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## Self-adjusting surrogate-assisted optimization techniques for expensive constrained black box problems

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