two-field analysis of this class of models and find surprisingly that due to the underlying hyperbolic geometry, the universal predictions of single field α-attractors are robust, even when multifield effects are significant. This work, together with geometric destabilisation [63], hyperinflation [68] and several others by other authors around the same time, initiated the revival of interest in multi-field models with curved field space. It is a collaborative project with Ana Achúcarro, Renata Kallosh, Andrei Linde and Yvette Welling [75].

  Chapter 3 proposes a new class of multi-field attractors called shift-symmetric orbital inflation, where the inflaton trajectory is turning significantly in field space, but the model predictions still mimic the single-field ones. In particular, we have demonstrated that, contrary to expectations in the literature, the primordial non-Gaussianity for these models is small and compatible with current bounds. This work was done with Ana Achúcarro, Ed Copeland, Oksana Iarygina, Gonzalo Palma and Yvette Welling [79].

Chapter 4 tackles a more general question: are there observational signatures related to the scale of field space curvature? Here I approach this question in the context of "quasi-single field inflation/ cosmological collider physics". Meanwhile the relation between the EFT of background fields and inflationary curved field spaces are discussed. Remarkably, I have found that the field space curvature can naturally lead to the running of the scaling index in the squeezed scalar bispectrum, and thus modify the collider signals in non-Gaussianity. This project was done by myself [105].

- **Part II. Tracing primordial triangles.**

  Chapter 5 revisits non-Gaussianities of non-attractor inflation. We show that previous calculations of the primordial bispectrum in non-attractor inflation were incomplete. Through careful analysis, we find that the transition process after the non-attractor phase, which had been previously neglected, always plays an important role. By examining the violation of Maldacena's consistency relation in this class of models, we worked out the first complete and detailed calculation of non-Gaussianity in the "non-attractor to slow-roll" transition. This was a joint project with Yi-Fu Cai, Xingang Chen, Mohammad Hossein Namjoo, Misao Sasaki and Ziwei Wang [106].

  Chapter 6 is about non-Gaussianities in alternatives to inflation, focusing on the distinctive features. In particular, we have calculated the primordial bispectrum in the matter bounce scenario with a general single scalar field. Here the non-Gaussian phenomenology of the matter bounce cosmology is extended to the cases with a small sound speed. Our results also lead to a "no-go" theorem which rules out many alternative models just using current observational constraints. This was a collaboration with Yi-Fu Cai, Yubin Li and Jerome Quintin [107].