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Self-adjusting surrogate-assisted optimization techniques for expensive constrained black box problems

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Citation

Bagheri, S. (2020, April 8). *Self-adjusting surrogate-assisted optimization techniques for expensive constrained black box problems*. Retrieved from https://hdl.handle.net/1887/87271

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Author: Bagheri, S.
Title: Self-adjusting surrogate-assisted optimization techniques for expensive constrained black box problems
Issue Date: 2020-04-08

Acknowledgements

I would like to genuinely thank TH Köln – University of Applied Sciences for giving me the opportunity to pursue a PhD program in parallel to my job by providing financial support as well as offering a great atmosphere for research. This work would not have been possible without the tireless assistance of my supervisors both from TH Köln and University of Leiden. Also, I thank Dr. Bas van Stein for helping me writing a Dutch summary of my dissertation, and my dear friends Atossa Setoodegan and her husband Dr. Rutger Fick who proofread it.

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